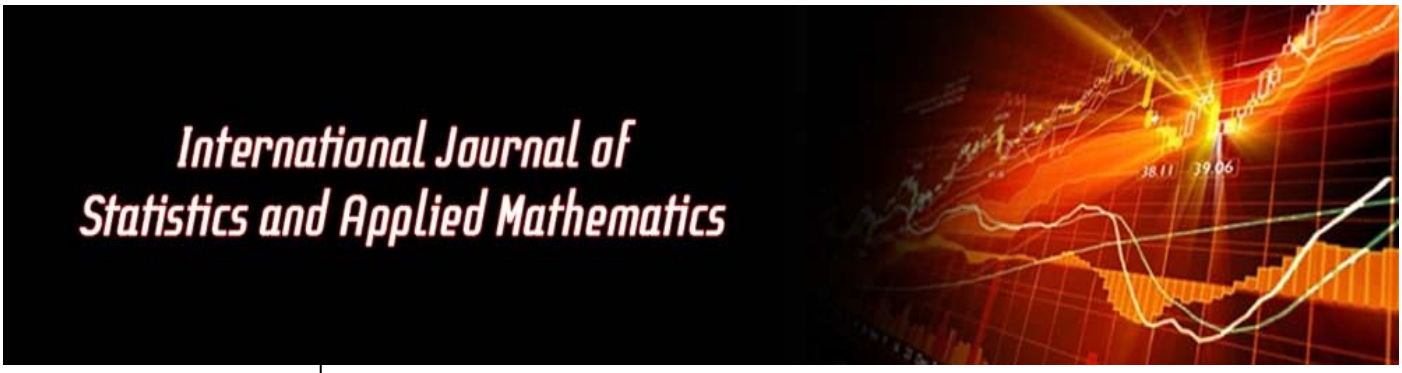
International Journal of Statistics and Applied Mathematics 2018; 3(2): 178-184



ISSN: 2456-1452
Maths 2018; 3(2): 178-184

www.mathsjournal.com
Received: 15-01-2018
Accepted: 17-02-2018

Anil Kumar
Department of Mathematics, Maharaja Agarsen Institute of Technology, Rohini, Delhi, India

Nagendra Kumar
Department of Mathematics, M.M.H. College, Ghaziabad, U.P., India

Correspondence
Anil Kumar
Department of Mathematics, Maharaja Agarsen Institute of Technology, Rohini, Delhi, India

Effect of time dependent background temperature on slow waves in viscous coronal plasma

Anil Kumar and Nagendra Kumar

Abstract
We study the effect of varying background temperature on small amplitude oscillations interpreted in terms of slow magneto hydrodynamic waves in solar coronal structures like prominences and coronal loops. We consider two damping mechanisms radiation and viscosity to study the behavior of slow MHD waves. We solve the MHD equations numerically to examine the effects of radiation and viscosity on velocity amplitude. It is found that amplitude of perturbed velocity decreases in case of increasing background temperature, whereas the perturbed velocity amplitude increases in case of decaying background temperature.

Keywords: Sun, MHD, Slow wave, Damping

1. Introduction
Solar observations made by SUMER on board SoHO (Wang *et al*., 2002 [1], 2003a [2]) have shown that solar atmosphere is composed of numerous magnetic structures and is dynamic in nature. These structures support a wide range of magnetohydrodynamic waves. Small amplitude oscillations observed in solar coronal loops can be interpreted in terms of linear magnetohydyrodynamic waves (De Moortel et. al 2002) [3]. The propagating intensity disturbances that were interprated as slow manetoacoustic waves have been detected in polar plumes firstly by Ofman *et al*. (1997) [4] using UVCS on board SoHO. The slow (propagating and standing) MHD waves have been seen to suffer a rapid damping (De Moortel 2009 [5], Wang *et al*. 2003a [2], 2003b [6]).
The fast damping of slow MHD waves has become a subject of remarkable attention to find out dominant damping mechanism. Thermal conduction, radiation and viscosity are the most studied damping mechanisms theoretically as well as numerically. The damping of slow MHD waves in a homogeneous medium has been discussed by De Moortel and Hood (2003) [7] by taking into account thermal conduction and compressive viscosity as the damping mechanisms. Numerical simulation performed by Ofman and Wang (2002) [8] shows that thermal conduction is dominant damping mechanism for standing slow waves.
In the absence the coronal heating, the plasma starts to cool by thermal conduction and radiation (Klimchuk., 2012) [9]. The effect of time dependent background temperature on standing slow MHD waves in hot coronal loop has been studied by Al-Ghafri and Erdelyi (2013) [10]. Recently, Ballester *et al*. (2016) [11] has studied the effect of a time dependent background temperature on prominence oscillations taking into account thermal conduction and radiative losses as damping mechanisms.
The present study investigates the behavior of slow MHD waves in an unbounded solar plasma in which temperature varies with time. We consider viscosity and radiation as damping mechanisms for slow MHD waves. The paper is structured as follows: In section 2, the basic equations governing the plasma motion in the atmosphere of the Sun are described. In section 3, numerical results are obtained. We draw our conclusions in section 4.

2. The model and governing equations
The basic MHD equations that describe the background plasma motion are

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \mathbf{v}) = 0 \tag{1}$$

$$\rho\left[\frac{\partial}{\partial t} + \mathbf{v}.\nabla\right]\mathbf{v} + \nabla p = \frac{(\nabla \times \mathbf{B}) \times \mathbf{B}}{\mu_0} + F_\upsilon \tag{2}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) \tag{3}$$

$$\frac{R}{\tilde{\mu}}\rho^\gamma\left[\frac{\partial}{\partial t}\frac{T}{\rho^{\gamma-1}} + (\mathbf{v}.\nabla)\frac{T}{\rho^{\gamma-1}}\right] = -(\gamma-1)L \tag{4}$$

$$p = \frac{R}{\tilde{\mu}}\rho T \tag{5}$$

Here $\mathbf{v}$, $\mathbf{B}$, ρ, p and T denote the velocity, magnetic field, density, gas pressure and temperature respectively; R is gas constant, $\tilde{\mu}$ is mean molecular weight, γ is the ratio of specific heats and μ_0 is the magnetic permeability of free space. The symbol F_υ represents the viscous force due to compressive viscosity taken to act along z-axis so that $F_\upsilon = \frac{4}{3}\frac{\partial}{\partial z}\left(\upsilon\frac{\partial v}{\partial z}\right)\hat{z}$ (Ofman *et al*. 1994 [12], Sigalotti *et al*. 2007 [13] Ofman and Wang 2002 [8]), where $v = 10^{-17}\ T^{5/2}$ kg m^{-1} s^{-1} is the compressive viscosity that is, viscosity parallel to magnetic field (Hollweg 1985 [14], Hood *et al*. 1989 [15]).

The symbol L is given by

$$L = L_r - H \tag{6}$$

encompassing radiation and heating terms L_r and H respectively. The magnetic field is considered to be uniform and horizontal, $\mathbf{B} = B_0\hat{z}$. Therefore the background state can be described as follows:

$$T_0 = T_0(t),\ p_0 = p_0(t),$$
$$\rho_0 = \text{const.},\ B_0 = \text{const.} \tag{7}$$

Here T_0, P_0, ρ_0 and B_0 are the background quantities identifying the temperature, pressure, density and magnetic field respectively. Assuming that there is no background flow, the MHD equations determining the background plasma state reduce to

$$\rho_0 = \text{const.}, v_0 = 0,\ \nabla p_0 = 0,$$
$$p_0 = \frac{R}{\tilde{\mu}}\rho_0 T_0,$$
$$\frac{R}{\tilde{\mu}}\rho_0\frac{dT_0}{dt} = -(\gamma-1)L. \tag{8}$$

Following Ballester *et al*. (2016) [11], we assume that radiation term is proportional to plasma temperature, $L_r = aT_0(t)$ and heating term is constant. Imposing T_{0i} as the initial temperature, we obtain from equation (8)

$$T_0(t) = \frac{H}{a} + \left(T_{0i} - \frac{H}{a}\right)\exp\left(-\frac{t}{\tau}\right), \tag{9}$$

where $\tau = \frac{R\rho_0}{(\gamma-1)\tilde{\mu}a}$ is the characteristic time that governs the rate at which plasma temperature is changing.

Considering small perturbations from the equilibrium, the field quantities can be written as

$$\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_1(\mathbf{r},t),\ \mathbf{v} = \mathbf{0} + \mathbf{v}_1(\mathbf{r},t),\ p = p_0(t) + p_1(\mathbf{r},t),$$
$$\rho = \rho_0 + \rho_1(\mathbf{r},t),\ T = T_0(t) + T_1(\mathbf{r},t)$$

where the subscripts "0" and "1" refer to equilibrium and perturbed quantities respectively. Taking into consideration the motion and propagation in the XZ plane, the linearised MHD equations are

$$\frac{\partial \rho_1}{\partial t} + \rho_0\left(\frac{\partial v_{1x}}{\partial x} + \frac{\partial v_{1z}}{\partial z}\right) = 0 \tag{10}$$

$$\rho_0\frac{\partial v_1}{\partial t} = -\nabla p_1 + \frac{1}{\mu_0}(\nabla \times B_1) \times B_0 + \frac{4}{3}\upsilon\frac{\partial^2 v_1}{\partial z^2} \tag{11}$$

$$\frac{\partial B_{1x}}{\partial t} = \frac{\partial(v_{1x}B_0)}{\partial z} \tag{12}$$

$$\frac{\partial B_{1z}}{\partial t} = -\frac{\partial\left(v_{1x}B_0\right)}{\partial x} \tag{13}$$

$$\frac{R}{\tilde{\mu}}\left(\rho_1\frac{\partial T_0}{\partial t} + \rho_0\frac{\partial T_1}{\partial t} + \left(\gamma-1\right)\rho_0 T_0\left(\frac{\partial v_{1x}}{\partial x} + \frac{\partial v_{1z}}{\partial z}\right)\right) = \left(\gamma-1\right)T_1 \tag{14}$$

$$p_1 = \frac{R}{\mu}\left(\rho_0 T_1 + \rho_1 T_0\right) \tag{15}$$

As we aim to study slow waves, we will consider the perturbed velocity component v_{1z}, parallel wave number k_z and $T_0(t)$ only. Hence all perturbations can be expressed as

$$f_1\left(x,z,t\right) = f_1(t)\exp^{ik_z} \tag{16}$$

where $f_1(t)$ is time dependent amplitude of the perturbation. The linearised equations (10) to (15) become

$$\frac{\partial \rho_1}{\partial t} = -ik_z\rho_0 v_{1z} \tag{17}$$

$$\frac{\partial v_{1z}}{\partial t} = \frac{-ik_z p_1}{\rho_0} - \frac{4}{3}k_z{}^2\upsilon v_{1z} \tag{18}$$

$$\rho_1\frac{\partial T_0}{\partial t} + \rho_0\frac{\partial T_1}{\partial t} + \left(\gamma-1\right)\rho_0 ik_z v_{1z} = \frac{\rho_0 T_1}{\tau} \tag{19}$$

$$p_1 = \frac{R}{\mu}\left(\rho_0 T_1 + \rho_1 T_0\right) \tag{20}$$

The above equations (17) - (20) govern the slow waves propagating along the magnetic field.

3. Results

We solve equations (17)-(20) numerically by using the following set of parameters:

$\rho_0 = 5\times10^{-11}\,\text{kgm}^{-3}, T_{01} = 10000\text{K}, B_0 = 10^{-3}T$ and initial conditions $v_{1z}\left(0\right) = 1, \rho_1\left(0\right) = 0$ and $T_1\left(0\right) = 0$.

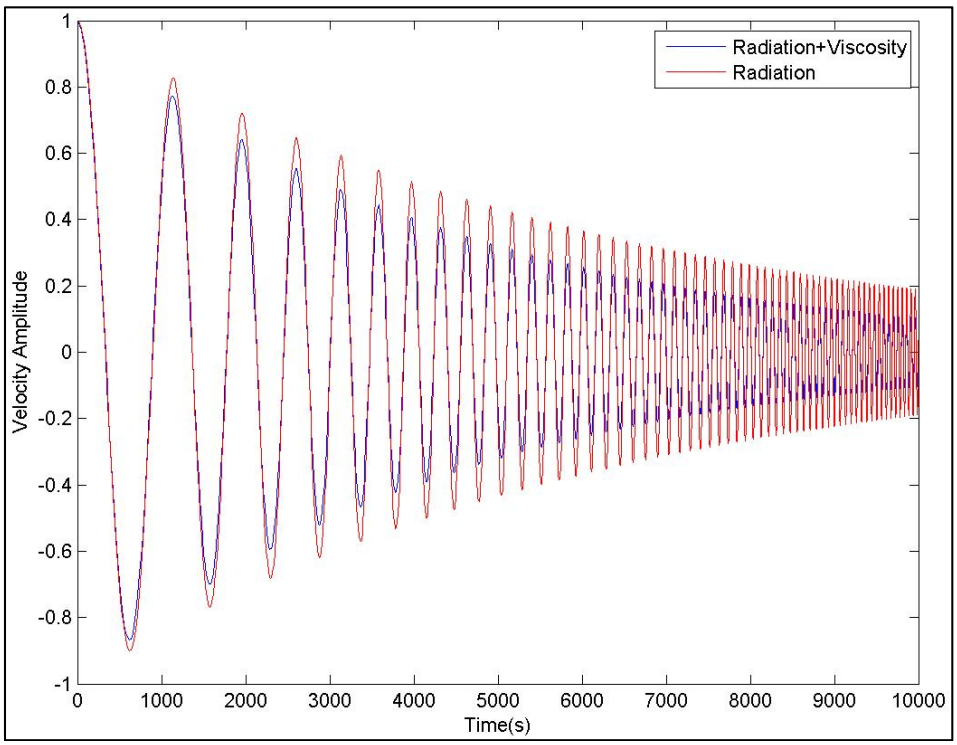


Fig 1: Temporal evolution of perturbed velocity for $\tau = 1500, T_{0i} = 10000\text{K}, k_z = 3\times10^{-7}\,\text{m}^{-1}$ H=$3.5\times10^{-5}\text{Wm}^{-3}$ with background temperature increasing with time.

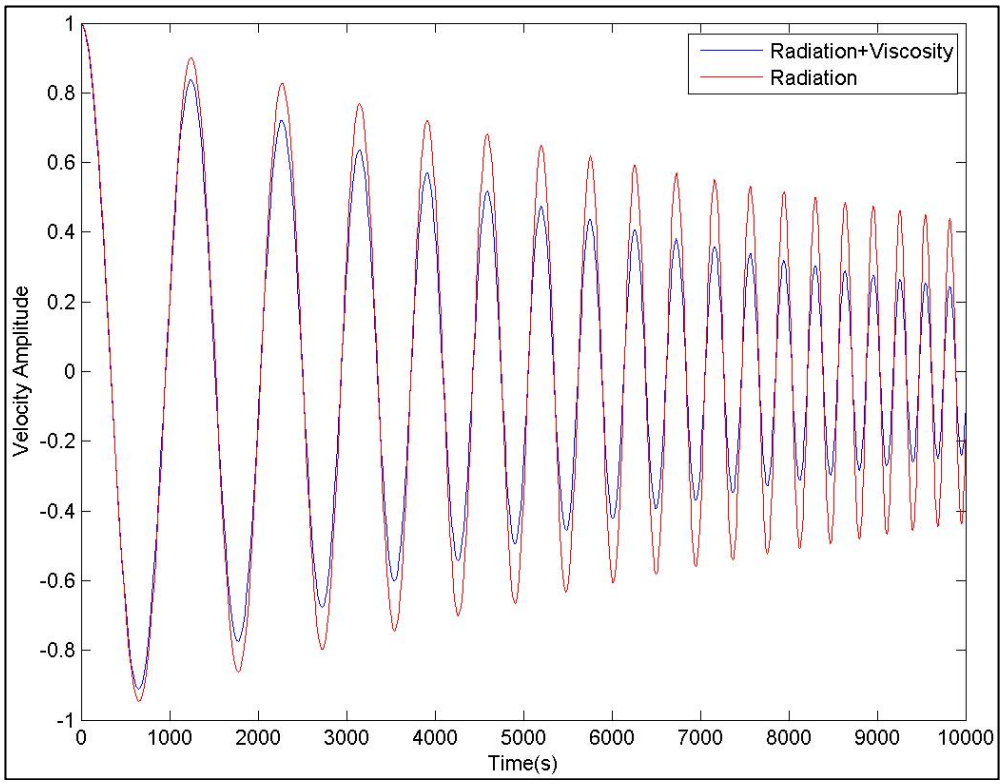


Fig 2: Temporal evolution of perturbed velocity for $\tau = 3000, T_{0i} = 10000\text{K}, k_z = 3\times10^{-7}\,\text{m}^{-1}$ $H = 3.5\times10^{-5}\,\text{Wm}^{-3}$ with background temperature increasing with time.

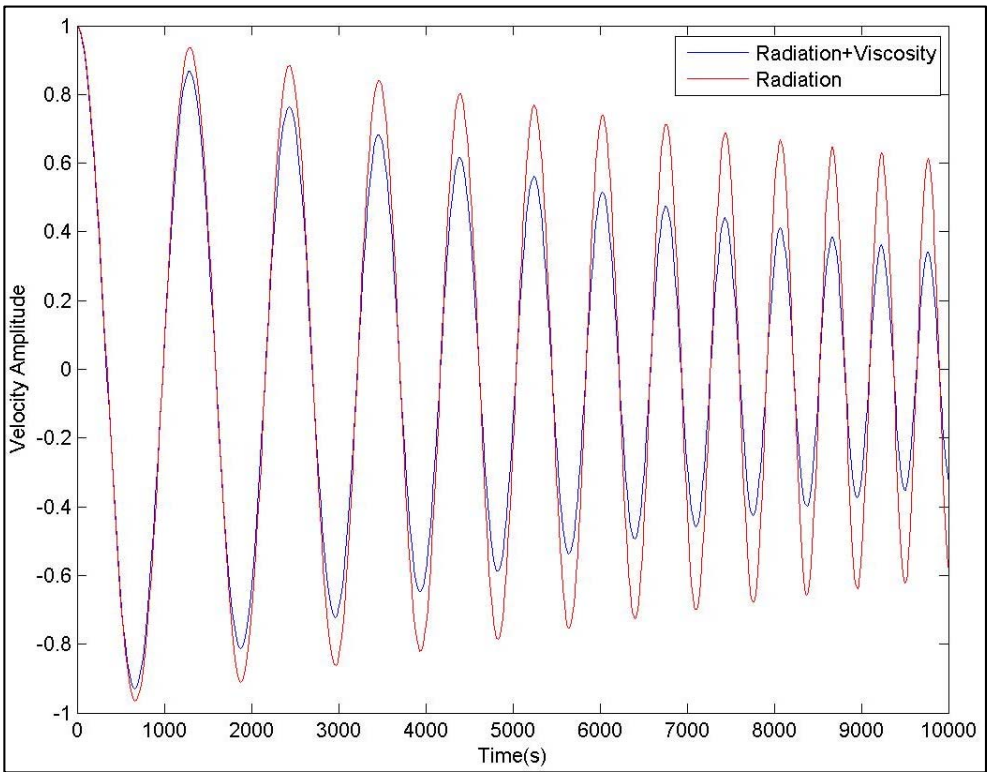


Fig 3: Temporal evolution of perturbed velocity for $\tau = 5000, T_{0i} = 10000\text{K}, k_z = 3\times10^{-7}\,\text{m}^{-1}$ $H = 3.5\times10^{-5}\,\text{Wm}^{-3}$ with background temperature increasing with time.

We plot amplitude of perturbed velocity vs. time to study the combined effects of radiation and viscosity on the propagation on slow MHD waves in figures 1 to 6. In each figure combined effect of viscosity and radiation is compared with the effect of radiation alone. Figures 1, 2 and 3 show the temporal behavior of normalized perturbed velocity for coronal plasma structure whose background temperature increases with time for different values of characteristic time τ. It is observed that inclusion of viscosity in addition to radiation further reduces the velocity amplitude and thus enhances the damping. The reduction in velocity amplitude is more prominent for lower values of characteristic time. The period of slow wave decreases with the increase in characteristic time τ. Inclusion of viscosity as additional damping mechanism does not alter the period.

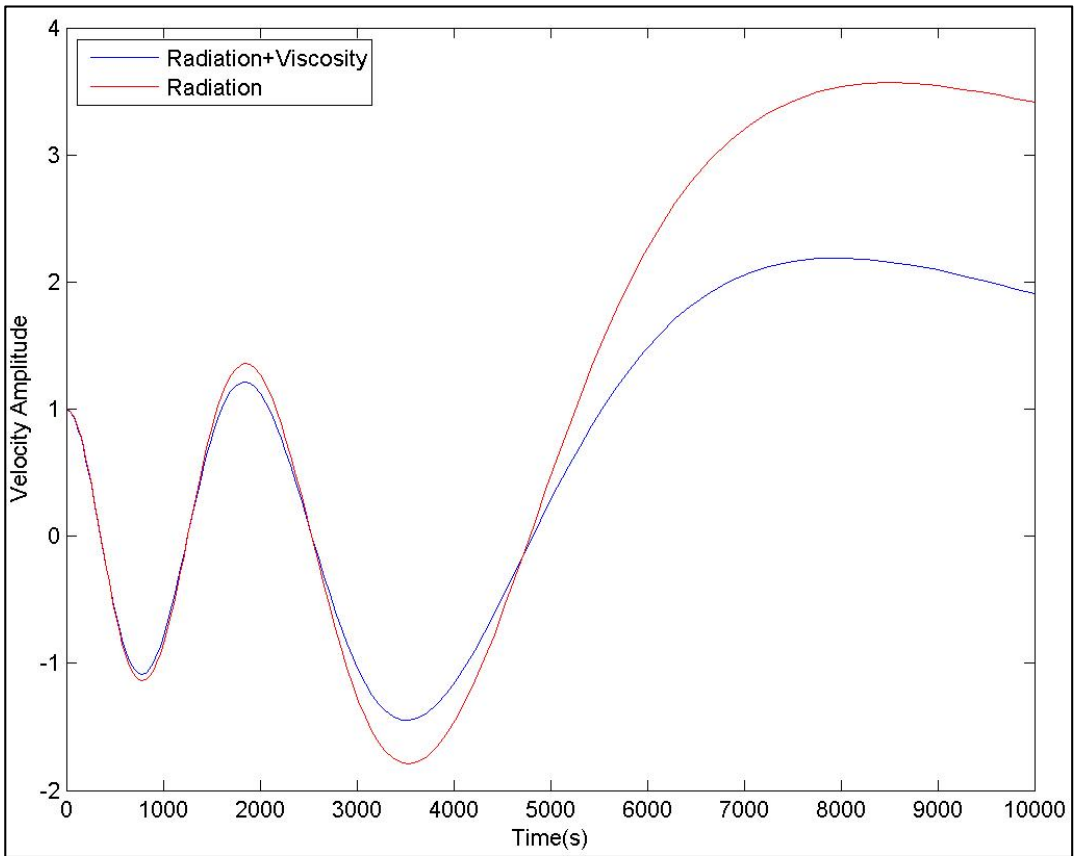


Fig 4: Temporal evolution of perturbed velocity for $\tau = 1500, T_{0i} = 10000\text{K}, k_z = 3\times10^{-7}\,\text{m}^{-1}$ $H = 3.5\times10^{-5}\,\text{Wm}^{-3}$ with background temperature decreasing with time.

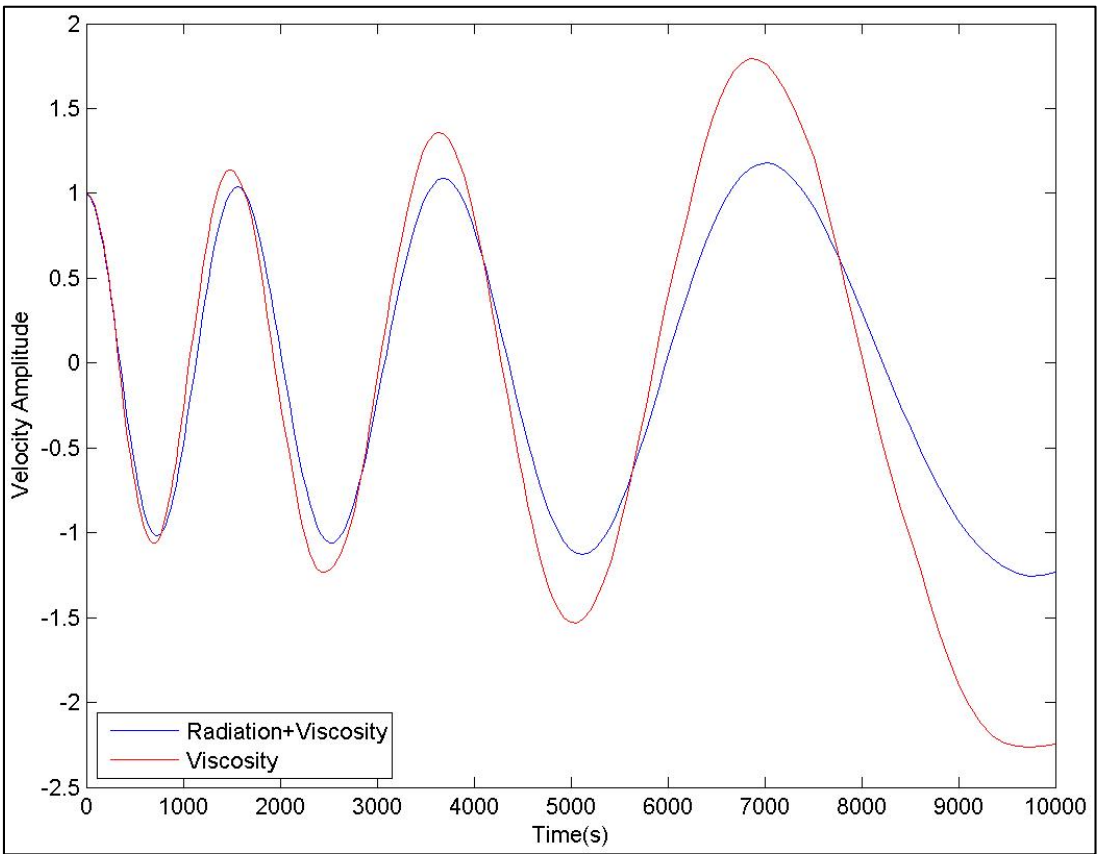


Fig 5: Temporal evolution of perturbed velocity for $\tau = 3000, T_{0i} = 10000\text{K}, k_z = 3\times10^{-7}\,\text{m}^{-1}$ $H = 3.5\times10^{-5}\,\text{Wm}^{-3}$ with background temperature decreasing with time.

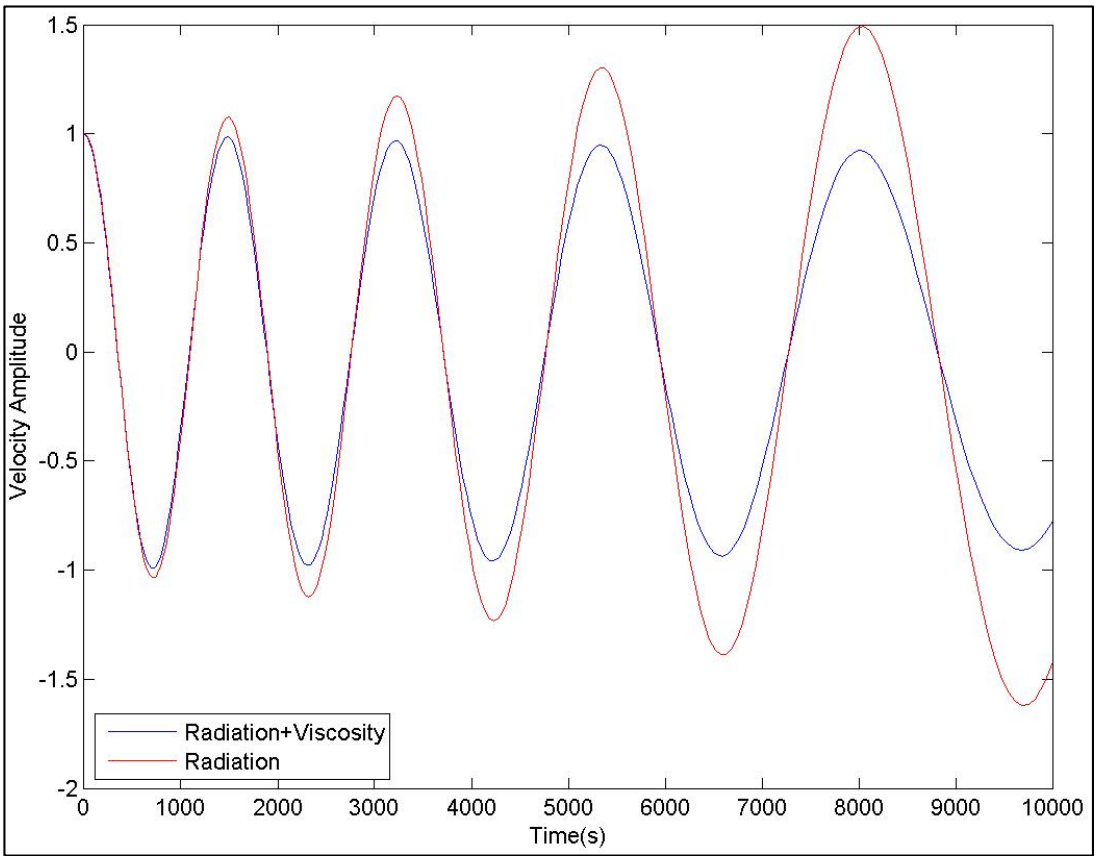


Fig 6: Temporal evolution of perturbed velocity for $\tau = 5000, T_{0i} = 10000\text{K}, k_z = 3\times10^{-7}\text{m}^{-1}$ $H = 3.5\times10^{-5}\text{Wm}^{-3}$ with background temperature decreasing with time.

Figures 4, 5 and 6 show the temporal behavior of normalized perturbed velocity for a coronal plasma structure having decreasing background temperature with time for different values of characteristic time τ. It can be seen from figures 4, 5 and 6 that viscosity contributes to damping in a similar way as in the case of increasing background temperature. The amplitude of perturbed velocity increases slowly with time. From figures 4, 5 and 6 we observe that perturbed velocity amplitude increases more rapidly for smaller values of characteristic time τ. Period of slow wave also increases with the decrease in background temperature.

4. Conclusions

In this paper, we have studied the propagation and damping of slow MHD waves propagating in solar coronal plasma with varying background temperature with time. Radiation and viscosity have been taken into account as damping mechanisms. The period of slow wave and velocity amplitude decrease with the increase in background temperature with time. The decrease in period and perturbed velocity amplitude is more prominent for smaller values of characteristic time τ. On the other hand, when background temperature decreases with time, both period and perturbed velocity amplitude increase with time. The period and perturbed velocity amplitude increase with the decrease in characteristic time τ.

5. References

1. Wang TJ, Solanki SK, Curdt W, Innes DE, Dammasch IE. Doppler shift oscillations of hot Solar coronal plasma seen by SUMER: A signature of loop oscillations?, Astrophys. J. 2002; 574:L101-L104.
2. Wang TJ, Solanki SK, Curdt W, Innes DE, Dammasch IE, Kliem B. Hot coronal loop oscillations observed with SUMER: Examples and statistics, Astron. Astrophys. 2003a; 406:1105-1121.
3. De Moortel I, Ireland J, Hood AW, Walsh RW. The detection of 3 & 5 min period oscillations in coronal loops, Astron. Astrophys. 2002; 387:L13-L16.
4. Ofman L, Romoli M, Poletto G, Noci G, Kohl JL. Ultraviolet coronagraph spectrometer observations of density fluctuations in the solar wind, Astrophys. J. 1997; 491:L111-L114.
5. De Moortel I. Longitudinal waves in coronal loops, Space Sci. Rev. 2009; 149:65-81.
6. Wang TJ, Solanki SK, Innes DE, Curdt W, Marsch E. Slow-mode standing waves observed by SUMER in hot coronal loops, Astron. Astrophys. 2003b; 402:L17-L20.
7. De Moortel I, Hood AW. The damping of slow MHD waves in solar coronal magnetic fields, Astron. Astrophys. 2003; 408:755-765.
8. Ofman L, Wang TJ. Hot coronal loop oscillations observed by SUMER: Slow magnetosonic wave damping by thermal conduction, Astrophys. J. 2002; 580:L85-L88.
9. Klimchuk JA. The role of type II spicules in the upper solar atmosphere. J Geophys. Res. 2012; 117:A12102.
10. Al-Ghafri KS, Erdélyi R. Effect of variable background on an oscillating hot coronal loop, Solar Phys. 2013; 283:413-428.

11. Ballester JL, Carbonell M, Soler R, Terradas J. Prominence oscillations: Effect of a time-dependent background temperature, Astron. Astrophys. 2016; 591:A109.
12. Ofman L, Davila JM, Steinolfson RS. Coronal heating by the resonant absorption of Alfven waves: The effect of viscous stress tensor, Astrophys. J. 1994; 421:360-371.
13. Sigalotti L, Di G, Mendoza-Briceño CA, Luna-Cardozo M. Dissipation of standing slow magnetoacoustic waves in hot coronal loops, Solar Phys. 2007; 246:187-212.
14. Hollweg JV. Viscosity in a magnetized plasma - Physical interpretation. J Geophys. Res. 1985; 90:7620-7622.
15. Hood AW, van der Linden R, Goossens M. A formulation of non-ideal localized (or ballooning) modes in the solar corona, Solar Phys. 1989; 120:261-283.