

International Journal of Statistics and Applied Mathematics



ISSN: 2456-1452
 Maths 2016; 1(1): 37-42
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 www.mathsjournal.com
 Received: 06-03-2016
 Accepted: 07-05-2016

Dr. Mohammad Miyan
 Associate Professor, Department
 of Mathematics Shia P. G.
 College, University of Lucknow,
 Lucknow-226020

The distribution of pressure for fitted bearings of hydrodynamic lubrication under low and high rotation number

Dr. Mohammad Miyan

Abstract

The second order rotatory theory of hydrodynamic lubrication was founded on the expression obtained by retaining the terms containing first and second powers of rotation number in the extended generalized Reynolds equation. In the present paper, there is analysis about the pressure distribution for low rotation number and high rotation number. The comparisons of the pressures have been done with the help of geometrical figures, expressions, calculated tables and graphs for the fitted bearings in the second order rotatory theory of hydrodynamic lubrication. The analysis of equations for pressures, tables and graphs reveal that pressures increase exponentially with increasing values of rotation number. For high rotation number the pressure increases much rapidly than that of the low rotation number. The relevant tables and graphs confirm these important investigations in the present paper.

Keywords: Rotation number, Reynolds equation, Film thickness.

1. Introduction

In fully hydrodynamic or "full-film" lubrication ^[1, 3], the moving surface of the journal is completely separated from the bearing surface by a very thin film of lubricant, was given by *Cameron* ^[4] in 1981. The applied load causes the centerline of the journal to be displaced from the centerline of the bearing. This eccentricity creates a circular "wedge" in the clearance space.

The lubricant, by virtue of its viscosity, clings to the surface of the rotating journal, and is drawn into the wedge, creating a very high pressure, which acts to separate the journal from the bearing to support the applied load.

The bearing eccentricity is expressed as the centerline displacement divided by the radial clearance. The bearing eccentricity increases with applied load and decreases with greater journal speed and viscosity. The hydrodynamic pressure has no relationship at all to the engine oil pressure, except that if there is insufficient engine oil pressure to deliver the required copious volume of oil into the bearing, the hydrodynamic pressure mechanism will fail and the bearing and journal will be destroyed. The pressure distribution in the hydrodynamic region of a fluid film bearing increases from quite low in the large clearance zone to its maximum at the point of minimum film thickness for the incompressible fluid like oil is pulled into the converging "wedge" zone of the bearing, given by *Cameron* ^[4]. However, this radial profile does not exist homogeneously across the axial length of the bearing. If the bearing has sufficient width, the profile will have a nearly flat shape across the high-pressure region ^[5].

The second mode of bearing operation is boundary lubrication. In boundary lubrication, the "peaks" of the sliding surfaces i.e., journal and bearing, are touching each other, but there is also an extremely thin film of the lubricant only a few molecules thick which is located in the surface "valleys". That thin film tends to reduce the friction from what it would be if the surfaces were completely dry.

The mixed mode is a region of transition between boundary and full-film lubrication. The surface peaks on the journal and bearing surfaces partially penetrate the fluid film and some surface contact occurs, but the hydrodynamic pressure is starting to increase. When motion starts, the journal tries to climb on the wall of the bearing due to the metal-to-metal friction between the two surfaces.

Correspondence:
Dr. Mohammad Miyan
 Associate Professor, Department
 of Mathematics Shia P. G.
 College, University of Lucknow,
 Lucknow-226020

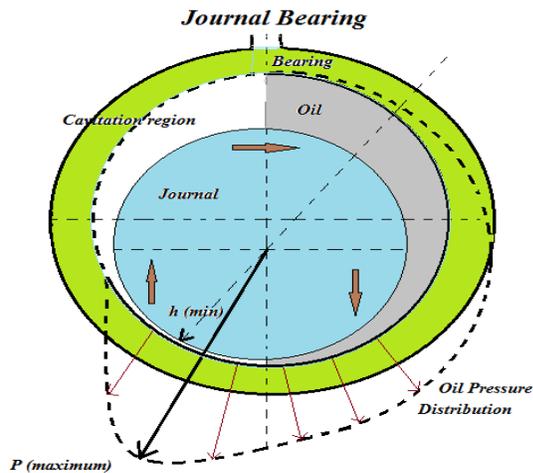


Fig 1: (Motion of fluid in Hydrodynamic journal bearing)

If there is an adequate supply of lubricant, the motion of the journal starts to drag the lubricant into the wedge area and hydrodynamic lubrication begins to occur along with the boundary lubrication. If we assume that the load and viscosity remain relatively constant during this startup period, then as revolution per minute increases, the hydrodynamic operation strengthens until it is fully developed and it moves the journal into its steady state orientation. The direction of the eccentricity and the minimum film thickness, do not occur in line with the load vector and are angularly displaced from the load.

According to Hori [6], there are also some other form of fluid-film lubrication, which includes the squeeze-film lubrication i.e., the piston engine etc. Squeeze-film action is based on the fact that a given amount of time is required to squeeze the lubricant out of a bearing axially, thereby adding to the hydrodynamic pressure, and therefore to the load capacity. Since there is little or no significant rotating action in the wrist-pin bores, squeeze-film hydrodynamic lubrication is the prevailing mechanism which separates wrist pins from their bores in the rods and pistons.

The figure-1 shows a hydrodynamic journal bearing and a journal, are rotating in the clockwise direction. The rotation of the journal causes pumping of the lubricant that flows around the bearing in the direction of rotation. If there is no force applied to the journal then its position remains unaltered and concentric to the bearing position. The loaded journal moves from the concentric position and forms converging gap between the journal surfaces and bearing. Now the movement of journal forced the lubricant to squeeze through the gap generating the pressure. Hori *et al.* [6] had said that the pressure falls to the cavitations pressure i.e., closer to the atmospheric pressure in the gap in which the cavitations forms. In general the two types of cavitations are form in the journal bearing.

(a) Gaseous cavitations: This is associated with air and gases mixed with lubricant. If the pressure of lubricant falls below the atmospheric pressure then the gases come out to form the cavitations.

(b) Vapors cavitations: This is formed when the load applied to the bearing fluctuates at the high frequency. The pressure of fluid falls rapidly and causes the cavitations due to fast evaporation.

Now the fluid pressure creates the supporting force which separates the journal from the surface of the bearing.

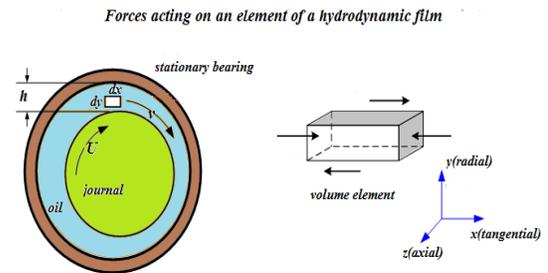


Fig 2: (Forces acting on a unit volume of an element of hydrodynamic lubrication film)

The hydrodynamic force of friction and force of fluid pressure counterbalance the external load. So the position of journal can be determined by these forces. In the hydrodynamic regime, the journal climbs in the rotational direction. If the working of journal is in the boundary and mixed lubrication then the hydrodynamic pressure ends and the journal climbs in the opposite to the rotational direction.

In the theory of hydrodynamic lubrication, two dimensional classical theories were first given by Osborne Reynolds. In 1886, in the wake of a classical Beauchamp Tower's experiment given by Reynolds, he formulated an important differential equation, which was known as: Reynolds Equation given by Reynolds in 1886. The formation and basic mechanism of fluid film was analyzed by that experiment on taking some important assumptions given as:

- The fluid film thickness is very small as compare to the axial and longitudinal dimensions of fluid film.
- If the lubricant layer is to transmit pressure between the shaft and the bearing, the layer must have varying thickness.

Later Osborne Reynolds himself derived an improved version of Reynolds Equation known as: "Generalized Reynolds Equation" [3, 5], which depends on density, viscosity, film thickness, surface and transverse velocities. The concept of rotation was discussed by Banerjee *et al.* [1, 2] in 1981 that the rotation of the fluid film which lies across the film gives some new results in lubrication problems of fluid mechanics. The origin of rotation can be traced by certain general theorems related to vorticity in the rotating fluid dynamics. The rotation induces a component of vorticity in the direction of rotation of fluid film and the effects arising from it are predominant, for large Taylor's Number, it results in the streamlines becoming confined to plane transverse to the direction of rotation of the film.

The new extended version of "Generalized Reynolds Equation" is said to be "Extended Generalized Reynolds Equation" given by Banerjee *et al.* [1, 2], which takes into account of the effects of the uniform rotation about an axis that lies across the fluid film and depends on the rotation number M , i.e. the square root of the conventional Taylor's Number. The generalization of the classical theory of hydrodynamic lubrication is known as the "Rotatory Theory of Hydrodynamic Lubrication" given by Banerjee *et al.* [1, 2]. The "First Order Rotatory Theory of Hydrodynamic Lubrication" and the "Second Order Rotatory Theory of Hydrodynamic Lubrication" was given by retaining the terms containing up to first and second powers of M respectively by neglecting higher powers of M , was given by Banerjee *et al.* [1-3].

The bearings having its diameter equal to the journal are known as fitted bearings or non clearance bearings. In these bearings the radial clearance is zero. The figure-3 shows the motion of fluid in fitted bearing.

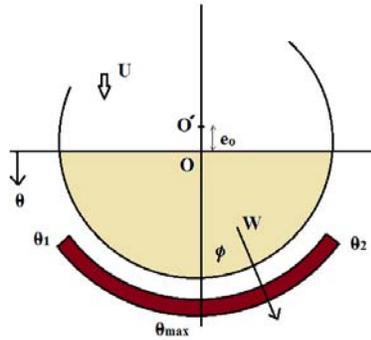


Fig 3: (Geometry of Fitted Bearing)

Where U is the sliding velocity, θ is the angular coordinate, ϕ is the permeability, R is the radius of bearing, e_0 is the eccentricity and F is the outward force of the bearing

2. Governing Equations

In the second order rotatory theory of hydrodynamic lubrication the “Extended Generalized Reynolds Equation” [5] is given as:

$$\frac{\partial}{\partial x} \left[\sqrt{\frac{2\mu}{M\rho}} \frac{1}{M} \left(\frac{\sinh h \sqrt{\frac{M\rho}{2\mu}} - \sin h \sqrt{\frac{M\rho}{2\mu}}}{\cosh h \sqrt{\frac{M\rho}{2\mu}} + \cos h \sqrt{\frac{M\rho}{2\mu}}} \right) \frac{\partial P}{\partial x} \right] + \frac{\partial}{\partial y} \left[\sqrt{\frac{2\mu}{M\rho}} \frac{1}{M} \left(\frac{\sinh h \sqrt{\frac{M\rho}{2\mu}} - \sin h \sqrt{\frac{M\rho}{2\mu}}}{\cosh h \sqrt{\frac{M\rho}{2\mu}} + \cos h \sqrt{\frac{M\rho}{2\mu}}} \right) \frac{\partial P}{\partial y} \right] + \frac{\partial}{\partial x} \left[-\frac{h}{M} + \sqrt{\frac{2\mu}{M\rho}} \frac{1}{M} \left(\frac{\sinh h \sqrt{\frac{M\rho}{2\mu}} + \sin h \sqrt{\frac{M\rho}{2\mu}}}{\cosh h \sqrt{\frac{M\rho}{2\mu}} + \cos h \sqrt{\frac{M\rho}{2\mu}}} \right) \frac{\partial P}{\partial y} \right] - \frac{\partial}{\partial y} \left[-\frac{h}{M} + \sqrt{\frac{2\mu}{M\rho}} \frac{1}{M} \left(\frac{\sinh h \sqrt{\frac{M\rho}{2\mu}} + \sin h \sqrt{\frac{M\rho}{2\mu}}}{\cosh h \sqrt{\frac{M\rho}{2\mu}} + \cos h \sqrt{\frac{M\rho}{2\mu}}} \right) \frac{\partial P}{\partial x} \right] - \frac{U}{2} \frac{\partial}{\partial x} \left[\rho \sqrt{\frac{2\mu}{M\rho}} \left(\frac{\sinh h \sqrt{\frac{M\rho}{2\mu}} + \sin h \sqrt{\frac{M\rho}{2\mu}}}{\cosh h \sqrt{\frac{M\rho}{2\mu}} + \cos h \sqrt{\frac{M\rho}{2\mu}}} \right) \right] - \frac{U}{2} \frac{\partial}{\partial y} \left[-\rho \sqrt{\frac{2\mu}{M\rho}} \left(\frac{\sinh h \sqrt{\frac{M\rho}{2\mu}} - \sin h \sqrt{\frac{M\rho}{2\mu}}}{\cosh h \sqrt{\frac{M\rho}{2\mu}} - \cos h \sqrt{\frac{M\rho}{2\mu}}} \right) \right] - \rho W^* \tag{1}$$

Where x, y and z are coordinates, P is the pressure, ρ is the fluid density, μ is the viscosity and W^* is fluid velocity in z -direction. The Extended Generalized Reynolds Equation in view of second order rotatory theory of hydrodynamic lubrication, in ascending powers of rotation number M and by retaining the terms containing up to second powers of M and neglecting higher powers of M , can be written as:

$$\frac{\partial}{\partial x} \left[-\frac{h^3}{12\mu} \left(1 - \frac{17M^2\rho^2h^4}{1680\mu^2} \right) \rho \frac{\partial P}{\partial x} \right] + \frac{\partial}{\partial y} \left[-\frac{h^3}{12\mu} \left(1 - \frac{17M^2\rho^2h^4}{1680\mu^2} \right) \rho \frac{\partial P}{\partial y} \right] + \frac{\partial}{\partial x} \left[-\frac{M\rho^2h^3}{120\mu^2} \left(1 - \frac{31M^2\rho^2h^4}{3024\mu^2} \right) \frac{\partial P}{\partial y} \right] - \frac{\partial}{\partial y} \left[-\frac{M\rho^2h^3}{120\mu^2} \left(1 - \frac{31M^2\rho^2h^4}{3024\mu^2} \right) \frac{\partial P}{\partial x} \right] - \frac{\partial}{\partial x} \left[\frac{\rho U}{2} \left\{ h - \frac{M^2\rho^2h^3}{120\mu^2} \left(1 - \frac{31M^2\rho^2h^4}{3024\mu^2} \right) \right\} \right] - \frac{\partial}{\partial y} \left[\frac{M\rho^2U}{2} \left\{ -\frac{h^3}{12\mu} \left(1 - \frac{17M^2\rho^2h^4}{1680\mu^2} \right) \right\} \right] - \rho W^* \tag{2}$$

For the case of pure sliding $W^* = 0$, so we have the equation as given:

$$\frac{\partial}{\partial x} \left[-\frac{h^3}{12\mu} \left(1 - \frac{17M^2\rho^2h^4}{1680\mu^2} \right) \rho \frac{\partial P}{\partial x} \right] + \frac{\partial}{\partial y} \left[-\frac{h^3}{12\mu} \left(1 - \frac{17M^2\rho^2h^4}{1680\mu^2} \right) \rho \frac{\partial P}{\partial y} \right] + \frac{\partial}{\partial x} \left[-\frac{M\rho^2h^3}{120\mu^2} \left(1 - \frac{31M^2\rho^2h^4}{3024\mu^2} \right) \frac{\partial P}{\partial y} \right] - \frac{\partial}{\partial y} \left[-\frac{M\rho^2h^3}{120\mu^2} \left(1 - \frac{31M^2\rho^2h^4}{3024\mu^2} \right) \frac{\partial P}{\partial x} \right] - \frac{\partial}{\partial x} \left[\frac{\rho U}{2} \left\{ h - \frac{M^2\rho^2h^3}{120\mu^2} \left(1 - \frac{31M^2\rho^2h^4}{3024\mu^2} \right) \right\} \right] - \frac{\partial}{\partial y} \left[\frac{M\rho^2U}{2} \left\{ -\frac{h^3}{12\mu} \left(1 - \frac{17M^2\rho^2h^4}{1680\mu^2} \right) \right\} \right] \tag{3}$$

3. Formulation of Problem

Let we assume the bearing to be infinitely long in x-direction, which implies that the variation of pressure in y-direction is very small as compared to the variation of pressure in x-direction i.e., $\frac{\partial P}{\partial x} \gg \frac{\partial P}{\partial y}$, then the equation (3), [7-14] will be

$$\frac{\partial}{\partial x} \left[-\frac{h^3}{12\mu} \left(1 - \frac{17M^2\rho^2h^4}{1680\mu^2} \right) \rho \frac{\partial P}{\partial x} \right] - \frac{\partial}{\partial y} \left[-\frac{M\rho^2h^3}{120\mu^2} \left(1 - \frac{31M^2\rho^2h^4}{3024\mu^2} \right) \frac{\partial P}{\partial x} \right] = -\frac{\partial}{\partial x} \left[\frac{\rho U}{2} \left\{ h - \frac{M^2\rho^2h^5}{120\mu^2} \left(1 - \frac{31M^2\rho^2h^4}{3024\mu^2} \right) \right\} \right] - \frac{\partial}{\partial y} \left[\frac{M\rho^2U}{2} \left\{ -\frac{h^3}{12\mu} \left(1 - \frac{17M^2\rho^2h^4}{1680\mu^2} \right) \right\} \right] \quad (4)$$

Taking h, U, P has given

$$h = h(x), U = -U, P = P(x) \quad (5)$$

The film thickness in angular coordinates is given as:

$$h = e_0 \cos\theta \quad (6)$$

Here e_0 is the eccentricity.

By rotating the angular coordinate 90° , in the direction of motion, we have

$$h = e_0 \sin\theta, x = R\theta \quad (7)$$

In view of above conditions, the equation (4) can be written as:

$$\frac{\partial}{\partial x} \left[-\frac{h^3}{12\mu} \left(1 - \frac{17M^2\rho^2h^4}{1680\mu^2} \right) \rho \frac{\partial P}{\partial x} \right] = -\frac{\partial}{\partial x} \left[\frac{\rho U}{2} \left\{ h - \frac{M^2\rho^2h^5}{120\mu^2} \left(1 - \frac{31M^2\rho^2h^4}{3024\mu^2} \right) \right\} \right] \quad (8)$$

4. Discussion and Results

For the determination of pressure distribution excluding the negative regions, the boundary conditions are as follows:

$$P = K \frac{dP}{d\theta} = 0, \text{ at } \theta = \theta_2, K = \text{constant} \quad (9)$$

Where θ_1 and θ_2 are connected by the condition that

$$P = 0 \text{ at } \theta = \theta_1. \quad (10)$$

On integrating and using the boundary conditions (9), (10) the equation of pressure after neglecting the higher powers of M and retaining the terms up to M^2 [13] is given as:

$$P = \frac{3\mu UR}{e_0^2} [F_1(\theta_1) - F_1(\theta)] + \frac{M^2\rho^2e_0^2RU}{280\mu} [F_2(\theta_1) - F_2(\theta)] \quad (11)$$

Where $F_1(\theta)$ and $F_2(\theta)$ are given by expressions:

$$F_1(\theta) = 2\cot\theta - \sin\theta_2 \left(\operatorname{cosec}\theta \cot\theta - \log\tan\frac{\theta}{2} \right) \quad (12)$$

$$F_2(\theta) = 17 \left(\frac{\sin 2\theta}{4} - \frac{\theta}{2} - \sin\theta_2 \cos\theta \right) - 7 \left\{ \sin^2\theta_2 \left(\log\tan\frac{\theta}{2} - \operatorname{cosec}\theta \cot\theta \right) - \theta + \frac{\sin 2\theta}{2} \right\} \quad (13)$$

5. Numerical Simulation

Let us take the values of mathematical terms in C.G.S. system as follows:

$$\rho = 0.9, \mu = 0.0002, e_0 = 0.3, R = 3.35, U = 500, \theta_1 = 20^\circ, \theta = 50^\circ, \theta_2 = 160^\circ.$$

The calculated values of pressures are given by the table-1 and table-2.

Table 1: (The variation of Pressure Distribution with respect to Low Rotation Number M)

S. No.	M	P
1.	0.1	80.85488142
2.	0.2	327.5560264
3.	0.3	738.7246014
4.	0.4	1314.360606
5.	0.5	2054.464041
6.	0.6	2959.034906
7.	0.7	4028.073201
8.	0.8	5261.578926
9.	0.9	6659.552081
10.	1.0	8221.992666

Table 2: (The variation of Pressure Distribution with respect to High Rotation Number M)

S. No.	M	P
1.	1	8221.992666
2.	2	32892.10717
3.	3	74008.96467
4.	4	131572.5652
5.	5	205582.9087
6.	6	296039.9952
7.	7	402943.8247
8.	8	526294.3972
9.	9	666091.7127
10.	10	822335.7712

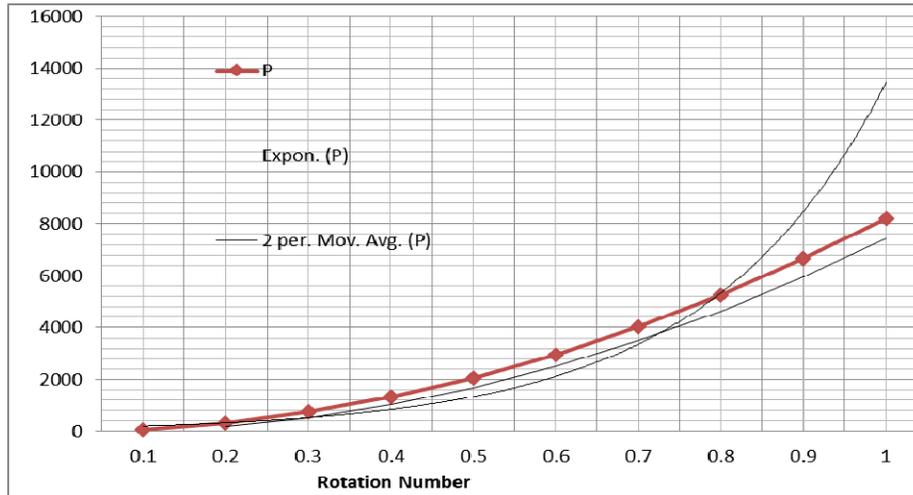


Fig 4: Variation of Pressure with respect to Low Rotation Number

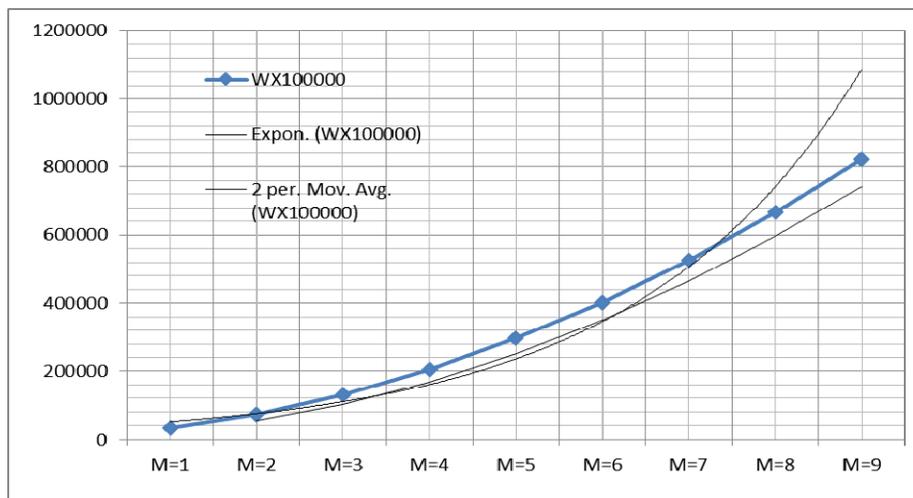


Fig 5: Variation of Pressure with respect to High Rotation Number

6. Conclusions

The derived equation of pressure is given by equation (11). The graphical representation for the variation of pressures is also shown by figure-4 to figure-5. The comparisons of the pressures have been done with the help of geometrical figures, expressions, calculated tables and graphs for the fitted bearings in the second order rotatory theory of hydrodynamic lubrication. The analysis of equations for pressures, tables and graphs reveal that pressures increase exponentially with increasing values of rotation number. For high rotation number the pressure increases much rapidly than that of the low rotation number. The relevant tables and graphs confirm these important investigations in the present paper.

7. References

1. Banerjee MB, Gupta RS, Dwivedi AP. The Effects of Rotation in Lubrication Problems, WEAR. 1981; 69:205-220.
2. Banerjee MB, Chandra P, Dube GS. Effects of Small Rotation in Short Journal Bearings, Nat. Acad. Sci. Letters. 1981; 4(9):112-125.
3. Banerjee MB, Dube GS, Banerjee K. The Effects of Rotation in Lubrication Problems: A New Fundamental Solutions, WEAR. 1982; 79:311-323.
4. Cameron A. Basic Lubrication Theory, Ellis Harwood Limited, Coll. House, Watergate, Chicester. 1981, 45-162.
5. Dube GS, Chatterjee A. Proc. Nat. Acad. Sci. India. 1988;

(58)I:79-85.

6. Hori, Y. *Hydrodynamic Lubrication*, Springer Science & Business Media. 2005, 23-31.
7. Miyan M. Pressure Variation in Porous Bearings under Second Order Rotation, American International Journal of Research in Science, Technology, Engineering & Mathematics. (AIJRSTEM). 2013; 13-310:25-29.
8. Miyan M. Inclined Slider Bearing Under the Effects of Second Order Rotatory Theory of Hydrodynamic Lubrication, International Journal of Science and Research (IJSR). 2013; 4(2):2089-2093.
9. Miyan M. Analysis of Short Journal Bearings under the Effect of Second Order Rotation, International Journal of Emerging Technologies in Computational and Applied Sciences. (IJETCAS). 2014; 14-823:294-298.
10. Miyan M. Behavior of Porous Bearings Under Second Order Rotatory Theory Of Hydrodynamic Lubrication, International Journal of Scientific Research and Engineering Studies. (JSRES). 2014; 1(6):14-18.
11. Miyan M. Infinitely Long Journal Bearing Under the Effect of Second Order Rotatory Theory of Hydrodynamic Lubrication, International Journal of Mechanical Engineering and Information Technology. 2015; 3(2):997-1003.
12. Miyan M. Analysis of Step Bearing of Hydrodynamic Lubrication, American International Journal of Research in Science, Technology, Engineering & Mathematics. (AIJRSTEM). 2015; 15-577:148-152.
13. Miyan M. Analysis of Fitted Bearings under Second Order Rotatory Theory of Hydrodynamic Lubrication, International Journal of Multidisciplinary and Current Research. (July/Aug issue). 2015; 3:765-771.
14. Miyan M. Different phases of Reynolds equation, International Journal of Applied Research. (IJAR). 2016; 2(1):140-148.