A scientific model of finite element technique to study spiral heat regulation in human limbs

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Abstract
We consider this paper as the one-dimensional bio-heat exchange condition with the perfusion of blood, reliant on temperature which oversees physiological heat dispersion issues relating to limbs and the organic properties are supposed to be distinctive along the radial direction. By methods for an extensive technique with metaphorical estimation, we acquire the correct analytical answer for inspect the heat circulation in the tissues. The dermal region is comprised of three layers, to be specific epidermis, dermis, and subcutaneous tissues. The model consolidates critical varieties of physical and physiological parameters like blood mass stream rate, rate of metabolic heat generation, and thermal conductivity in each layer. Numerical outcomes have been acquired for different instances of functional interest.

Keywords: Rate of metabolism, blood mass stream rate, warm conductivity, warm era, limited component method, Pennes Bio - Heat Model

Introduction
The heat transmit methodology through human dermal locales is an extremely unpredictable phenomena because of various reasons. At lower environmental temperatures, the one dimensional models for heat distribution in limbs have wide degree as the center temperature of the human appendages fluctuates broadly at lower air temperatures. For falling climatic temperature, the body center therapists quickly and isotherm shells in the appendage change their separate position progressively. This might be because of the way that the blood vessel blood has chilled off while streaming towards the furthest points. Likewise the two inverse side of internal center of a human appendage might be at various temperature. This might be on the grounds that one side of the appendage contains real veins (corridors) with blood originating from fundamental trunk at body center temperature and the external side of the appendage contains veins with blood coming back from the furthest points at lower temperature.

Perl [12] took collectively the forms of differential equations and Fick’s perfusion principle with heat conduction and matter diffusion equations and metabolic term to get equation. He used equation to solve its simple cases by taking all parameters as constant throughout the region. Perl and Hirsch [7] used this equation to test the transient response for measuring local tissue blood flow on dog and rabbit kidney. Trezek and Cooper [1] computed thermal conductivity of tissue by taking all parameters as constant. Cooper and Trezek [2] obtain solution of equation in SST region by taking all parameters as constant. Patterson [6] made experimental attempts, to determine temperature profiles in skin and subcutaneous region. This paper employs a variational finite element approach to study the temperature distribution in a normal cross-sectional region of a limb. Due to unsymmetric situations of large blood vessels passing through the core of the limb the inter-face has angular variation. The peripheral part of limb is directly exposed to atmosphere. Different types of variations of parameters have been considered for different natural subregions such as stratum corneum, stratum germinativum, dermis and underlying tissue (Montagana, Jarrett and Gray) [5, 4, 3]. Finite element formulation provides necessary flexibility in taking care of different behavior of distinctly different subregions.
2. Material and Methods

The heat flow in SST region is given by the following partial differential equation

\[
div \left( K \, \text{grad} \, T \right) + m_s \, C_s \, (T_s - T) + S = \rho \, c \, \frac{\partial T}{\partial t}
\]

(2.1)

Where T and Tb are Temperature and Body center temperature, S is Rate of metabolic warmth generation, m_s Blood mass stream rate in the tissue, is thickness of the tissue. Ct=Specific warmth of blood, K is Warm conductivity of the tissue. Above condition has been adjusted and extensively used by Saxena [11], Saxena and Arya [8], Saxena and Bindra [9, 10] in the warm investigation of human skin and subcutaneous tissue.

In this, skin and fundamental tissues of round and hollow locales, for example, appendages of a human are partitioned into four annular layers. They can be considered as multi layered districts. Each layer has diverse physical and physiological properties. The external body surface is layered districts. Each layer has diverse physical and physiological properties. The properties and temperature conveyance surface happens because of conduction, convection, radiation and evaporation. Here we utilize for a human appendage with the nodal values is given by ai (i = 0, 1, 2, 3, 4) called as nodal values. The distance between the nodal values is given by ai (i = 0, 1, 2, 3, 4) from the outermost layer of epidermis to innermost layer of subdermallayer. LetT^0(i) (i = 1, 2, 3, 4) denote the linear values of T (r) for a_i < r < a_(i+1)

Now, applying the shape function to approximate the solution of the problem.

\[
T^i = A_i + B_i 
\]

where,

\[
A_i = \frac{T_i - T_{i-1}}{a_i - a_{i-1}}, \quad B_i = \frac{T_{i+1} - T_i}{a_i - a_{i+1}} \quad \text{for} \quad i = 2, 3, 4
\]

Thus the equation in one dimensional unsteady state case for each layer to the following cylindrical form

\[
\frac{1}{r} \frac{d}{dr} \left( K_r \frac{dT}{dr} \right) + M \left( T_s - T \right) + S = \rho \, c \, \frac{\partial T}{\partial t}
\]

After camparing with Euler’s Lagrange’s equation, we get the variational forms

\[
I = \int \left( \frac{1}{2} \left[ K_r \frac{dT}{dr} \right]^2 + m_c \, C_s \left( T' - T \right)^2 + \rho \, c \, \frac{\partial T}{\partial t} \right) \, dr
\]

We have,

\[
I = \int \left( T^0, T^1, T^2, T^3, T^4 \right)
\]

Now minimizing I with respect to parameters T^0, T^1, T^2, T^3, T^4 ,therefore

\[
\frac{\partial I}{\partial T^0} + \frac{\partial I}{\partial T^1} + \frac{\partial I}{\partial T^2} + \frac{\partial I}{\partial T^3} + \frac{\partial I}{\partial T^4} = 0
\]

Finally taking La-place transform, we get four non homogenous simultaneous equations as

\[
x^1 T^0 + y^1 T^1 = n^1
\]

\[
x^2 T^0 + y^2 T^1 + z^2 T^2 = n^2
\]

\[
y^3 T^1 + z^3 T^2 + w^3 T^3 = n^3
\]

\[
z^4 T^2 + w^4 T^4 = n^4
\]

solving for \( \overline{T}_0, \overline{T}_1, \overline{T}_2, \overline{T}_3, \overline{T}_4 \) by using matrix method, we get the values of \( \overline{T}_i \) in the form of polynomials

\[
\begin{bmatrix}
\bar{x}_1 & \bar{y}_1 & 0 & 0 & \eta_1 \\
\bar{x}_2 & \bar{y}_2 & \bar{z}_2 & 0 & \eta_2 \\
0 & \bar{y}_3 & \bar{z}_3 & \omega_3 & \eta_3 \\
0 & 0 & \bar{z}_4 & \omega_4 & \eta_4
\end{bmatrix}
\]

\[
\overline{T}_i = \frac{X_i(p)}{Y_i(p)}
\]
Where \( x^j \ (p) \) is a polynomial of degree less than \( n \) that of \( Y^j \ (p) \), i.e., \((n-1)\). The value of nodal temperature \( T_0, T_1, T_2, T_3 \), can be obtained by taking the inverse La-place transform

\[
T = e^{-\frac{p}{t}} \sum_{i=0}^{N} \frac{X_i(P)}{Y_i(P)} e^{\frac{p}{t}}
\]

Therefore, we can see that nodal values are dependent on time.

4. Tables
The numerical result have been obtained with the help of following values –

<table>
<thead>
<tr>
<th>Thermal Conductivity (cal/cm min °C)</th>
<th>Heat Transfer Coefficient ( h ) (cal/cm(^2) min °C)</th>
<th>Specific Heat of Tissues ( c ) (cal/gm °C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_1=0.060, K_2=0.045, K_3=0.030 )</td>
<td>0.009</td>
<td>0.830</td>
</tr>
<tr>
<td>Blood Density of Tissues ( \rho ) (gm/cm(^3))</td>
<td>Latent Heat ( L ) (cal/gm)</td>
<td>Body Core Temperature ( T_b ) (°C)</td>
</tr>
<tr>
<td>1.090</td>
<td>579.0</td>
<td>37</td>
</tr>
</tbody>
</table>

The numerical result have been computed for three case of atmospheric temperatures \( T = 15 °C, 23 °C \) and \( 33 °C \). The following sets of numerical values have been taken and graphical representation obtained for temperature distribution in SST region. The three different set of values for thickness of layers in SST region are taken for \( a_0, a_1, a_2, a_3 \) and \( a_4 \).

<table>
<thead>
<tr>
<th>Thickness of Skin</th>
<th>( a_0 )</th>
<th>( a_1 )</th>
<th>( a_2 )</th>
<th>( a_3 )</th>
<th>( a_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set I</td>
<td>8</td>
<td>7</td>
<td>7</td>
<td>6</td>
<td>5.5</td>
</tr>
<tr>
<td>Set II</td>
<td>7.5</td>
<td>7</td>
<td>6</td>
<td>5</td>
<td>4.5</td>
</tr>
<tr>
<td>Set III</td>
<td>7</td>
<td>6</td>
<td>6.5</td>
<td>5</td>
<td>5.5</td>
</tr>
</tbody>
</table>

5. Conclusion
Distinctive \( T(i) \) \((i = 0, 1, 2, 3)\) versus time \( t \) for various thickness of skin and diverse estimations of climatic temperatures have been ascertained. On looking at the nodal temperature on external surface and in each sub district, it is watched that these nodal temperatures differ extensively with the change in environmental temperatures and rate of sweat dissipation. Clearly sharpness of slope is more set apart in the epidermal sub-layer which is specifically presented to condition. In addition blood compliment in the dermal and sub dermal sub layers.

6. Reference
1. Cooper TE, Trezek GJ. A Probe technique for determining the thermal conductivity of tissues, J Heat transfe. 1972a, 133-140. ASME 94.
2. Cooper JE, Trezek GT. on the freezing of tissues, J Heat transfer.1972b, 251-253. ASME, 94.