Comparision of profit analysis of a two – Unit cold standby system with instruction and preparation time for repair

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Abstract
The comparative study of the profit of a two-unit cold standby system with instruction and preparation time for repair is carried out. The repair man (who is an expert) does not come alone but he comes with his assistant. If the expert repair man is busy in repairing a failed unit and second unit fails then the assistant repairman repairs the latter unit after getting instructions from expert repairman. The paper consists of two models. In model-1 expert repairman takes the unit for repair from his assistant if he completes the repair earlier than his assistant whereas in model-2 either of the repairman (expert or his assistant) do not leave the repair till its completion.

Keywords: Unit cold standby system, assistant repairman, expert repairman

Introduction
Many investigations concerning the reliability of two-unit system are being made. Various authors including [1-3] discussed two-unit cold standby systems assuming that whenever the operative unit fails, it goes under repair immediately i.e. preparation time for repair is negligible. Guo Tong De [4] and some others studied two-unit systems with preparation time for repair and with one repairman serving at a time. However, practically there may be situations when on failure of a unit the expert repairman does not come alone but he comes with his assistant. The assistant repairman repairs the failed unit only after getting instructions from expert which are given at the time of system failure so that both the failed unit go under repair of both the repairman – one under expert and the other under his assistant.

So, in this paper, we have investigated a two-unit cold stand by system with instruction and preparation time for repair. Whenever a unit fails, we call an expert repairman immediately to repair the failed unit. He comes with his assistant who does the repair of failed unit perfectly only if instructions are given to him by the expert. The expert repairman first prepare himself for repair i.e. he makes the arrangement of tools and other essential materials which are necessary for repair the failed unit and then he starts repair. If a unit is under repair/preparation for repair of the expert and at that time a second unit fails the repairman leaves the repair/preparation for repair of former unit and starts giving instruction to his assistant. It is assumed that after getting instructions the assistant repairman repairs perfectly. Former unit goes under repair of expert and the later under repair of his assistant with the assumptions that both the repairmen first prepare themselves for repair. The paper consists of two models. In model-1 expert repairman takes the unit for repair from his assistant if he completes the repair earlier than his assistant whereas in model-2 either of the repairman (expert or his assistant) do not leave the repair till its completion.

The system is analysed by making use of semi-Markov-Renewal Processes and regenerative processes and determine the expressions for the various measures of system effectiveness such as mean time to system failure (MTSF), steady-state availability, total fraction of busy time of the repairman, expected number of visits by the expert repairman and expected profit earned by the system. Graphs are plotted and comparison between models is made through graphs.
**Notations**

- $o/c_s$: operative/cold-standby
- $F_o$: unit in F mode and under preparation for repair
- $F_s$: unit in F-mode and under repair of expert/assistant.
- $F_e$: unit in F-mode and repair is continued by an expert repairman from earlier state.
- $F_w$: unit in F-mode and waiting for repair when instructions are being given to assistant.
- $\lambda$: Constant failure rate of an operative unit.
- $\mu$: Constant repair rate of assistant repairman.
- $h(t)$, $H(t)$: pdf and cdf of time to preparation for repair of failed unit.
- $g(t)$, $G(t)$: pdf and cdf of time to repair by expert.
- $i(t)$, $I(t)$: pdf and cdf of time when expert gives instructions to his assistant.

For rest of the notations see reference [5].

**Model - 1**

In this model, the expert takes the unit for repair from his assistant if he completes the repair earlier than his assistant. Thus considering the above symbols, possible states of the system and the transitions into the states are shown in Fig. 1. The epoch of entrance into the states 6 from 5 is non regenerative.

![State Transition Diagram](image)

Fig 1: State Transition Diagram

- Up State
- Failed State
- Regenerative Point
Transition Probabilities and Sojourn Times
The non-zero elements $p_{ij}$ of the transition probability matrix (t.p.m.) for the system are as follows:

$p_{01} = 1, \ p_{12} = h^*(\lambda), \ p_{13} = 1 - h^*(\lambda), \ p_{20} = g^*(\mu),
\ p_{23} = 1 - g^*(\mu), \ p_{34} = 1, \ p_{45} = 1$, and

$p_{52} = g^*(\mu),
\ p_{53} = [\mu g^*(\lambda) - \lambda g^*(\mu)] / (\lambda - \mu)

By these transition probabilities, it can be verified that

$p_{01} = p_{34} = p_{45} = 1, \ p_{20} + p_{23} = p_{12} + p_{13} = 1

Let $T_i$ be the sojourn time in state $S_i$ and $\mu_i = E(T_i)$, then using the formula

$\mu_i = \int_0^\infty p(T_i > t)dt$

then for $\phi_0^\mu(s)$, we have

$\phi_0^\mu(s) = \frac{Q_{01}^\mu(s)Q_{13}^\mu(s) + Q_{12}^\mu(s)Q_{23}^\mu(s)}{1 - Q_{01}^\mu(s)Q_{13}^\mu(s)Q_{23}^\mu(s)}

Now the MTTSF, given that the system started at the beginning of state 0 is

$T_0 = \lim_{s \to 0} \frac{1 - \phi_0^\mu(s)}{s}$

Using L'Hospital's rule and substituting the value of $\phi_0^\mu(s)$ from equation (4), we have

$T_0 = \frac{\mu_0 + \mu_1 + \mu_2 p_{12}}{1 - p_{12}p_{20}}

$\textit{Availability Analysis}$
As defined, $M_i(t)$ denotes the probability that the system starting in up state (regenerative state) is up at time $t$ without passing through any regenerative state. Thus we have

$M_0(t) = e^{-\lambda t}, \ M_1(t) = e^{-\lambda t} \bar{H}(t)
\ M_2(t) = e^{-\lambda t} \bar{G}(t), \ M_5(t) = \left[\mu e^{-\mu t} \oplus e^{-\lambda t}\right] \bar{G}(t)$

and

$A_0(t) = M_0(t) \oplus q_{01}(t) \oplus A_1(t)
\ A_1(t) = M_1(t) \oplus q_{12}(t) \oplus A_2(t) \oplus q_{13}(t) \oplus A_3(t)
\ A_2(t) = M_2(t) \oplus q_{20}(t) \oplus A_0(t) \oplus q_{23}(t) \oplus A_3(t)
\ A_3(t) = q_{34}(t) \oplus A_4(t)
\ A_4(t) = q_{45}(t) \oplus A_5(t)
\ A_5(t) = M_5(t) \oplus q_{52}(t) \oplus A_2(t) \oplus q_{53}(t) \oplus A_3(t) \oplus q_{50}(t) \oplus A_0(t) \oplus q_{51}(t) \oplus A_1(t)$

Taking the Laplace transform of the above equations and solving them for $AV_0^\mu(s)$ and then steady-state availability of the system is given by

$A_0 = \lim_{t \to \infty} A_0(t) = \lim_{s \to 0} A_0^\mu(s) = \frac{N_1}{D_1}$

where

$N_1 = (\mu_0 + \mu_1)\left(1 - p_{34}^{(6)} - p_{23}p_{52}\right) + \mu_2\left(p_{12}p_{50}^{(6)} + p_{52}\right) + \epsilon_1(1 - p_{12}p_{20})$

and

$M_5'(0) = \epsilon_1 = \mu(\mu_4 - \mu_2) / (\lambda - \mu)$

(say)
\[ D_1 = (\mu_2 + (\mu_0 + \mu_1) p_{20}) p_{52} + (\mu_0 + \mu_1 + \mu_2 p_{12}) p_{50}^{(6)} + (\mu_3 + \mu_4 + m_i)(1 - p_{12} p_{20}) \]  
where \( m_i = m_{52} + m_{50}^{(6)} + m_{53}^{(6)} = (\lambda \mu_5 - \mu_1) / (\lambda - \mu) \) \hfill (19)

**Busy-Period Analysis of an Expert Repairman**

\[ B_0(t) = q_{01}(t) \odot B_1(t) \]
\[ B_1(t) = W_1(t) + q_{12}(t) \odot B_2(t) + q_{13}(t) \odot B_3(t) \]
\[ B_2(t) = W_2(t) + q_{20}(t) \odot B_0(t) + q_{21}(t) \odot B_1(t) \]
\[ B_3(t) = W_3(t) + q_{32}(t) \odot B_2(t) \]
\[ B_4(t) = W_4(t) + q_{43}(t) \odot B_3(t) \]
\[ B_5(t) = W_5(t) + q_{54}(t) \odot B_4(t) \]

where

\[ W_1(t) = \sum e^{-\lambda t} H(t), \quad W_2(t) = \sum e^{-\mu t} G(t), \quad W_3(t) = \sum e^{-\lambda t} \]
\[ W_4(t) = \sum e^{-\mu t} G(t) \]

Solving equations (20-25) for \( B^*(s) \) with the help of Laplace-transform and then in a steady-state, the total fraction of the time for which the expert repairman is busy, is given by

\[ B_0 = \lim_{s \to 0} sB_0^*(s) = \frac{N_2}{D_1} \]  
\[ N_2 = \mu_i (1 - p_{53}^{(6)} - p_{23} p_{52}) + \mu_2 (p_{52} + p_{12} p_{50}^{(6)}) + \mu_4 + m_i (1 - p_{12} p_{20}) \]  
where \( D_1 \) is already specified.

**Expected Number of Visits by the Expert Repairman**

\[ V_0(t) = Q_{01}(t) \odot [1 + V_1(t)] \]
\[ V_1(t) = Q_{12}(t) \odot V_2(t) + Q_{13}(t) \odot V_3(t) \]
\[ V_2(t) = Q_{20}(t) \odot V_0(t) + Q_{21}(t) \odot V_1(t) \]
\[ V_3(t) = Q_{32}(t) \odot V_2(t) \]
\[ V_4(t) = Q_{43}(t) \odot V_3(t) \]
\[ V_5(t) = Q_{54}(t) \odot V_4(t) + Q_{50}^{(6)}(t) \odot V_0(t) + Q_{53}^{(6)}(t) \odot V_3(t) \]

Taking Laplace-Stieltjes transform and solving the above equations for \( V_0^{**}(s) \) and in steady-state the number of visits per unit time is given by

\[ V_0 = \lim_{t \to \infty} \left[ \frac{V_0(t)}{t} \right] = \lim_{s \to 0} s V_0^{**}(s) = \frac{N_3}{D_1} \]  
\[ N_3 = p_{20} p_{52} + p_{50}^{(6)} \]  
where \( D_1 \) is already specified.

**Profit Analysis**

The expected total profit incurred to the system in a steady-state is

\[ P = K_0 A V_0 - K_1 B_0 - K_2 V_0 \]  
where \( K_0 \) is the revenue per unit up-time of the system, \( K_1 \) is the cost per unit time for which the expert repairman is busy, \( K_2 \) is the cost per unit visits by the expert repairman.

**Particular Case**

Assume that the preparation time for repair, repair time of expert repairman and instruction time are exponentially distributed as follows:

\[ h(t) = \alpha \exp(-\alpha t) \quad g(t) = \beta \exp(-\beta t) \quad i(t) = \gamma \exp(-\gamma t) \]
Then, we have

\[ p_{01} = 1, \quad p_{12} = \alpha / (\alpha + \lambda), \quad p_{13} = \lambda / (\alpha + \lambda), \quad p_{20} = \beta / (\beta + \lambda) \]

\[ p_{23} = \lambda / (\beta + \lambda), \quad p_{34} = p_{45} = 1, \quad p_{52} = \beta / (\beta + \mu), \quad p_{56} = \beta \mu / (\beta + \mu)(\beta + \lambda), \quad p_{57} = \lambda \mu / (\beta + \mu)(\beta + \lambda), \]

\[ \mu_0 = 1/\lambda, \quad \mu_1 = 1 / (\alpha + \lambda), \quad \mu_2 = 1 / (\beta + \lambda), \quad \mu_3 = 1 / \gamma = \text{(Mean instruction time)}, \]

\[ \mu_4 = 1 / \alpha = \text{(Mean preparation time)}, \quad \mu_5 = 1 / (\beta + \mu) \]

\[ \text{MTSF} = \frac{(\alpha + \beta)[(\beta + \lambda)(\alpha + \lambda) + \alpha \lambda]}{\lambda^2 (\alpha + \lambda)(\alpha + \beta + \lambda)} \]

Using the above equations and equations (17), (26), (34) and finally equation (36), we get the expected total profit in study state.

**Model 2**

This model is discussed with the additional assumption that either of the repairman (expert and his assistant) do not leave the repair of unit till its completion. The transition diagram showing the various rate of transition is given in Fig. 2. The various characteristics are obtained as follows:

![State Transition Diagram for Model – 2](image)

By probabilistic arguments, the non-zero elements \( p_{ij} \) are given below:

\[ p_{01} = p_{34} = p_{48} = 1, \quad p_{12} = h^*(\lambda), \quad p_{13} = (1 - h^*(\lambda)), \quad p_{20} = g^*(\lambda), \quad p_{23} = (1 - g^*(\lambda)) \]

\[ p_{50}^{(*)} = \mu [g^*(\mu) - g^*(\lambda)] / (\lambda - \mu), \quad p_{52}^{(*)} = [\mu g^*(\lambda) - \lambda g^*(\mu) + (\lambda - \mu)] / (\lambda - \mu) \]

\[ p_{57}^{(*)} = \mu g^*(\mu), \quad p_{57} = \mu / (\lambda + \mu), \quad p_{58} = \lambda / (\lambda + \mu), \quad p_{59}^{(*)} = \mu [h^*(\mu) - h^*(\lambda)] / (\lambda - \mu) \]

\[ p_{58}^{(*)} = \mu [h^*(\lambda) - \lambda h^*(\mu) + (\lambda - \mu)] / (\lambda - \mu), \quad p_{85} = h^*(\mu) \]

It can be verified that

\[ p_{12} + p_{13} = p_{20} + p_{23} = 1, \quad p_{50}^{(*)} + p_{57}^{(*)} = 1, \quad p_{52}^{(*)} + p_{53}^{(*)} + p_{57} = 1 \]

\[ p_{52}^{(*)} + p_{53}^{(*)} + p_{57} = 1 \]

Also \( \mu_1 \), the mean sojourn in state \( S_1 \) are

\[ \mu_0 = 1 / \lambda, \quad \mu_1 = [1 - h^*(\lambda)] / \lambda, \quad \mu_2 = [1 - g^*(\lambda)] / \lambda \]
\[ \mu_2 = \int_0^\infty I(t) dt = \int_0^\infty t \mu(t) dt = \text{(Mean instruction time)} \]

\[ \mu_4 = \int_0^\infty H(t) dt = \int_0^\infty t H(t) dt = \text{(Mean preparation time)} \]

\[ \mu_5 = [1 - g^*(\mu)] / \mu, \quad \mu_7 = 1 / (\lambda + \mu), \quad \mu_8 = [1 - h^*(\mu)] / \mu \] \hspace{1cm} (54–61)

The unconditional mean time taken by the system to transit for any regenerative state I, when it is counted from the epoch of entrance into that state is, mathematically, stated as

\[ m_i = \int_0^\infty t Q_i(t) dt = q_i(s) \bigg|_{s=0} \]

\[ m_{11} = 1 / \lambda, \quad m_{24} = i^*'(0), \quad m_{46} = -h^*(0) \]

\[ m_{12} = -h^*'(\lambda), \quad m_{13} = h^*'(\lambda) + [(1 - h^*(\lambda)) / \lambda] \]

\[ m_{20} = -g^*'(\lambda), \quad m_{23} = g^*'(\lambda) + [(1 - h^*(\lambda)) / \lambda] \]

\[ m_{50}^{(6)} = \mu [g^*'(\lambda) - g^*'(\lambda)] + [(1 - g^*(\lambda)) / \lambda] \]

\[ m_{53}^{(6)} = \left[ \frac{\lambda g^*'(\mu) - \mu g^*'(\mu)}{\lambda} - \frac{\mu (1 - g^*(\lambda))}{\lambda} \right] / (\lambda - \mu) \]

\[ m_{75} = g^*'(\lambda), \quad m_{70} = \mu / (\lambda + \mu)^2, \quad m_{78} = \lambda / (\lambda + \mu)^2 \]

\[ m_{82}^{(9)} = \mu [h^*'(\lambda) - h^*'(\mu)] / (\lambda - \mu) \]

\[ m_{83}^{(9)} = \left[ \lambda h^*'(\mu) - \mu h^*'(\mu) + \frac{\lambda (1 - h^*(\mu))}{\mu} - \frac{\mu (1 - h^*(\lambda))}{\lambda} \right] / (\lambda - \mu), \quad m_{85} = -h^*(\mu) \]

From these are conclude that

\[ m_{11} = \mu_0, \quad m_{34} = \mu_3, \quad m_{45} = \mu_4, \quad m_{12} + m_{13} = \mu_1, \quad m_{20} + m_{23} = \mu_2 \]

\[ m_{50}^{(6)} + m_{33}^{(6)} + m_{53} = [\lambda \mu_4 - \mu_2] / (\lambda - \mu) = m_1 (\text{say}) \]

\[ m_{70} + m_{75} = \mu_7, \quad m_{82}^{(9)} + m_{83}^{(9)} + m_{85} = [\lambda \mu_8 - \mu_4] / (\lambda - \mu) = m_2 (\text{say}) \] \hspace{1cm} (65–70)

\textbf{Mean Time to System Failure}

The expression for MTSF will remain same as explained in previous Model –1

\textbf{Availability Analysis}

\[ A_0(t) = M_0(t) + q_{01}(t) \odot A_1(t) \]

\[ A_1(t) = M_1(t) + q_{12}(t) \odot A_2(t) + q_{13}(t) \odot A_3(t) \]

\[ A_2(t) = M_2(t) + q_{20}(t) \odot A_0(t) + q_{23}(t) \odot A_3(t) \]

\[ A_3(t) = q_{34}(t) \odot A_4(t) \]

\[ A_4(t) = q_{45}(t) \odot A_5(t) \]

\[ A_5(t) = M_5(t) + q_{50}^{(6)}(t) \odot A_0(t) + q_{53}^{(6)}(t) \odot A_3(t) + q_{54}(t) \odot A_4(t) \]

\[ A_6(t) = M_6(t) + q_{60}(t) \odot A_0(t) + q_{63}(t) \odot A_3(t) + q_{64}(t) \odot A_4(t) \]

\[ A_7(t) = M_7(t) + q_{70}(t) \odot A_0(t) + q_{73}(t) \odot A_3(t) + q_{74}(t) \odot A_4(t) \] \hspace{1cm} (71–78)

Taking Laplace transform of equations and the letting \( s \to 0 \), we get

\[ M'_0(0) = \mu_0, \quad M'_1(0) = \mu_1, \quad M'_2(0) = \mu_2, \quad M'_7(0) = \mu_7 \]

\[ M'_5(0) = \mu [\mu_5 - \mu_2] (\lambda - \mu) = \epsilon_1 \text{ (say)} \]

\[ M'_8(0) = \mu [\mu_8 - \mu_4] / (\lambda - \mu) = \epsilon_2 \text{ (say)} \] \hspace{1cm} (79–84)

The steady-state availability of the system is given by

\[ A_0 = \lim_{s \to 0} [sA_0^*(s)] = N_1 / D_1 \] \hspace{1cm} (85)
where
\[
N_1 = [1 - p_{57} p_{78} p_{85} - (p_{53}^{(6)} + p_{57} p_{78} p_{85}^{(9)})] [\mu_0 + \mu_2 p_{12}] + [e_1 + p_{57} (\mu_7 + e_2 p_{78})]
\]
\[
\times [p_{12} p_{23} + p_{13}] - p_{57} p_{78} p_{85}^{(9)} [\mu_0 p_{23} + \mu_1 p_{23} - \mu_2 p_{13}]
\]
and
\[
D_1 = (\mu_0 + \mu_1) (p_{20} p_{57} p_{78} p_{85}^{(9)} + p_{57} p_{78} + p_{50}^{(8)})_+ + \mu_2 (p_{57} p_{78} p_{85}^{(9)} + p_{12} p_{57} p_{78} + p_{12} p_{50}^{(6)} +
\]
\[
(1 - p_{12} p_{20})_+ (\mu_3 + e_4) (1 - p_{57} p_{78} p_{85}) + m_1 + p_{57} (\mu_7 + m_2 p_{78})]
\]  
(86–87)

**Busy–Period Analysis For The Expert Repairman**

\[
B_0(t) = q_{01}(t) \odot B_1(t)
\]
\[
B_1(t) = W_1(t) + q_{12}(t) \odot B_2(t) + q_{13}(t) \odot B_3(t)
\]
\[
B_2(t) = W_2(t) + q_{20}(t) \odot B_0(t) + q_{23}(t) \odot B_3(t)
\]
\[
B_3(t) = W_3(t) + q_{34}(t) \odot B_4(t)
\]
\[
B_4(t) = W_4(t) + q_{45}(t) \odot B_5(t)
\]
\[
B_5(t) = W_5(t) + q_{50}^{(6)}(t) \odot B_0(t) + q_{53}^{(6)}(t) \odot B_3(t) + q_{57}^{(6)}(t) \odot B_7(t)
\]
\[
B_7(t) = W_7(t) + q_{70}(t) \odot B_0(t) + q_{73}(t) \odot B_3(t)
\]
\[
B_8(t) = W_8(t) + q_{82}^{(9)}(t) \odot B_2(t) + q_{83}^{(9)}(t) \odot B_3(t) + q_{85}(t) \odot B_5(t)
\]
(88–95)

Taking Laplace transform of equations and letting \(s \to 0\), we have
\[
W_1^*(0) = \mu_1, \quad W_2^*(t) = \mu_2, \quad W_3^*(0) = \mu_3, \quad W_4^*(0) = \mu_4, \quad W_5^*(0) = (\mu_5 + e_1), \quad W_6^*(0) = \mu_7
\]
\[
W_8^*(0) = (\mu_5 + e_2)
\]
(96–102)

In steady–state the total fraction of time for which the system is under repair is given by
\[
B_0^* = \lim_{s \to 0} [sB_0(s)] = N_2 / D_1
\]
(103)

Where
\[
N_2 = (1 - p_{57} p_{78} p_{85}) [\mu_1 + p_{12}(\mu_2 + p_{23}(\mu_3 + \mu_4)) + p_{13}(\mu_3 + \mu_4)] - [(p_{53}^{(6)} + p_{57} p_{78} p_{85}^{(9)})
\]
\[
\times (\mu_1 + \mu_2 p_{12} + (\mu_5 + e_1 + p_{57}(\mu_7 + (\mu_8 + e_2) p_{78})]) - p_{57} p_{78} p_{85}^{(9)} [\mu_0 p_{23} + \mu_1 p_{23} + \mu_2 p_{13} + p_{13}]
\]
(104)

and \(D_1\) is already specified.

**Expected Number Of Visits By The Expert Repairman**

\[
V_0(t) = Q_{01}(t) \odot [1 + V_1(t)]
\]
\[
V_1(t) = Q_{12}(t) \odot V_2(t) + Q_{13}(t) \odot V_3(t)
\]
\[
V_2(t) = Q_{20}(t) \odot V_0(t) + Q_{23}(t) \odot V_3(t)
\]
\[
V_3(t) = Q_{34}(t) \odot V_4(t)
\]
\[
V_4(t) = Q_{45}(t) \odot V_5(t)
\]
\[
V_5(t) = Q_{50}^{(6)}(t) \odot V_0(t) + V_3^{(6)}(t) \odot V_3(t) + Q_{57}(t) V_7(t)
\]
\[
V_7(t) = Q_{70}(t) \odot V_0(t) + Q_{73}(t) \odot [1 + V_3(t)]
\]
\[
V_8(t) = Q_{82}^{(9)}(t) \odot V_2(t) + Q_{83}^{(9)}(t) \odot V_3(t) + Q_{85}(t) \odot V_5(t)
\]
(105–112)

In steady–state, the expected number of visits per unit time is given by
\[
V_0 = \lim_{t \to \infty} \left[ \frac{V_0(t)}{t} \right] = \lim_{s \to 0} [sV_0^{**}(s)] = N_3 / D_1
\]
(113)

Where
\[
N_3 = 1 - p_{53}^{(6)} + p_{20} p_{57} p_{78} (p_{82}^{(9)} - p_{12}^{(6)}
\]
(114)
**Profit analysis**

The expected total profit incurred to the system in a steady-state is

\[ P_2 = K_0 A_0 - K_1 B_0 - K_2 V_0 \]

Where

- \( K_0 \) = Revenue per unit up time of the system
- \( K_1 \) = Cost per unit time for which the expert repairman is busy
- \( K_2 \) = Cost per visit by the expert repairman.

**Particular Case**

Assume that the preparation time to repair time of expert repairman and instruction time are exponentially distributed as follows:

\[ h(t) = \alpha \exp(-\alpha t), \quad g(t) = \beta \exp(-\beta t), \quad i(t) = \gamma \exp(-\gamma t) \]

Then, we have

\[ p_{01} = 1, \quad p_{12} = \frac{\alpha}{\alpha + \lambda}, \quad p_{13} = \frac{\lambda}{\alpha + \lambda}, \quad p_{20} = \frac{\beta}{\beta + \lambda}, \quad p_{23} = \frac{\lambda}{\beta + \lambda}, \]

\[ p_{34} = \frac{\beta}{\beta + \mu}, \quad p_{35} = \frac{\mu}{\beta + \mu} \]

\[ p_{47} = \beta / (\beta + \mu), \quad p_{70} = \mu / (\mu + \lambda), \quad p_{78} = \lambda / (\mu + \lambda) \]

\[ p_{82} = \alpha \mu / (\alpha + \mu)(\alpha + \lambda), \quad p_{83} = \lambda \mu / (\alpha + \mu)(\alpha + \lambda), \quad p_{85} = \mu / (\alpha + \mu) \]

Also, mean sojourn times are:

\[ \mu_0 = 1 / \lambda, \quad \mu_1 = 1 / (\alpha + \lambda), \quad \mu_2 = 1 / (\beta + \lambda), \quad \mu_3 = 1 / (\alpha + \mu) \]

\[ \mu_4 = 1 / (\beta + \mu), \quad \mu_5 = 1 / (\lambda + \mu), \quad \mu_7 = 1 / (\lambda + \mu) \]

\[ m_1 = (\alpha + \lambda + \mu) / (\alpha + \mu)(\alpha + \lambda), \quad m_2 = (\beta + \lambda + \mu) / (\beta + \mu)(\beta + \lambda) \]

\[ \varepsilon_1 = \mu / (\beta + \mu)(\beta + \lambda), \quad \varepsilon_2 = \mu / (\alpha + \mu)(\alpha + \lambda) \]

**Comparative Study of Profit for Model–1 and Model–2**

Fig. 3 shows the behaviour of profits of Model–1 and Model–2 with respect to failure rate (\( \lambda \)) for two values of instructions rate (\( \gamma = 1 \& 10 \)), keeping other parameters fixed. From the graph, it is interpreted that:

(i) Profit of each model decreases with an increase in failure rate (\( \lambda \)) while it increases with the increases in instructions rate (\( \gamma \)).

(ii) When \( \gamma = 1 \): If \( \lambda = 0.260585 \), the profits of two models coincide and equal to 340.836. If \( \lambda < 0.260585 \) then \( P_2 > P_1 \) i.e. Model–2 is better than Model–1. If \( \lambda > 0.260585 \) then \( P_1 > P_2 \) i.e. Model–1 is better than Model–2.

(iii) When \( \gamma = 10 \): If \( \lambda = 0.230832 \), the profits of both the models coincide and equal to 387.063. If \( \lambda < 0.230832 \) then \( P_2 > P_1 \) i.e. Model–2 is better than Model–1. If \( \lambda > 0.230832 \) then \( P_1 > P_2 \) i.e. Model–1 is better than Model–2.

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**Fig 3:** Comparison between profit of Model 1 and 2.
References
7. Ashok Kumar, Suresh K Gupta, Tuteja RK. Cost benefit analysis of a two-unit cold standby system with instruction time, IAPQR Transactions, 1997; 22(2):127-133.