Hall effect on MHD flow of visco-elastic fluid layer heated for below saturating a porous medium

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Abstract
In this paper the problem of visco-elastic fluid layer heated from below in the presence of uniform vertical magnetic field with Hall current in porous medium is discussed here and obtained a dispersion relation. Using normal mode analysis, from this dispersion relation, we observed that the medium permeability $k_1$ has destabilizing effect, the magnetic field has stabilizing effect and this stabilizing effect is independent of Hall current, Hall parameter has destabilizing effect and the sufficient condition for the non-existence of over-stability are given as $\beta_e < \frac{1}{6\pi^2 P_m}, P_m < \sqrt{2}$, and $2P_m < EPr < \frac{1}{\sqrt{2}}$.

Keywords: magnetic field, hall current, porous medium, visco-elastic, MHD flow

Introduction
In technological areas there are some important class of fluid, called non-Newtonian fluid, are also being studied extensively because of their practical applications, such as fluid film lubrication, analysis of Polymers in chemical engineering etc. the micropolar fluid is famous case for non-Newtonian fluid as El-Bory (2005). Also another example for non-Newtonian fluid is viscoelastic fluid. A detailed theoretical investigation has recently begun for the viscoelastic prototype designated liquid B. Walters (1964) \cite{1} and Beard and Walters (1964) \cite{20}. Sen (1978) \cite{13} studied the behavior of unsteady free convection flow of a viscoelastic fluid past an infinite porous plate with constant section. Singh and Singh (1983) \cite{18} have studied the magneto-hydrodynamic flow of viscoelastic fluid past an accelerated plate. The flow of viscoelastic and electrically conducting fluid past an infinite plate has been studied by Sherief and Ezaat (1994) \cite{16}.

The viscoelastic fluid subclass of microstructure flows, exhibits a great deal of influence on the normal and shear stresses in flow films. Some interesting flow characteristics of the following film, just to name a few, include:
1. The current state of stresses is a function of past history.
2. Various phenomena including elastic recoiling, creeping and stress relaxation can occur.
3. The relation between the stress and velocity field is highly, non-linear, even in situation where the history of the strain is highly repetitive.

The stability problem of a falling film of viscoelastic fluid has been studied Gupta (1967) \cite{7} studied the stability of a small amplitude falling fluid of second order. Shagfle \textit{et al.} (1989) demonstrated that the viscoelastic property has destabilizing effect on the film flow for small Reynolds number. However, the viscoelastic property possesses a primarily stabilizing effect on the film flow for moderate Reynolds number. Andersson and Dahi (1999) \cite{11} studied the gravity-driven flow of a viscoelastic liquid film along a vertical wall.

Saleh \textit{et al} (2010) \cite{12} examined heat and mass transfer in MHD viscoelastic fluid flow through a porous medium over a stretching sheet with chemical reaction. Sonth \textit{et al.} (2012) \cite{19} studied heat and mass transfer in a viscoelastic fluid over an accelerated surface with heat source/ sink and viscous dissipation. E. Omokhuaele \textit{et al.} (2012) \cite{10} studied the effects of concentration and Hall current on unsteady flow of a viscoelastic fluid in a fixed plate.

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In view of the fact that the study of viscoelastic fluid in a porous medium may find applications in geophysics and chemical technology. However, in this paper I have made an attempt to examine the effect of Hall current on MHD flow of visco-elastic (Rivlin-Ericksen) be fluid layer heated from below on porous medium and to the best of my knowledge this problem is uninvestigated so far.

2. Mathematical Formulation

Consider an infinite, horizontal, incompressible electrically non-conducting visco-elastic fluid layer of thickness \( d \). A Cartesian coordinate system \((x, y, z)\) is chosen such that origin it at the lower boundary and the \( z \)-axis is vertically upward. This fluid layer is assumed to be flowing through on isotropic and homogeneous porous medium of porosity \( \epsilon \) and medium permeability \( k \). The lower boundary at \( z = 0 \) and the upper boundary at \( z = d \) are maintained at constant but different temperature \( T_0 \) and \( T_1 \) such that a steady adverse temperature gradient \( \beta = \frac{dT}{dz} \) is maintained. The whole system is acted upon by a gravity field \( \mathbf{g} = (0, 0, -g) \) and a strong uniform magnetic field \( \mathbf{H} = (0, 0, H_0) \) is applied along \( z \)-axis.

Here, we have taken Rivlin-Ericksen visco-elastic fluid in which when the fluid parameters a porous medium, the gross effect is a represented by Darcy’s low and the usual viscous term in the momentum equation is replaced by the resistance term

\[
\frac{1}{k} \left[ \mu + \mu' \frac{\partial}{\partial t} \right] \mathbf{q}
\]

Also both boundaries are considered to be free and perfect conductor of heat. For an isotropic medium the surface porosity is \( \epsilon \) so that \( 1 - \epsilon \) is the fraction that is occupied by solid.

Within Boussinesq approximation, the equation governing the motion of micropolar fluid saturating. Porous medium following G. Lebon (1981)\(^9\), Lukaszewicz (1999), Kirti Prakash et al (1999) for above model are as follows:

The equation of continuity for an incompressible fluid is

\[
\nabla \cdot \mathbf{q} = 0
\]  \(\ldots(1)\)

The equation of momentum is

\[
\frac{\rho_0}{\epsilon} \left[ \frac{\partial}{\partial t} \mathbf{q} + \frac{1}{\epsilon} (\mathbf{q} \cdot \nabla) \mathbf{q} \right] = -\nabla p + \mu \nabla^2 \mathbf{q} - \frac{1}{k} \left( \mu + \mu' \frac{\partial}{\partial t} \right) \mathbf{q} - \rho \mathbf{g} \hat{e}_z + \frac{H_0}{4\pi} \left( \nabla \times \mathbf{H} \right) \times \mathbf{H}
\]  \(\ldots(2)\)

Where, \( P, \rho, \rho_0, \mathbf{q}, \mu, \mu', k, H_0 \) and \( \hat{e}_z \) denote respectively, pressure, fluid density, reference density, filter velocity, viscosity, visco-elasticity, medium permeability, magnetic permeability and unit vector in \( z \)-direction.

The equation of energy is
\[
\left[ \rho_o C_v + \rho_s C_s (1 - \varepsilon) \right] \frac{\partial T}{\partial t} + \rho_o C_v (\dot{q}, \nabla) = \chi_T \nabla^2 T
\]  
... (3)

And the equation of state is
\[
\rho = \rho_o [1 - \alpha (T - T_0)]
\]  
... (4)

Where \( C_v, C_s, \chi_T, \rho_s, \alpha, T \) and \( T_0 \) denote respectively specific heat at constant volume, heat capacity of solid (Porous Material Matrix), thermal conductivity, density of solid matrix, coefficient of thermal expansion, temperature and reference temperature.

The Maxwell’s equation in the presence of Hall current yield
\[
\frac{\varepsilon}{\partial t} \mathbf{H} = \nabla \times (\dot{q} \times \mathbf{H}) + \varepsilon \gamma_m \nabla^2 \mathbf{H} - \frac{\varepsilon}{4\pi \varepsilon_0 n_e} \nabla \times [\nabla \times \mathbf{H} \times \mathbf{H}]
\]  
... (5)

and \( \nabla \cdot \mathbf{H} = 0 \)  
... (6)

Where \( \mathbf{H} = (0, 0, H_0) \), \( H_0 \) is a constant, \( n_e = \) electron density and \( e = \) charge on electron and \( \gamma_m \) is the magnetic viscosity.

3. Basic State of the Problem

The basic state is given by
\[
\mathbf{q} = \mathbf{q}_b (0, 0, 0), \quad \rho = \rho_b (z) \quad \text{and} \quad P = P_b (z)
\]

Under this basic state equation (1) to (6) become
\[
\frac{d\rho_b}{dz} + \rho_b \varepsilon = 0
\]
... (7)

\[
T = -\beta z = T_0
\]
... (8)

and \( \rho_b = \rho_o (1 + \alpha \beta z) \)  
... (9)

4. Perturbation Equations

\[
\mathbf{q} = \mathbf{q}_b + \mathbf{q}', \quad P = \rho_b + P', \quad \rho = \rho_b + \rho', \quad \mathbf{H} = \mathbf{H}_b + \mathbf{h}, \quad T = T_b + \theta
\]  
... (10)

Where \( \mathbf{q}', P', \rho', \mathbf{h} \) and \( \theta \) are the perturbations in \( \mathbf{q}, P, \rho, \mathbf{H} \) and \( T \) respectively.

Using (7) to (10), equations (1) to (6) becomes
\[
\nabla \mathbf{q}' = 0
\]  
... (11)

\[
\rho_o \left[ \frac{\partial \mathbf{q}'}{\partial t} + \frac{1}{\varepsilon} (\mathbf{q}', \nabla) \mathbf{q}' \right] = -\nabla P' + \mu \nabla^2 \mathbf{q}' - \frac{1}{k} \left( \mu + \mu' \frac{\partial}{\partial t} \right) \mathbf{q}'
\]
\[
-\rho' \varepsilon \dot{\mathbf{z}} + \frac{\mu \varepsilon}{4\pi} (\nabla \times \mathbf{h}) \times \mathbf{H}_b + \frac{\mu \varepsilon}{4\pi} (\nabla \times \mathbf{h}) \times \mathbf{h}
\]  
... (12)

\[
[\rho_o C_v + \rho_s C_s (1 - \varepsilon)] \frac{\partial \mathbf{h}}{\partial t} + \rho_o C_v (\mathbf{q}', \nabla) \theta + \rho_o C_v (\mathbf{q}', \nabla) \mathbf{H}_b = \chi_T \nabla^2 \mathbf{h}
\]  
... (13)

\[
\frac{\varepsilon}{\partial t} \mathbf{h} = \nabla \times (\mathbf{q}' \times \mathbf{h}) + \nabla \times (\mathbf{q} \times \mathbf{H}_b) + \varepsilon \gamma_m \nabla^2 \mathbf{h}
\]
\[
-\frac{\varepsilon}{4\pi \varepsilon_0 n_e} \nabla \times [\nabla \times (\mathbf{h} \times \mathbf{h}) \times \mathbf{H}_b]
\]  
... (15)

and \( \rho' = -\rho_o \alpha \theta \)  
... (16)

In order to linearize above equation, ignoring the terms \( (\mathbf{q}', \nabla) \mathbf{q}', (\mathbf{q}', \nabla) \theta, (\nabla \times \mathbf{h}) \times \mathbf{h}, \nabla \times (\mathbf{q}' \times \mathbf{h}) \) we have
\[
\nabla \mathbf{q}' = 0
\]  
... (17)
\[
\frac{\rho_0}{\varepsilon} \left[ \frac{\partial \varphi}{\partial t} + \rho g \right] = -\nabla P + \mu \nabla^2 \varphi - \frac{1}{k} \left( \mu + \mu' \right) \varphi' \varepsilon \tilde{e}_z + \frac{\mu e H_0}{4\pi} (\nabla \times \hat{h}) \times \tilde{e}_z \quad \text{(18)}
\]

\[
\left[ \varepsilon + \frac{\rho_0 c_v (1 - \varepsilon)}{\rho c_v} \right] \frac{\partial \theta}{\partial t} - \omega' \beta = \frac{\alpha T \nabla^2 \theta}{\rho c_v} \quad \text{(19)}
\]

\[
\varepsilon \frac{\partial \hat{h}}{\partial t} = H_0 \nabla \times (\varphi \times \varepsilon \tilde{e}_z) + \gamma_m \nabla^2 \hat{h} - \frac{\varepsilon H_0}{4\pi \epsilon_0} (\nabla \times \hat{h}) \times \varepsilon \tilde{e}_z \quad \text{(20)}
\]

\[
\nabla \times \hat{h} = 0 \quad \text{(21)}
\]

and \( \rho' = -\rho_0 \beta \theta \) \( \text{(22)} \)

Converting the equation (17) to (22) into non-dimensional form by the following transformation and dropping the stars,

\[
x = dx^*, \quad y = dy^*, \quad z = dz^*, \quad \varphi' = kT d q^*, \quad t = \frac{\rho_0 d^2}{\mu} t^*, \quad \theta = \beta d \theta^*, \quad P' = \frac{\mu kT}{d^2} P^*, \quad \hat{h} = H_0 \hat{h}^*, \quad \text{where} \quad kT = \frac{\alpha T}{\rho_0 c_v} \quad \text{is the thermal diffusivity, we have}
\]

\[
\nabla \cdot \varphi^* = 0 \quad \text{(23)}
\]

\[
\frac{1}{\varepsilon} \frac{\partial \varphi^*}{\partial t} = -\nabla P + \nabla^2 \varphi^* - \frac{1}{k_1} \left[ 1 + F \frac{\partial}{\partial t} \right] \varphi^* + \beta \omega' \varepsilon \tilde{e}_z + Q (\nabla \times \hat{h}) \times \varepsilon \tilde{e}_z \quad \text{(24)}
\]

Similarly

\[
\frac{\partial \theta^*}{\partial t} = \frac{\nabla^2 \theta^*}{\varepsilon} + \omega' \quad \text{(25)}
\]

\[
\varepsilon \frac{\partial \hat{h}^*}{\partial t} = \frac{\partial \varphi^*}{\partial \varepsilon} + \varepsilon \frac{P^*}{P_m} \nabla^2 \hat{h}^* - \varepsilon \beta e^2 \frac{\partial}{\partial \varepsilon} (\nabla \times \hat{h})^* \quad \text{(26)}
\]

Where \( R = \frac{\rho_0 \alpha^2 \beta A}{\mu kT} \) is the thermal Rayleigh number

\[
Q = \frac{\mu_0 H_0^2 d^2}{4\pi \mu kT} \quad \text{is the Chandrasekhar number}
\]

\[
k_1 = \frac{k}{d^2}, \quad E = \frac{P c_v (1 - \varepsilon)}{\rho_0 c_v}, \quad P_r = \frac{\mu}{\rho_0 kT} \quad \text{is the Prandtl number,} \quad P_m = \frac{\mu}{\rho_0 \gamma m} \quad \text{is the magnetic Prandtl number,} \quad F = \frac{\mu}{\rho_0 kT} \quad \text{is the viscoelastic parameter} \quad \beta e = \left( \frac{H_0}{4\pi kT \varepsilon_0} \right)^2 \quad \text{is the Hall parameter and} \quad W = \frac{\partial}{\partial t} q^* e^*_z
\]

5. Boundary Conditions

Both boundary are taken to be free and perfectly heat conducting, then we have \( W = \frac{d^2 W}{d z^2} = 0, \theta = 0, \) at \( z = 0 \) and \( z = 1 \) \( \text{(27)} \)

6. Dispersion Relation

Taking curl on the sides (24), we have

\[
\frac{1}{\varepsilon} \frac{\partial (\nabla \times \varphi^*)}{\partial t} = \nabla^2 (\nabla \times \varphi^*) - \frac{1}{k_1} \left[ 1 + F \frac{\partial}{\partial t} \right] (\nabla \times \varphi^*) + R \frac{\partial \theta^*}{\partial y} \tilde{e}_x + \frac{\partial \theta^*}{\partial x} \tilde{e}_y + Q \frac{\partial}{\partial \varepsilon} (\nabla \times \hat{h})^* \quad \text{(28)}
\]

Again applying curl on both sides of (28) and taking \( z \)-component on both sides, we have

\[
\left[ \frac{1}{\varepsilon} + \frac{1}{k_1} \left( 1 + F \frac{\partial}{\partial t} \right) - \nabla^2 \right] \nabla^2 W = RW^2 \theta^* + Q D (\nabla^2 h_z) \quad \text{(29)}
\]

Where \( \nabla^2 = \frac{\partial^2}{\partial x^2}, \quad \frac{\partial^2}{\partial y^2}, \quad \hat{h} = \hat{h}^* e_z, \quad D = \frac{\partial}{\partial \varepsilon}
\]

Taking \( z \)-component on both sides of equation (28)
\[
\begin{bmatrix}
1 + & \frac{F}{k_1} \\
\frac{\partial}{\partial t} & \frac{\partial}{\partial t}
\end{bmatrix} - \nabla^2 \mathbf{z} = Q \mathbf{m}_z
\] ...
(30)

Where \( \zeta_z = (\nabla \times \mathbf{q}) \dot{\zeta}_z \) and \( \mathbf{m}_z = (\nabla \times \nabla) \dot{\zeta}_z \)

\[
\frac{E Pr \partial}{\partial t} - \nabla^2 \theta = W
\] ...
(31)

Taking curl on both side of equation (26) and taking z-component on both side, we have

\[
e P_r \frac{\partial m_z}{\partial t} = D \zeta_z + e P_r \nabla^2 m_z + e \beta e^{1/2} D \nabla^2 h_z
\] ...
(32)

Taking z-component on both sides of equation (26), we have

\[
e P_r \frac{\partial h_z}{\partial t} = DW + e P_r \nabla^2 h_z - e \beta e^{1/2} D m_z
\] ...
(33)

Boundary condition (27) now becomes

\[
W = D^2 W = 0 = \zeta_z = D \zeta_z = h_z = D m_z = \theta \text{ at } z = 0 \text{ and } z = 1
\] ...
(34)

7. Normal Mode Analysis

Consider

\[
[W, \zeta_z, \Theta, h_z, m_z] = [W(z), X(z), \Theta(z), B(z), M(z)] \exp \left[ ik_x x + ik_y y + \sigma t \right]
\]

Applying above normal mode analysis to the equation (29) to (33) we have

\[
\begin{bmatrix}
\frac{\sigma}{k_1} + \frac{1}{k_1} (1 + F \sigma) - (D^2 - a^2) \\
\frac{\partial}{\partial t}
\end{bmatrix} (D^2 - a^2) W = -Ra^2 \Theta + QD(D^2 - a^2) B 
\] ...
(35)

\[
\begin{bmatrix}
\frac{\sigma}{k_1} + \frac{1}{k_1} (1 + F \sigma) - (D^2 - a^2) \\
\frac{\partial}{\partial t}
\end{bmatrix} X = QDM
\] ...
(36)

\[
\begin{bmatrix}
EP_r \sigma - (D^2 - a^2) \\
\frac{\partial}{\partial t}
\end{bmatrix} \Theta = W
\] ...
(37)

\[
\begin{bmatrix}
e P_r \sigma - \frac{P_r}{P_m} (D^2 - a^2) \\
\frac{\partial}{\partial t}
\end{bmatrix} M = DX + e \beta e^{1/2} D(D^2 - a^2) B
\] ...
(38)

\[
\begin{bmatrix}
e P_r \sigma - \frac{P_r}{P_m} (D^2 - a^2) \\
\frac{\partial}{\partial t}
\end{bmatrix} B = DW - e \beta e^{1/2} DM
\] ...
(39)

Where \( a^2 = k_x^2 + k_y^2 \) is the wave number and \( \sigma = \sigma_r + i \sigma_i \) is the stability parameter.

Now, the boundary condition become

\[
W = D^2 W = 0 = X = DX = B = M = DM, \Theta = 0 \text{ at } z = 0 \text{ and } z = 1
\] ...
(40)

Assuming (40) we get

\[
D(2n)W = 0 \text{ at } z = 0, z = 1, \text{ where } n \text{ is a positive integer}
\]

Thus the proper solution satisfying (40) can be taken as

\[
W = W_o \sin \pi z, \text{ where } W_o \text{ is a constant}
\] ...
(41)

Eliminating \( \Theta, B, X \) and \( M \) from (35) to (39), we have

\[
\begin{bmatrix}
\frac{\sigma}{k_1} + \frac{1}{k_1} (1 + F \sigma) - (D^2 - a^2) \\
\frac{\partial}{\partial t}
\end{bmatrix} \times \begin{bmatrix}
e P_r \sigma - \frac{P_r}{P_m} (D^2 - a^2) \\
\frac{\partial}{\partial t}
\end{bmatrix} B + \frac{1}{k_1} (L + F \sigma) - (D^2 - a^2)
\] ...
(42)

From (35) and (37), we get

\[
\begin{bmatrix}
\frac{\sigma}{k_1} + \frac{1}{k_1} (1 + F \sigma) - (D^2 - a^2) \\
\frac{\partial}{\partial t}
\end{bmatrix} \times \begin{bmatrix}
EP_r \sigma - (D^2 - a^2) \\
\frac{\partial}{\partial t}
\end{bmatrix} (D^2 - a^2) W = -Ra^2 W
\]

\[
+ QD(D^2 - a^2) (EP_r \sigma - (D^2 - a^2)) B
\] ...
(43)

From (42) and (43)
\[
W \left[ \frac{\sigma}{1 + k_1} (1 + F\sigma) - (D^2 - a^2) \right] \times \left[ \frac{\sigma}{1 + k_1} (1 + F\sigma) - (D^2 - a^2) \right] = \frac{\sigma}{1 + k_1} (1 + F\sigma) - (D^2 - a^2) + \beta_e D^2 \left( D^2 - a^2 \right) - QD^2 \left[ \frac{\sigma}{1 + k_1} (1 + F\sigma) - (D^2 - a^2) \right] \times \left[ \frac{\sigma}{1 + k_1} (1 + F\sigma) - (D^2 - a^2) \right] - \left( EP_r \sigma - \frac{P_r}{P_m} \right) \left( D^2 - a^2 \right) + Ra^2
\]

\[
= \frac{\sigma}{1 + k_1} (1 + F\sigma) - (D^2 - a^2) \times \left[ \frac{\sigma}{1 + k_1} (1 + F\sigma) - (D^2 - a^2) \right] - \left( EP_r \sigma - \frac{P_r}{P_m} \right) \left( D^2 - a^2 \right) - \left( EP_r \sigma - \frac{P_r}{P_m} \right) \left( D^2 - a^2 \right)
\]

\[
. QD^2 \left( D^2 - a^2 \right) \left[ EP_r \sigma - \frac{P_r}{P_m} \right] W
\]

\[(44)\]

Substituting the value of \( W \) from (41) into (44) and using \( b = \pi^2 + a^2 \), we have

\[
\left[ \frac{\sigma}{1 + k_1} (1 + F\sigma) + b \right] \times \left[ \frac{\sigma}{1 + k_1} (1 + F\sigma) + b \right] + \pi^2 \beta_e b \left[ \frac{\sigma}{1 + k_1} (1 + F\sigma) + b \right]
\]

\[
\times \left[ \frac{\sigma}{1 + k_1} (1 + F\sigma) + b \right] \left[ EP_r \sigma + b \right] \left( -b \right) + Ra^2
\]

\[
= \frac{\sigma}{1 + k_1} (1 + F\sigma) + b \times \left[ \frac{\sigma}{1 + k_1} (1 + F\sigma) + b \right] + \theta \pi^2 b (EP_r \sigma + b)
\]

\[(45)\]

8. Stationary Convection

In order to examine the stationary convection

We put \( \sigma = 0 \) in (45)

\[
R = \frac{b^2 \left[ \frac{\sigma}{P_m} \left( 1 + b \right) + Q \pi^2 \right]^2 + \pi^2 b^3 \beta_e \left( \frac{1}{k_1} + b \right)^2}{a^2 \left[ \frac{1}{k_1} + b \right] \left[ \frac{\sigma}{P_m} \right]^2 + \pi^2 b \beta_e} + Q \pi^2 \frac{\sigma}{P_m} \left( 1 + b \right)
\]

\[(46)\]

In the absence of magnetic field i.e., \( (H_o = 0) \), i.e., \( Q = 0 \) and \( \beta_e = 0 \) then equation (46)

\[
R = \frac{b^2 \left( \frac{1}{k_1} + b \right)}{a^2}
\]

\[(47)\]

In particular, in non-porous medium \( [k_1 \rightarrow \infty] \) equation (47) reduces to

\[
R = \frac{b^3}{a^2}
\]

Which is classical value of \( R \) for Newtonian fluid as that proposed by G. Lebon and C. Perez-Garcia.

Equation (46) can be written as

\[
R = \frac{b^2 \left[ \frac{\sigma}{P_m} \left( 1 + b \right) + Q \pi^2 \right]^2 + \pi^2 b^3 \beta_e \left( \frac{1}{k_1} + b \right)^2}{a^2 \left[ \frac{1}{k_1} + b \right] \left[ \frac{\sigma}{P_m} \right]^2 + \pi^2 b \beta_e} + Q \pi^2 \frac{\sigma}{P_m} \left( 1 + b \right)
\]

\[(48)\]

In order to investigate the behavior of \( k_1 \) (Medium Permeability), \( Q \) (Magnetic field), \( \beta_e \) (Hall current), we find the nature of \( \frac{dR}{dk_1}, \frac{dR}{dQ}, \frac{dR}{d\beta_e} \) respectively.

From (48)
Thus, the medium permeability $k_1$ has destabilizing effect when $Q < \frac{P_r b^2}{P_m \pi^2}$.

In particular, $(H_0 = 0)$, i.e., $Q = 0$ and $\beta_e = 0$

$$\frac{dR}{dk_1} = -\frac{b^2}{a^2 k_1^2}$$

Which is always negative, the medium permeability has destabilizing effect without any condition.

From (48)

$$\frac{dR}{dQ} = \frac{\pi^2 b^2}{a^2} \left[ 2 \left( \frac{1}{k_1 + b} \right)^2 \left( \frac{3 P_r b^3}{P_m^3} \right) + 2 \left( \frac{1}{k_1 + b} \right)^2 \left( \frac{\pi^2 b^2 \beta_e P_r}{P_m} \right) \right.$$  

$$\left. + 2 \left( \frac{1}{k_1 + b} \right) \left( Q \pi^2 \frac{2 P_r b^2}{P_m^2} + Q^2 \pi^4 \frac{2 P_r b^2}{P_m^2} \right) + 2 \left( \frac{1}{k_1 + b} \right) Q^2 \pi^4 \frac{2 P_r b^2}{P_m^2} \right]$$

$$\frac{dR}{dQ} > 0$$

Thus, the magnetic field has stabilizing effect and this stabilizing effect is independent of the presence of Hall current.

From (48)

$$\frac{dR}{d\beta_e} = -b^2 Q^2 \pi^2 \left[ 2 \left( \frac{1}{k_1 + b} \right)^2 \left( \frac{3 P_r b^3}{P_m^3} \right) + 2 \left( \frac{1}{k_1 + b} \right)^2 \left( \frac{\pi^2 b^2 \beta_e P_r}{P_m} \right) \right.$$  

$$\left. + 2 \left( \frac{1}{k_1 + b} \right) \left( Q \pi^2 \frac{2 P_r b^2}{P_m^2} + Q^2 \pi^4 \frac{2 P_r b^2}{P_m^2} \right) + 2 \left( \frac{1}{k_1 + b} \right) Q^2 \pi^4 \frac{2 P_r b^2}{P_m^2} \right]$$

$$\frac{dR}{d\beta_e} < 0$$

Thus, the Hall current parameter has destabilizing effect

9. Oscillatory Convection

Putting $\sigma = i\sigma_i$ in (45) and separate real and imaginary parts, we have

$$\left[ b + \frac{1}{k_1} + i \left( \frac{1}{k_1} \right) \sigma_i \right] \times \left[ \left( \frac{e P_r b}{P_m} + i \frac{e P_r b}{P_m} \right) + e^2 \frac{\pi^2 b^2 \beta_e}{P_m} \right] + Q^2 \left( i \frac{e P_r b}{P_m} + i \frac{e P_r b}{P_m} \right)$$
The real part is given by

\[ R = \frac{a_1 \sigma_i^4 + a_2 \sigma_i^2 + a_3}{b_1 \sigma_i^2 + b_2} \]

Where

\[ a_1 = \left( b + \frac{1}{k_1} \right) \left( \frac{1}{e + \frac{F}{k_1}} \right) \left( -e^2 E F^3 b - e^2 E F^3 b \right) + \left( \frac{1}{e + \frac{F}{k_1}} \right) \left( -2 e^2 E F^3 b^2 P_m - e^2 E F^3 b^2 P_m \right) \]

\[ a_2 = e^2 E F^3 b^2 \left( b + \frac{1}{k_1} \right)^2 + \left( \frac{1}{e + \frac{F}{k_1}} \right) \left( b + \frac{1}{k_1} \right) \left( 2 e^2 E F^3 b^2 P_m \right) \]

\[ + \left( b + \frac{1}{k_1} \right) \left( 1 + \frac{F}{e} \right) \left( e^2 E F^3 b^3 \right) + \left( b + \frac{1}{k_1} \right) \left( 1 + \frac{F}{e} \right) \left( e^2 \pi^2 E b^2 P_r \beta_e \right) \]

\[ + \left( \frac{1}{e + \frac{F}{k_1}} \right) \left( Q \pi^2 E P_m b^2 \right) + \left( b + \frac{1}{k_1} \right) \left( 1 + \frac{F}{e} \right) \left( 2 e^2 E F^3 b^2 P_m \right) \]

\[ + \left( b + \frac{1}{k_1} \right) \left( 1 + \frac{F}{e} \right) \left( \frac{E e^2 P_r^3 b^3}{P_m} \right) + \left( b + \frac{1}{k_1} \right) \left( 1 + \frac{F}{e} \right) \left( e^2 \pi^2 P_r b^2 \beta_e \right) \]

\[ + \left( b + \frac{1}{k_1} \right) \left( Q \pi^2 E P_m b^2 \right) + \left( b + \frac{1}{k_1} \right) \left( 1 + \frac{F}{e} \right) \left( 2 e^2 E F^3 b^2 P_m \right) \]

\[ + \left( \frac{1}{e + \frac{F}{k_1}} \right) \left( e^2 P_r^2 b^4 \right) + \left( 1 + \frac{F}{e} \right) \left( e^2 \pi^2 b^3 \beta_e \right) + \left( \frac{1}{e + \frac{F}{k_1}} \right) Q \pi^2 E P_r b^2 \]

\[ \cdots (49) \]
\[ a_3 = - \left[ \left( b + \frac{1}{k_1} \right) \left( \frac{e^2 P_r^2 b^4}{P_m^2} \right) + \left( b + \frac{1}{k_1} \right) \left( \frac{Q \pi^2 e P_r b^3}{P_m} \right) \right] \]

And \[ b_1 = a^2 \left[ \left( b + \frac{1}{k_1} \right) \left( e^2 P_r^2 \right) + \left( \frac{1}{e} + \frac{F}{k_1} \right) \left( \frac{2 e^2 P_r^2 b}{P_m} \right) \right] \]

\[ b_2 = - a^2 \left[ \left( e^2 P_r^2 b^2 + e^2 \pi^2 b \beta_e \right) \left( b + \frac{1}{k_1} \right) + \frac{Q \pi^2 e P_r b}{P_m} \right] \]

The imaginary part is given by
\[ R = \frac{P_1 \sigma_i^5 + P_2 \sigma_i^3 + P_3 \sigma_i}{q_1 \sigma_i^3 + q_2 \sigma_i} \quad \ldots (50) \]

Where
\[ P_1 = \left( \frac{1}{e} + \frac{F}{k_1} \right)^2 e^2 E P_r^3 b \]

\[ P_2 = - \left[ \left( b + \frac{1}{k_1} \right)^2 e^2 E P_r^3 b + \left( b + \frac{1}{k_1} \right) \left( \frac{1}{e} + \frac{F}{k_1} \right) \left( \frac{2 e^2 E P_r^3 b^2}{P_m} \right) \right] \]

\[ \quad \quad + \left( \frac{1}{e} + \frac{F}{k_1} \right) \left( b + \frac{1}{k_1} \right) \left( e^2 P_r^2 b^2 \right) + \left( \frac{1}{e} + \frac{F}{k_1} \right)^2 \left( \frac{2 e^2 P_r^2 b^3}{P_m} \right) \]

\[ \quad + \left( b + \frac{1}{k_1} \right) \left( \frac{1}{e} + \frac{F}{k_1} \right) \left( e^2 P_r^2 b^2 \right) + \left( b + \frac{1}{k_1} \right) \left( \frac{1}{e} + \frac{F}{k_1} \right) \left( \frac{2 e^2 E P_r^3 b^2}{P_m} \right) \]

\[ \quad + \left( \frac{1}{e} + \frac{F}{k_1} \right) \left( e^2 P_r^3 b^3 \right) + \left( \frac{1}{e} + \frac{F}{k_1} \right) \left( e^2 \pi^2 E P_r b \beta_e \right) + 2 \left( \frac{1}{e} + \frac{F}{k_1} \right) \left( Q \pi^2 e P_r^2 b \right) \]

\[ P_3 = \left( b + \frac{1}{k_1} \right)^2 \left( \frac{e^2 E P_r^3 b^3}{P_m} \right) + \left( b + \frac{1}{k_1} \right)^2 \left( e^2 \pi^2 E P_r b \beta_e \right) \]

\[ \quad + \left( b + \frac{1}{k_1} \right) \left( \frac{Q \pi^2 E P_r^2 b^2}{P_m^2} \right) + \left( b + \frac{1}{k_1} \right) \left( \frac{1}{e} + \frac{F}{k_1} \right) \left( e^2 P_r^2 b^4 \right) \]

\[ \quad + \left( b + \frac{1}{k_1} \right) \left( \frac{1}{e} + \frac{F}{k_1} \right) \left( e^2 \pi^2 b^3 \beta_e \right) + \left( \frac{1}{e} + \frac{F}{k_1} \right) \left( \frac{Q \pi^2 e P_r b^3}{P_m} \right) \]

\[ \quad + \left( b + \frac{1}{k_1} \right) \left( \frac{e^2 P_r^3 b^3}{P_m} \right) + \left( b + \frac{1}{k_1} \right) \left( \frac{1}{e} + \frac{F}{k_1} \right) \left( \frac{2 e^2 P_r^2 b^4}{P_m^2} \right) \]

\[ \quad + \left( b + \frac{1}{k_1} \right) \left( \frac{1}{e} + \frac{F}{k_1} \right) \left( e^2 \pi^2 b^3 \beta_e \right) + \left( b + \frac{1}{k_1} \right) \left( Q \pi^2 e P_r b^2 \right) \]

\[ \quad + \left( b + \frac{1}{k_1} \right) \left( Q \pi^2 e P_r^2 b + \left( \frac{1}{e} + \frac{F}{k_1} \right) \left( \frac{Q \pi^2 e P_r b^3}{P_m} \right) + \left( b + \frac{1}{k_1} \right) \left( \frac{Q \pi^2 e P_r^2 b^2}{P_m} \right) \right) \]

\[ q_1 = - a^2 \left( \frac{1}{e} + \frac{F}{k_1} \right) \left( e^2 P_r^2 \right), \]
\[ q_2 = a^2 \left( \frac{2 e^2 P_r^2 b}{P_m} \left( \frac{b}{k_1} + 1 \right) \right) + \left( \frac{1}{e} + \frac{F}{k_1} \right) \left( \frac{e^2 P_r^2 b^2}{P_m^2} + e^2 \pi^2 b \beta_e \right) + Q \pi^2 \in P_r \]

Eliminating \( R \) between (49) and (50)

For \( f_0 \sigma_i^6 + f_1 \sigma_i^4 + f_2 \sigma_i^2 + f_3 = 0 \)

Where \( s = \sigma_i^2 \)

\[ f_0 s^3 + f_1 s^2 + f_2 s + f_3 = 0 \]

\[ f_0 = a_1 q_1 - R b_1, \quad f_1 = a_2 q_1 + a_1 q_2 - P_2 b_1 - P_2 b_2, \]
\[ f_2 = a_3 q_1 + a_2 q_2 - P_3 b_1 - P_3 b_2, \quad f_3 = a_3 q_2 - P_3 b_2 \]

Where \( f_0 = a_1 q_1 - R b_1 \)

\[ f_0 = a_2 q_1 + a_1 q_2 - P_2 b_1 - P_2 b_2 > 0 \]

when

\[ \left( \frac{1}{e} + \frac{1}{4\sqrt{2}} \right) \frac{1}{e} + \frac{F}{k_1} < \min \left\{ \frac{1}{3}, \frac{P_r}{\pi k_1 \sqrt{\beta_e}} \right\}, k_1 < \frac{1}{2} \]

\[ \beta_e < \frac{1}{6 \pi^2 P_m}, \quad P_m < \sqrt{2}, E > 1, P_r \in \left( \frac{1}{2}, 1 \right), 2 P_m < EP_r < \frac{1}{\sqrt{2}} \]

From (51), we notice that \( s = \sigma_i^2 \) which is always positive, therefore the sum of roots equation of (51) is positive but this is impossible if \( f_0 > 0 \) and \( f_1 > 0 \), because the sum of roots of equation (51) is \( \left( \frac{f_0}{f_1} \right) \). Thus, \( f_0 > 0 \) and \( f_1 > 0 \) are the sufficient condition for the non-existence of over-stability.

Now \( f_0 > 0 \)

and \( f_1 > 0 \)

when

\[ \left( \frac{1}{e} + \frac{1}{4\sqrt{2}} \right) \frac{1}{e} + \frac{F}{k_1} < \min \left\{ \frac{1}{3}, \frac{P_r}{\pi k_1 \sqrt{\beta_e}} \right\}, k_1 < \frac{1}{2} \]

\[ \beta_e < \frac{1}{6 \pi^2 P_m}, \quad P_m < \sqrt{2}, E > 1, P_r \in \left( \frac{1}{2}, 1 \right), 2 P_m < EP_r < \frac{1}{\sqrt{2}} \]

Hence for the conditions given in (52), overstability cannot occur and the principle of exchange of stability (PES) is valid.

10. Conclusions

1. For Stationary Convection
   (i) When \( Q < \frac{e P_r^2 b^2}{P_m \pi^2}, \quad \frac{dR}{dQ} < 0 \), which implies that the medium permeability \( k_1 \) has destabilizing effect under the above condition. In the absence of Hall parameter and magnetic field, the medium permeability has destabilizing effect without any condition.
   (ii) \( \frac{dR}{dQ} > 0 \), Thus the magnetic field has stabilizing effect and this stabilizing effect is independent of Hall current.
   (iii) \( \frac{dR}{d\beta_e} < 0 \), thus the Hall parameter has destabilizing effect.

2. For Oscillatory Convection: The sufficient condition for the non-existence of over-stability are given by condition

\[ \left( \frac{1}{e} + \frac{1}{4\sqrt{2}} \right) \frac{1}{e} + \frac{F}{k_1} < \min \left\{ \frac{1}{3}, \frac{P_r}{\pi k_1 \sqrt{\beta_e}} \right\}, k_1 < \frac{1}{2} \]

\[ \beta_e < \frac{1}{6 \pi^2 P_m}, \quad P_m < \sqrt{2}, E > 1, P_r \in \left( \frac{1}{2}, 1 \right), 2 P_m < EP_r < \frac{1}{\sqrt{2}} \]
References