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Fuzzy pgprw-continuous maps and fuzzy pgprw-irresolute in fuzzy topological spaces

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Abstract
In this paper, we introduce the concept of fuzzy pgprw-continuous maps and fuzzy pgprw-irresolute maps in fts. We prove that the composition of two fuzzy pgprw-continuous maps need not be fuzzy pgprw-continuous and study some of their properties.

Keywords: Fuzzy pgprw-continuous maps, fuzzy pgprw-irresolute maps.

Introduction

The concept of a fuzzy subset was introduced and studied by L.A. Zadeh in the year 1965. The subsequent research activities in this area and related areas have found applications in many branches of science and engineering. In the year 1965, L.A.Zadeh [1]. introduced the concept of fuzzy subset as a generalization of that of an ordinary subset. The introduction of fuzzy subsets paved the way for rapid research work in many areas of mathematical science. In the year 1968, C.L.Chang [2] introduced the concept of fuzzy topological spaces as an application of fuzzy sets to topological spaces . Subsequently several researchers contributed to the development of the theory and applications of fuzzy topology. The theory of fuzzy topological spaces can be regarded as a generalization theory of topological spaces. An ordinary subset A or a set X can be characterized by a function called characteristic function

$\mu_A : X \longrightarrow [0,1]$ of A, defined by

$\mu_A(x) = 1$, if $x \in A$.

$= 0$, if $x \notin A$.

Thus an element $x \in X$ is in A if $\mu_A(x) = 1$ and is not in A if $\mu_A(x) = 0$. In general if X is a set and A is a subset of X then A has the following representation. $A = \{ (x, \mu_A(x)) : x \in X\}$, here $\mu_A(x)$ may be regarded as the degree of belongingness of x to A, which is either 0 or 1. Hence A is the class of objects with degree of belongingness either 0 or 1 only. Prof. L.A.Zadeh [1] introduced a class of objects with continuous grades of belongingness ranging between 0 and 1 ; he called such a class as fuzzy subset. A fuzzy subset A in X is characterized as a membership function

$\mu_A : X \longrightarrow [0,1]$, which associates with each point in x a real number $\mu_A(x)$ between 0 and 1 which represents the degree or grade membership of belongingness of x to A.

The purpose of this paper is to introduce a new class of fuzzy sets called fuzzy pgprw-closed sets in fuzzy topological spaces and investigate certain basic properties of these fuzzy sets. Among many other results it is observed that every fuzzy closed set is fuzzy pgprw-closed but not conversely. Also we introduce fuzzy pgprw-open sets in fuzzy topological spaces and study some of their properties.

1. Preliminaries

1.1Definition: :[1] A fuzzy subset A in a set X is a function $A : X \rightarrow [0, 1]$. A fuzzy subset in X is empty iff its membership function is identically 0 on X and is denoted by 0 or μ_ϕ. The set X can be considered as a fuzzy subset of X whose membership function is identically

1 on X and is denoted by μ_x or I_x. In fact every subset of X is a fuzzy subset of X but not conversely. Hence the concept of a fuzzy subset is a generalization of the concept of a subset.

1.2 Defnition: [1] If A and B are any two fuzzy subsets of a set X , then A is said to be included in B or A is contained in B iff A(x) ≤ B(x) for all x in X. Equivalently, A ≤ B iff A(x) ≤ B(x) for all x in X.

1.3 Defnition: [1] Two fuzzy subsets A and B are said to be equal if A(x) = B(x) for every x in X.
Equivalently A = B if A(x) = B(x) for every x in X.

1.4 Defnition: [1] The complement of a fuzzy subset A in a set X, denoted by A′ or 1 − A, is the fuzzy subset of X defined by A′(x) = 1 − A(x) for all x in X. Note that (A′)′ = A.

1.5 Defnition: [1] The union of two fuzzy subsets A and B in X, denoted by A ∨ B, is a fuzzy subset in X defined by (A ∨ B)(x) = Max{μ_A(x), μ_B(X)} for all x in X.

1.6 Defnition: [1] The intersection of two fuzzy subsets A and B in X, denoted by A ∧ B, is a fuzzy subset in X defined by (A ∧ B)(x) = Min{A(x), B(x)} for all x in X.

1.7 Defnition: [1] A fuzzy set on X is 'Crisp' if it takes only the values 0 and 1 on X.

1.8 Defnition: [2] Let X be a set and be a family of fuzzy subsets of (X, τ) is called a fuzzy topology on X iff τ satisfies the following conditions.
(i) μ_ϕ ; $\mu_X \in \tau$: That is 0 and 1 ∈ τ
(ii) If $G_i \in \tau$ for i ∈ I then $\vee G_i \in \tau$
i ∈ I
(iii) If G,H ∈ τ then G ∧ H ∈ τ
The pair (X, τ) is called a fuzzy topological space (abbreviated as fts). The members of τ are called fuzzy open sets and a fuzzy set A in X is said to be closed iff 1 − A is an fuzzy open set in X.

1.9 Remark: [2] Every topological space is a fuzzy topological space but not conversely.

1.10 Defnition: [2] Let X be a fts and A be a fuzzy subset in X. Then ∧ {B : B is a closed fuzzy set in Xand B ≥ A} is called the closure of A and is denoted by A or cl(A).

1.11 Defnition: [2]Let A and B be two fuzzy sets in a fuzzy topological space (X,τ)and let A ≥ B. Then B is called an interior fuzzy set of A if there exists G ∈τ such that A ≥ G ≥ B, the least upper bound of all interior fuzzy sets of A is called the interior of A and is denoted by A^0.

1.12 Definition: [3] A fuzzy set A in a fts X is said to be fuzzy semiopen if and only if there exists a fuzzy open set V in X such that V ≤ A ≤ cl(V).

1.13 Definition: [3] A fuzzy set A in a fts X is said to be fuzzy semi-closed if and only if there exists a fuzzy closed set V in X such that int(V) ≤ A ≤ V. It is seen that a fuzzy set A is fuzzy semiopen if and only if 1-A is a fuzzy semi-closed.

1.14 Theorem: [3] The following are equivalent:

(a) μ is a fuzzy semiclosed set,
(b) μ^C is a fuzzy semiopen set,
(c) int(cl(μ)) ≤ μ.
(b) int(cl(μ)) ≥ μ^c

1.15 Theorem: [3] Any union of fuzzy semiopen sets is a fuzzy semiopen set and (b) any intersection of fuzzy semi closed sets is a fuzzy semi closed.

1.16 Remark: [3]

(i) Every fuzzy open set is a fuzzy semiopen but not conversely.
(ii) Every fuzzy closed set is a fuzzy semi-closed set but not conversely.
(iii) The closure of a fuzzy open set is fuzzy semiopen set
(iv) The interior of a fuzzy closed set is fuzzy semi-closed set

1.17Definition: [3] A fuzzy set μ of a fts X is called a fuzzy regular open set of X if int(cl(μ).= μ.

1.18 Definition: [3] A fuzzy set μ of fts X is called a fuzzy regular closed set of X if cl (int(μ)) = μ.

1.19 Theorem: [3] A fuzzy set μ of a fts X is a fuzzy regular open if and only if μ^C fuzzy regular closed set.

1.20 Remark: [3]

(i) Every fuzzy regular open set is a fuzzy open set but not conversely.
(ii) Every fuzzy regular closed set is a fuzzy closed set but not conversely.

1.21 Theorem: [3]

(i) The closure of a fuzzy open set is a fuzzy regularclosed.
(ii) The interior of a fuzzy closed set is a fuzzy regular open set.

1.22 Definition: [4] A fuzzy set α in fts X is called fuzzy rwclosed if cl (α) $\leq$ μ whenever $\alpha \leq \mu$ and μ is regular semi-open in X.

1.23 Definition [5]: A fuzzy set α in fts X is called fuzzy pgprw closed if p-cl (α) $\leq$ μ whenever $\alpha \leq \mu$ and μ is rgα $-$open set in X.

1.24 Defintion [5]: A fuzzy set α of a fts X is fuzzy pgprw-open set, if it's complement α^c is a fuzzy pgprw-closed in fts X.

1.25 Definition [2]: Let X and Y be fts. A map f: X$\rightarrow$Y is said to be a fuzzy continuous mapping if $f^{-1}(\mu)$ is fuzzy open in X for each fuzzy open set μ in Y.

1.26 Definition [6]: Let X and Y be fts. A map f: X $\rightarrow$ Y is said to be a fuzzy almost continuous mapping if $f^{-1}(\mu)$ is fuzzy open in X for each fuzzy regular open set μ in Y.

1.27 Definition [6]: Let X and Y be fts. A map f: X $\rightarrow$ Y is said to be a fuzzy irresolute if $f^{-1}(\mu)$ is fuzzy semi-open set in X for each fuzzy semi-open set μ in Y.

1.28 Definition [2]: Let X and Y be fts. A map f: X $\rightarrow$ Y is said to be a fuzzy semi-continuous if
$f^{-1}(\mu)$ is fuzzy semi-open set in X for each fuzzy open set μ in Y.

1.29 Definition [4]: Let X and Y be fts. A map f: X $\rightarrow$ Y is said to be a fuzzy rw-continuous if
$f^{-1}(\mu)$ is fuzzy rw-open set in X for each fuzzy open set μ in Y.

1.30 Definition [7]: Let X and Y be fts. A map f: X $\rightarrow$ Y is said to be a fuzzy completely semi-continuous if $f^{-1}(\mu)$ is fuzzy regular semi-open set in X for each fuzzy open set μ in Y.

2. Fuzzy pgprw-continuous maps and fuzzy pgprw-irresolute maps in fuzzy topological spaces

Definition 2.1: Let X and Y be fts. A map f: X $\rightarrow$ Y is said to be fuzzy pgprw-continuous map if the inverse image of every fuzzy open set in Y is fuzzy pgprw-open in X.

Theorem 2.2: If a map f: (X, T) $\rightarrow$ (Y, T) is fuzzy continuous, then f is fuzzy pgprw-continuous map.

Proof: Let μ be a fuzzy open set in a fts Y. Since f is fuzzy continuous map $f^{-1}(\mu)$ is a fuzzy open set in fts x,as every open set is fuzzy pgprw-open ,we have $f^{-1}(\mu)$ is fuzzy a pgprw-open set in fts X. Therefore f is fuzzy continuous map.

The converse of the above theorem need not be true in general as seen from the following example

Example 2.3: Let X=Y= {a, b, c,d} and the functions $\alpha, \beta, \gamma, \delta$: X$\rightarrow$[0, 1] be defined as

α (x) = 1 if x = a
0 otherwise

β(x) = 1 if x = b
0 therwise
$\gamma(x)$ = 1 if x = a,b
0 otherwise
δ(x) = 1 if x = a,b,c
0 Otherwise.
Consider T= {1, 0,α,β,γ,δ},σ = {1,0,α,β,γ,δ}.Now (X,T) and (Y, σ) are the fts. Define a map
f:(X,T) $\longrightarrow$ (Y,T) by f(a)=c,f(b)=a,f(c)=b,f(d)=d, then f is fuzzy pgprw-continuous map but not fuzzy continuous map as the inverse image of the fuzzy set γ in (Y, σ) is μ:X $\longrightarrow$ [01}defined as
μ (x) = 1 if x = b,c
0 otherwise.
This is not a fuzzy open set in (X, T).

Theorem 2.4: A map f: (X,T) $\longrightarrow$ (Y, σ) is fuzzy pgprw-continuous map iff the inverse image of every fuzzy closed set in a fts Y is a fuzzy pgprw closed set in fts X.

Proof: Let δ be a fuzzy closed set in a fts Y then δ^c is fuzzy open in fts Y. Since f is fuzzy pgprw-continuous map,$f^{-1}(\delta^c)$ is pgprw-open in fts X but $f^{-1}(\delta^c)$ = 1- $f^{-1}(\delta^c)$ and so $f^{-1}(\mu)$ is a fuzzy pgprw-closed set in fts X.

Conversely, Assume that the inverse image of every fuzzy closed set in Y is fuzzy pgprw closed in fts X. Let μ be a fuzzy open set in fts Y then μ^c is fuzzy closed in Y;
by hypothesis $f^{-1}(\mu^c)$= 1- $f^{-1}(\mu)$ is fuzzy pgprw closed in X and so $f^{-1}(\mu)$ is a fuzzy pgprw-open set in fts X. Thus f is fuzzy pgprw-continuous map.

Theorem 2.5: If a function f: (X, T) $\longrightarrow$ (Y, σ) is fuzzy almost continuous map, then it is fuzzy pgprw-continuous map.

Proof: Let a function f: (X, T) $\longrightarrow$ (Y, σ) be a fuzzy almost continuous map and μ be fuzzy open set in fts Y. Then $f^{-1}(\mu)$ is a fuzzy regular open set in fts X. Now, $f^{-1}(\mu)$ is fuzzy pgprw-open in X, as every fuzzy regular open set is fuzzy pgprw-open .Therefore f is fuzzy pgprw-continuous map.

The converse of the above theorem need not be true in general as seen from the following example.

Example 2.6: consider the fts (X, T) and (Y, σ) as defined in example 2.3. define a map
f: (X,T) $\longrightarrow$ (Y, σ) by f(a)= c,f(b)=a,f(c)=b,f(d)=d. Then f is fuzzy pgprw-continuous map but it is not almost continuous map.

Example 2.7: fuzzy semi-continuous maps and fuzzy pgprw-continuous maps are independent as seen from the following examples.

Example 2.8: Let X= Y={a, b, c,d} and the functions α, β,γ,δ: X $\longrightarrow$ [0, 1] be defined as

α (x) = 1 if x = a
0 otherwise

β(x) = 1 if x = b
0 therwise

$\gamma(x)$ = 1 if x = a,b
0 otherwise

δ(x) = 1 if x = a, b, c
0 otherwise.

Consider T={1,0,α,β,γ,δ}, σ = {1,0,α} .Let map f: X $\longrightarrow$ Y defined by f(a)=b,f(b)=b,f(c)=a,f(d)=c, then f is fuzzy pgprw-continuous map but it is not fuzzy semi continuous map as the inverse image of fuzzy set α^c in (Y, σ) is

μ (x) = 1 if x = a, b, d
0 otherwise.

This is not a fuzzy semi closed set in fts X.

Example 2.9:
Let X= Y= {a, b, c} and the functionsα, β, : X $\rightarrow$ [0, 1] be defined as

α (x) = 1 if x = a
0 otherwise

β(x) = 1 if x = b, c
0 therwise

Consider T= {1,0,α,β} , σ = {1,0,α} .Let map f: X $\rightarrow$ Y defined by f(a)=b,f(b)=a,f(c)=c.
then f is fuzzy semi-continuous map but f is not fuzzy pgprw continuous map as the inverse image of fuzzy set α^c in (Y, σ) is

μ (x) = 1 if x = a,c
0 otherwise.

This is not fuzzy pgprw-closed set in fts X.

Remark 3.0: fuzzy rw-continuous maps and fuzzy pgprw-continuous maps are independent as seen from the following examples.

Example 3.1: Let X= {a, b, c,d},Y={a,b,c} and the functions α, β, γ, δ: X $\rightarrow$ [0, 1] be defined as

α (x) = 1 if x = a
0 otherwise

β(x) = 1 if x = b
0 therwise

γ(x) = 1 if x = a, b
0 Otherwise.
δ(x) = 1 if x = a, b, c
0 otherwise

Consider T= {1,0,α,β, γ, δ},σ= {1,0,α} .Let map f:X $\rightarrow$ Y defined by f(a)=b, f(b)=a, f(c)=a, f(d)=c, then f is fuzzy pgprw-continuous map but it is not fuzzy rw continuous map as the inverse image of fuzzy set α^c in (Y, σ) is

μ (x) = 1 if x = a ,d
0 otherwise.

This is not fuzzy rw-closed set in fts X.

Example 3.2: Let X= Y={a, b, c} and the functions α, β: X $\rightarrow$ [0, 1] be defined as

α (x) = 1 if x = a
0 otherwise

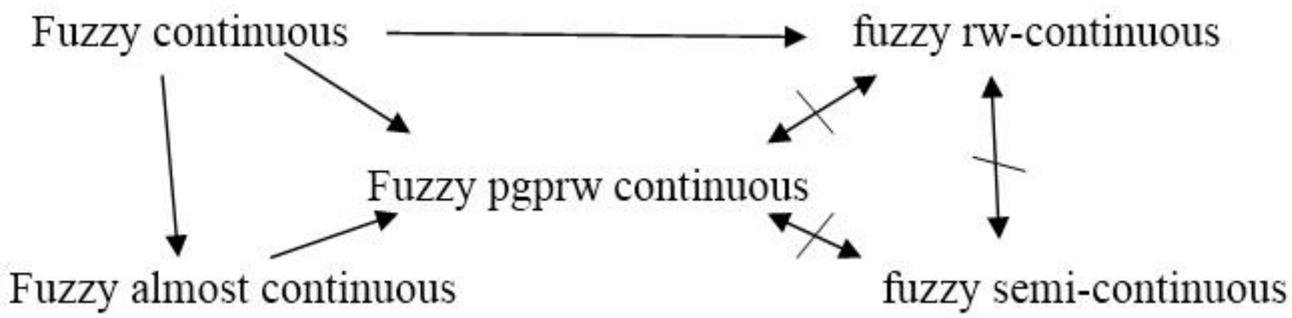
β(x) = 1 if x = b, c
0 therwise

Consider T= {1,0,α,β} , σ = {1,0,α} .Let map f: X $\rightarrow$ Y defined by f(a)=b, f(b)=a, f(c)=c.
Then f is fuzzy rw-continuous map but f is not fuzzy pgprw continuous map as the inverse image of fuzzy set α^c in (Y, σ) is

μ (x) = 1 if x = a,c
0 otherwise.

This is not fuzzy pgprw-closed set in fts X.

Remark 3.3: From the above discussion and known results we have the following implication



Theorem 3.4: If a function f : (X,T) → (Y, σ) is fuzzy pgprw-continuous map and fuzzy completely continuous map then it is fuzzy continuous map.

Proof: Let a function f : (X,T) → (Y, σ) be a fuzzy pgprw-continuous map and fuzzy completely semi-continuous map. Let μ be a fuzzy closed set in fts Y. Then $f^{-1}(\mu)$ is both fuzzy closed set in fts Y.Then $f^{-1}(\mu)$ is both fuzzy rgα-open and fuzzy pgprw-closed set in fts X .If a fuzzy set α of fts X is both fuzzy rgα and fuzzy pgprw-closed then α is a fuzzy closed in fts X thus $f^{-1}(\mu)$ is a fuzzy closed set in fts X .Therefore f is fuzzy continuous map.

Theorem 3.5: If f: (X,T) → (Y, σ) is fuzzy pgprw-continuous map and g: (Y, σ) → (Z,ρ) is fuzzy continuous map, then their composition gof:(X, T) → (Z,ρ) is fuzzy pgprw-continuous map.

Proof: Let μ be a fuzzy open set in fts Z. Since g is fuzzy continuous map, $g^{-1}(\mu)$ is fuzzy open set in fts Y. Since f is fuzzy pgprw-continuous map,$f^{-1}(g^{-1}(\mu))$ is a fuzzy pgprw-open set in fts X.
But $(gof)^{-1}(\mu)= f^{-1}(g^{-1}(\mu))$ thus gof is fuzzy pgprw-continuous map.

Definition 3.6: Let X and Y be fts. A map f: X → Y is said to be a fuzzy pgprw-irresoulute map if the inverse image of every fuzzy pgprw-open in Y is a fuzzy pgprw-open set in X.

Theorem 3.7: If a map f: X → Y is fuzzy pgprw- irresolute map then it is fuzzy pgprw-continuous map.

Proof: Let β be a fuzzy open set in Y. since every fuzzy open Set is fuzzy pgprw-open, β is a
Fuzzy pgprw-open set in Y. Since f is fuzzy pgprw- irresolute map,$f^{-1}(\beta)$ is fuzzy pgprw-open in x. Thus f is fuzzy pgprw-continuous map.

The converse of the above theorem need not be true in general as seen from the following example.

Example 3.8: Let X={a, b, c,d} , Y={a,b,c} the functions $\alpha, \beta, \gamma, \delta$: X → [0, 1] be defined as

α (x) = 1 if x = a
0 otherwise

β(x) = 1 if x = b
0 therwise

$\gamma(x)$ = 1 if x = a, b
0 otherwise
δ(x) = 1 if x = a, b, c
0 otherwise

Consider T= {1,0,α,β,γ, δ},σ = {1,0,μ}.Let map f: X → Y defined by f(a)=b, f(b)=a, f(c)=a,f(d)=c, then f is fuzzy pgprw-continuous map but it is not fuzzy pgprw-irresolute map.Since for the fuzzy pgprw-closed set μ: Y → [0,1] defined by
μ (x) = 1 if x = b
0 otherwise in Y
$f^{-1}(\mu) = \alpha$ is not fuzzy pgprw-closed in (X, T).

Theorem 3.9: Let X, Y, Z be fts. If f: X → Y is fuzzy pgprw-irresolute map and g: Y → Z is fuzzy pgprw-continuous map then their composition gof: X → Z is fuzzy pgprw-continuos map.

Proof: Let α be any fuzzy open set in fts Z, Since g is fuzzy pgprw-continuous map, ($g^{-1}(\alpha)$) is a fuzzy pgprw-irresolute map $f^{-1}((g^{-1}(\alpha))$ is a fuzzy pgprw-open set in fts X
But $(gof)^{-1}(\alpha) = f^{-1}(g^{-1}(\alpha))$.Thus gof is fuzzy pgprw-continuous map.

Theorem 3.10: Let X, Y, Z be fts and f: X→Y and g: Y → Z be fuzzy pgprw-irresolute maps then their composition maps then their composition gof: X → Z is fuzzy pgprw-irresoulte map.

Proof: Let α be a fuzzy pgprw-open set in fts Z. since g is fuzzy pgprw-irresolute map, $g^{-1}(\alpha)$
is a fuzzy pgprw-open set in fts Y. since f is fuzzy pgprw-irresolute map, $f^{-1}(g^{-1}(\alpha))$ is a fuzzy
Pgprw-open set in fts X But $(gof)^{-1}(\alpha) = f^{-1}(g^{-1}(\alpha))$. Thus gof is fuzzy pgprw-irresolute map.

Conclusion : In this paper, a new class of maps called Fuzzy Pgprw-Continuous maps and Fuzzy pgprw-irresolute in fuzzy topological spaces are introduced and investigated.In future the same process will be analyzed for Fuzzy pgprw-properties.

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