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On contra $\beta$wg-continuous functions in topological spaces

Abstract
The main aim of this paper is to define and study the notions of contra $\beta$wg-continuous, almost contra $\beta$wg-continuous functions and discussed the relationship with other contra continuous functions and obtained their characteristics. Further we introduce the concepts of contra $\beta$wg-irresolute, contra $\beta$wg-closed functions and obtain some of their properties.

Keywords: $\beta$wg-continuous, contra $\beta$wg-continuous, almost contra $\beta$wg-continuous, contra $\beta$wg-irresolute, contra $\beta$wg-closed functions, $\beta$wg-locally indiscrete space.

1. Introduction

Throughout this paper $\mathcal{(X, \tau)}$, $\mathcal{(Y, \sigma)}$ and $\mathcal{(Z, \eta)}$ (or simply $X$, $Y$, and $Z$) represent the non-empty topological spaces on which no separation axioms are assumed, unless otherwise mentioned. For a subset $A$ of $X$, the closure of $A$ and interior of $A$ will be denoted by $\text{Cl}(A)$ and $\text{Int}(A)$ respectively. The union of all $\beta$wg-open sets of $X$ contained in $A$ is called $\beta$wg-interior of $A$ and it is denoted by $\beta$wgInt $(A)$. The intersection of all $\beta$wg-closed sets of $X$ containing $A$ is called $\beta$wg-closure of $A$ and it is denoted by $\beta$wgCl $(A)$. Also the collection of all $\beta$wg-open subsets of $X$ containing a fixed point $x$ is denoted by $\beta$wg O $(X, x)$.

2. Preliminaries
We recall the following definitions which are useful in the sequel.

Definition 2.1 A subset $A$ of a topological space $\mathcal{(X, \tau)}$ is called
(i) Semi-open [11] if $A \subseteq \text{Cl}(\text{Int}(A))$ and semi-closed if $\text{Int}(\text{Cl}(A)) \subseteq A$.
(ii) Preopen [13] if $A \subseteq \text{Int}(\text{Cl}(A))$ and preclosed if $\text{Cl}(\text{Int}(A)) \subseteq A$.
(iii) $\alpha$-open [16] if $A \subseteq \text{Int}(\text{Cl}(\text{Int}(A)))$ and $\alpha$-closed if $\text{Cl}(\text{Int}(\text{Cl}(A))) \subseteq A$.
(iv) semi-preopen [1] (f-open) if $A \subseteq \text{Cl}(\text{Int}(\text{Cl}(A)))$ and
(v) Semi-preclosed (f-closed) if $\text{Int}(\text{Cl}(\text{Int}(A))) \subseteq A$.
(vi) Regular open [23] if $A = \text{Int}(\text{Cl}(A))$ and regular closed if $A = \text{Cl}(\text{Int}(A))$. 
Definition 2.2: A subset A of a topological space \((X, \tau)\) is called
(i) Generalized preclosed (briefly, gp-closed) \([12]\) if \(\text{pcl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open in \(X\).
(ii) Generalized semi-preclosed (briefly, gsp-closed) \([13]\) if \(\text{spcl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open in \(X\).
(iii) Generalized pre regular closed (briefly, gpr-closed) \([10]\) if \(\text{pcl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is regular open in \(X\).
(iv) Generalized star preclosed (briefly, g*p-closed set) \([23]\) if \(\text{pcl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is g-open in \(X\).
(v) Generalized pre star closed (briefly, gp*-closed set) \([10]\) if \(\text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is gp-open in \(X\).
(vi) \(\beta\)wg-closed \([14]\) if \(\beta cl(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is ag-open in \(X\).
(vii) Pre semi-closed \([25]\) if \(\text{spcl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is g-open in \(X\).

Definition 2.3: \([15]\) A function \(f: (X, \tau) \rightarrow (Y, \sigma)\) is called \(\beta\)wg-continuous if \(f^{-1}(V)\) is \(\beta\)wg-closed set in \((X, \tau)\) for every closed set \(V\) in \((Y, \sigma)\).

Definition 2.4: \([15]\) A function \(f: (X, \tau) \rightarrow (Y, \sigma)\) is called \(\beta\)wg-irresolute if \(f^{-1}(V)\) is \(\beta\)wg-closed set in \((X, \tau)\) for every \(\beta\)wg-closed set \(V\) in \((Y, \sigma)\).

Definition 2.5: A function \(f: (X, \tau) \rightarrow (Y, \sigma)\) is called
1. Contra continuous \([4]\) if \(f^{-1}(V)\) is closed in \(X\) for each open set \(V\) of \(Y\).
2. Contra pre-continuous \([10]\) if \(f^{-1}(V)\) is preclosed set in \(X\) for each open set \(V\) of \(Y\).
3. Contra semi-continuous \([5]\) if \(f^{-1}(V)\) is semi-closed set in \(X\) for each open set \(V\) of \(Y\).
4. Contra \(\alpha\)-continuous \([8]\) if \(f^{-1}(V)\) is \(\alpha\)-closed set in \(X\) for each open set \(V\) of \(Y\).
5. Contra pre-semi-continuous \([26]\) if \(f^{-1}(V)\) is pre semi-closed set in \(X\) for each open set \(V\) of \(Y\).
6. Contra \(g\)-continuous \([20]\) if \(f^{-1}(V)\) is \(g\)-closed set in \(X\) for each open set \(V\) of \(Y\).
7. Contra \(g\)-pre-continuous \([20]\) if \(f^{-1}(V)\) is \(g\)-pre-closed set in \(X\) for each open set \(V\) of \(Y\).
8. Contra \(rg\)-continuous if \(f^{-1}(V)\) is \(rg\)-closed set in \(X\) for each open set \(V\) of \(Y\).
9. Contra \(Ag\)-continuous if \(f^{-1}(V)\) is \(Ag\)-closed set in \(X\) for each open set \(V\) of \(Y\).
10. Contra \(Ag\)-pre-continuous if \(f^{-1}(V)\) is \(Ag\)-pre-closed set in \(X\) for each open set \(V\) of \(Y\).
11. Contra \(gsp\)-continuous \([20]\) if \(f^{-1}(V)\) is \(gsp\)-closed set in \(X\) for each open set \(V\) of \(Y\).
12. Contra \(gp\)-continuous \([21]\) if \(f^{-1}(V)\) is \(gp\)-closed set in \(X\) for each open set \(V\) of \(Y\).
13. Contra \((gsp)\)-continuous \([19]\) if \(f^{-1}(V)\) is \((gsp)\)-closed set in \(X\) for each open set \(V\) of \(Y\).
14. Contra \(g\)-pre-continuous \([18]\) if \(f^{-1}(V)\) is \(g\)-pre-closed set in \(X\) for each open set \(V\) of \(Y\).

Definition 2.6: A function \(f: (X, \tau) \rightarrow (Y, \sigma)\) is called
1. Perfectly continuous \([17]\) if \(f^{-1}(V)\) is clopen in \(X\) for every open set \(V\) of \(Y\).
2. Almost continuous \([22]\) if \(f^{-1}(V)\) is open in \(X\) for each regular open set \(V\) of \(Y\).
3. Almost \(\beta\)wg-continuous \([13]\) if \(f^{-1}(V)\) is \(\beta\)wg-open in \(X\) for each regular open set \(V\) of \(Y\).
4. Almost \(\beta\)wg-continuous \([13]\) if \(f^{-1}(V)\) is \(\beta\)wg-open in \(X\) for each regular open set \(V\) of \(Y\).
5. Pre-closed \([13]\) if \(f(U)\) is pre-closed in \(Y\) for each closed set \(U\) of \(X\).
6. Contra pre-closed \([2]\) if \(f(U)\) is pre-closed in \(Y\) for each open set \(U\) of \(X\).

Definition 2.7: Let \(A\) be a subset of a space \((X, \tau)\).
1. The set \(\bigcap \{U \in \tau | A \subseteq U\}\) is called the kernel of \(A\) and is denoted by \(\text{ker}(A)\).
2. The set \(\{F \in X/A \subseteq F, F\) is \(\beta\)-closed\} is called the \(\beta\)-closure of \(A\) and is denoted by \(\text{pcl}(A)\).

Lemma 2.8: The following properties hold for subsets \(A, B\) of a space \(X: \)
1. \(X \subseteq \text{ker}(A)\) if and only if \(U \cap A \neq \varnothing\) for any \(F \in C(X, x)\).
2. \(A \subseteq \text{ker}(A)\) and \(A = \text{ker}(A)\) if \(A\) is open in \(X\).
3. If \(A \subseteq B\), then \(\text{ker}(A) \subseteq \text{ker}(B)\).

Lemma 2.9: \([15]\) For \(x \in X, x \in [\beta \text{wg} \text{cl}(A))\) if and only if \(U \cap A \neq \varnothing\) for every \(\beta\)wg-open set \(U\) containing \(x\).
Proof: Necessary part: Suppose there exists \(\beta\)wg-open set \(U\) containing \(x\) such that \(U \cap A \neq \varnothing\). Since \(A \subseteq X - U\), \(\beta\)wg \(\text{cl}(A) \subseteq X-U\). This implies \(x \notin \beta\)wg \(\text{cl}(A)\). This is a contradiction.
Sufficiency part: Suppose that \(x \notin \beta\)wg \(\text{cl}(A)\). Then there exists \(\beta\)wg-closed subset \(F\) containing \(A\) such that \(x \notin F\). Then \(x \in X-F\) is \(\beta\)wg-open, \((X-F) \cap A = \varnothing\). This is a contradiction.

3. Contra \(\beta\)wg-Continuous Functions
In this section, we introduce and study new class of continuous functions called contra \(\beta\)wg-continuous functions and investigate some of their properties in the following.

Definition 3.1: A function \(f: X \rightarrow Y\) is called contra beta weakly generalised (briefly, \(\beta\)wg-continuous) continuous if \(f^{-1}(V)\) is \(\beta\)wg-closed set in \(X\) for every open set \(V\) in \(Y\).

Example 3.2: Let \(X = Y = \{a, b, c, d\}\) with topologies, \(\tau = \{X, \varnothing, \{a\}, \{b\}, \{a, b\}\}\) and

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\(\sigma = \{Y, \varphi, \{a,b,c\}, \{a,b,c,d\}\}. \text{ Now } \beta \omega g C(X) = \{X, \varphi, \{a\}, \{b\}, \{c\}, \{d\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a, c, d\}, \{b, c, d\}, \{a, c, d\}, X\}. \) Define a function \( f: X \rightarrow Y \) by \( f(a) = d, f(b) = a, f(c) = b \) and \( f(d) = c. \) Then \( f \) is contra \( \beta \omega g \)-continuous function, since every open set in \( Y \) is \( \beta \omega g \)-closed in \( X. \)

**Theorem 3.3:** Every contra continuous (resp. contra pre-continuous) function is contra \( \beta \omega g \)-continuous but not conversely.

**Proof:** Let \( U \) be an open set in \( Y \) then \( f^{-1}(U) \) is closed (resp. pre-closed) in \( X. \) Since every closed (resp. pre-closed) set is a \( \beta \omega g \)-closed set. Therefore \( f \) is contra \( \beta \omega g \)-continuous.

**Example 3.4:** Let \( X = \{a, b, c, d\} = Y \) with topologies \( \tau = \{X, \varphi, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}\} \) and \( \sigma = \{Y, \varphi, \{a\}, \{b\}, \{a,c\}, \{a,b,c\}\}. \) Define \( f: X \rightarrow Y \) by \( f(a) = c, f(b) = d, f(c) = b \) and \( f(d) = a. \) Then \( f \) is contra \( \beta \omega g \)-continuous but not contra continuous and contra pre-continuous. Since \( \{b,c\} \) is an open set in \( Y \) but \( f^{-1}(\{b,c\}) = \{a,c\} \) is \( \beta \omega g \)-closed but not closed and pre-closed in \( X. \)

We define the following

**Definition 3.5:** A function \( f: X \rightarrow Y \) is called contra \( \alpha g^* \)-continuous if \( f^{-1}(V) \) is \( \alpha \)-closed set in \( X \) for each open set \( V \) of \( Y \)

**Theorem 3.6:** Every contra \( \alpha g^* \)-continuous (resp. contra \( g^* \)-continuous) function is contra \( \beta \omega g \)-continuous but not conversely.

**Proof:** Let \( V \) be an open set in \( Y \) then \( f^{-1}(V) \) is \( \alpha g^* \)-closed (resp. \( g^* \)-closed) in \( X. \) Since every \( \alpha g^* \)-closed (resp. \( g^* \)-closed) set is an \( \beta \omega g \)-closed set. Therefore \( f \) is contra \( \beta \omega g \)-continuous.

**Example 3.7:** Let \( X = \{a, b, c, d\} = Y \) with topologies \( \tau = \{X, \varphi, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}\} \) and \( \sigma = \{Y, \varphi, \{a\}, \{b\}, \{a,c\}, \{a,b,c\}\}. \) Define \( f: X \rightarrow Y \) by \( f(a) = b, f(b) = a, f(c) = d \) and \( f(d) = b. \) Then \( f \) is contra \( \beta \omega g \)-continuous but not contra \( \alpha g^* \)-continuous and contra \( g^* \)-continuous, since \( \{b,c\} \) is an open set in \( Y \) but \( f^{-1}(\{b,c\}) = \{a,c\} \) is \( \alpha g^* \)-closed and \( g^* \)-closed in \( X. \)

**Theorem 3.8:** Every contra \( \beta \omega g \)-continuous function is contra pre-semi-continuous but not conversely.

**Proof:** Let \( U \) be an open set in \( Y \) then \( f^{-1}(U) \) is \( \beta \omega g \) - closed set in \( X. \) Since every \( \beta \omega g \)-closed set is pre semi-closed set then \( f^{-1}(U) \) is pre-semi-closed in \( X. \) Therefore \( f \) is contra pre-semi-continuous.

**Example 3.9:** Let \( X = \{a, b, c, d\} = Y \) with topologies \( \tau = \{X, \varphi, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}\} \) and \( \sigma = \{Y, \varphi, \{a\}, \{b\}, \{a,b\}, \{a,b,d\}\}. \) Define a function \( f: V \rightarrow Y \) by \( f(a) = a, f(b) = d, f(c) = c \) and \( f(d) = b. \) Then \( f \) is contra pre semi-continuous but not contra \( \beta \omega g \)-continuous, since \( \{a,b,d\} \) is an open set in \( Y \) but \( f^{-1}(\{a,b,d\}) = \{a,b,d\} \) is not \( \beta \omega g \)-closed in \( X. \)

**Theorem 3.10:** (i) Every contra \( \beta \omega g \)-continuous function is contra \( g^* \)-continuous.
1. Every contra \( \beta \omega g \)-continuous function is contra \( g^* \)-continuous.
2. Every contra \( \beta \omega g \) -continuous function is contra \( g \)-continuous.
3. Every contra \( \beta \omega g \)-continuous function is contra \( g^* \)-continuous.
4. Every contra \( \beta \omega g \)-continuous function is contra \( \alpha g^* \)-continuous (resp. \( g^* \)-continuous, \( g \)-continuous).

**Proof:** The proof is straightforward from the Definition 3.1 and Theorem 3.3.

**Remark 3.11:** The converses of Theorem 3.10, is not true as shown in the following examples.

**Example 3.12:** Let \( X = Y = \{a, b, c\}, \) \( \tau = \{X, \varphi, \{a\}, \{b\}, \{a,b\}\} \) and \( \sigma = \{Y, \varphi, \{a\}\}. \) Define \( f: X \rightarrow Y \) by \( f(a) = b, f(b) = b \) and \( f(c) = c. \) Then \( f \) is contra \( g^* \)-continuous but not contra \( \beta \omega g \)-continuous, since \( \{a,b\} \) is an open set in \( Y, f^{-1}(\{a,b\}) = \{a,b\} \) is \( g^* \)-closed but not \( \beta \omega g \)-closed in \( X. \)

**Example 3.13:** Let \( X = \{a,b,c,d\} = Y \) with topologies \( \tau = \{X, \varphi, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}\} \) and \( \sigma = \{Y, \varphi, \{a\}, \{a,b\}, \{a,b,d\}\}. \) Define \( f: X \rightarrow Y \) by \( f(a) = a, f(b) = a, f(c) = c \) and \( f(d) = d. \) Then \( f \) is contra \( g^* \)-continuous but not contra \( \beta \omega g \)-continuous, since \( \{a,b,d\} \) is open set in \( Y, f^{-1}(\{a,b,d\}) = \{a,b,d\} \) is \( g^* \)-closed but not \( \beta \omega g \)-closed in \( X. \)

**Example 3.14:** Let \( X = \{a, b, c\} \) with topologies, \( \tau = \{\varphi, \{a\}, X\} \) and \( \sigma = \{Y, \varphi, \{a\}, \{a,b\}, \{b\}\}. \) Define \( f: X \rightarrow Y \) by \( f(a) = a, f(b) = c \) and \( f(c) = b. \) Then \( f \) is contra \( g^* \)-continuous (resp. contra \( g^* \)-continuous, contra \( g^* \)-continuous, contra \( g^* \)-continuous, contra \( g^* \)-continuous) but not contra \( \beta \omega g \)-continuous, since \( \{a,b\} \) is an open set in \( Y, f^{-1}(\{a,b\}) = \{a,c\} \) is \( g^* \)-closed (resp. \( g^* \)-closed, \( g^* \)-closed, \( g^* \)-closed, \( g^* \)-closed) set but not \( \beta \omega g \)-closed in \( X. \)

Also, we define and obtain the following

**Definition 3.15:** A function \( f: X \rightarrow Y \) is called a
1. Contra \( rg \)-continuous if \( f^{-1}(V) \) is \( rg \)-closed set in \( X \) for each open set \( V \) of \( Y \)
2. Contra \( Ag \)-continuous if \( f^{-1}(V) \) is \( Ag \)-closed set in \( X \) for each open set \( V \) of \( Y \).
3. Contra \( Ag^* \)-continuous if \( f^{-1}(V) \) is \( Ag^* \)-closed set in \( X \) for each open set \( V \) of \( Y \).

**Theorem 3.16:** Every contra \( \beta \omega g \)-continuous function is contra \( rg \)-continuous but not conversely.

**Proof:** Let \( U \) be an open set in \( Y \) then \( f^{-1}(U) \) is \( \beta \omega g \)-closed set in \( X. \) Since every \( \beta \omega g \)-closed set is \( rg \)-closed set then \( f^{-1}(U) \) is \( rg \)-closed in \( X. \) Therefore \( f \) is contra \( rg \)-continuous.
Example 3.17: Let $X = \{a, b, c\}$, $\tau = \{\varnothing, \{a\}, X\}$ and $\sigma = \{\varnothing, \{c\}, \{a, c\}, Y\}$. Now $RGC(X) = P(X)$ and $\beta wgC (X) = \{X, \varnothing, \{b\}, \{c\}, \{a, b, c\}\}$. Define $f: X \rightarrow Y$ by $f(a) = c$, $f(b) = b$ and $f(c) = a$. Then $f$ is contra rg-continuous but not contra $\beta wg$-continuous, since $\{a, c\}$ is an open set in $X$, $f^{-1}(\{a, c\}) = \{a, c\}$ is rg-closed but not $\beta wg$-closed in $X$.

Theorem 3.17: If $f: X \rightarrow Y$ contra $(gsp)^*$-continuous then $f$ is contra $\beta wg$-continuous function but not conversely.

Proof: Let $G$ be an open set in $Y$. Since $f$ is contra $(gsp)^*$-continuous, then $f^{-1}(G)$ is $(gsp)^*$-closed set in $X$. Since every contra $(gsp)^*$-closed set is $\beta wg$-closed set then $f^{-1}(U)$ is $\beta wg$-closed in $X$. Therefore $f$ is contra $\beta wg$-continuous.

Example 3.18: Let $X = Y = \{a, b, c\}$, $\tau = \{\varnothing, \{c\}, \{a, c\}, X\}$ and $\sigma = \{\varnothing, \{a, b, c\}\}$. Define the function $f: X \rightarrow Y$ by $f(a) = b$, $f(b) = a$ and $f(c) = d$. Then $f$ is contra $\beta wg$-continuous but not contra $(gsp)^*$-continuous, since $\{c\}$ is an open set in $Y$, $f^{-1}(\{c\}) = \{c\}$ is $\beta wg$-closed but not $(gsp)^*$-closed in $X$.

Definition 3.19: A space $(X, \tau)$ is called $\beta wg$-locally indiscrete if every $\beta wg$-open set is closed.

Theorem 3.20: (i) If a function $f: X \rightarrow Y$ is $\beta wg$-continuous and $(X, \tau)$ is $\beta wg$-locally indiscrete then $f$ is contra continuous.

(ii) If a function $f: X \rightarrow Y$ is contra $\beta wg$-continuous and $(X, \tau)$ is $\beta wgT_\frac{1}{2}$ space then $f$ is contra continuous.

(iii) If a function $f: X \rightarrow Y$ is contra $\beta wg$-continuous and $(X, \tau)$ is $\beta wgT_\alpha$ space then $f$ is contra precontinuous.

Proof: (i) Let $G$ be open in $(Y, \sigma)$. By assumption, $f^{-1}(G)$ is $\beta wg$-open in $X$. Hence $f$ is contra continuous.

(ii) Let $G$ be open in $(Y, \sigma)$. By assumption, $f^{-1}(G)$ is $\beta wg$-closed in $X$. Since by definition, $X$ is $\beta wgT_\alpha$ space, $f^{-1}(G)$ is closed in $X$. Hence $f$ is contra continuous.

(iii) Let $G$ be open in $(Y, \sigma)$. By assumption, $f^{-1}(G)$ is $\beta wg$-closed in $X$. Since by definition, $X$ is $\beta wgT_\alpha$ space, $f^{-1}(V)$ is pre-closed in $X$. Hence $f$ is contra precontinuous.

Theorem 3.21: If a function $f: (X, \tau) \rightarrow (Y, \sigma)$ is contra $\beta wg$-continuous and $(X, \tau)$ is $\beta wgT_\alpha$-space then $f$ is contra g-continuous.

Proof: Let $V$ be open in $(Y, \sigma)$. By assumption, $f^{-1}(V)$ is $\beta wg$-closed in $X$. Since $X$ is $\beta wgT_\alpha$-space, $f^{-1}(V)$ is g-closed in $X$. Hence $f$ is contra g-continuous.

Theorem 3.22: If a function $f: (X, \tau) \rightarrow (Y, \sigma)$ is contra $\beta wg$-continuous and $(X, \tau)$ is $\beta wgT_\alpha$-space then $f$ is contra $\alpha$-continuous.

Proof: Let $V$ be open in $(Y, \sigma)$. By assumption, $f^{-1}(V)$ is $\beta wg$-closed in $X$. Since $X$ is $\beta wgT_\alpha$-space, $f^{-1}(V)$ is $\alpha$-closed in $X$. Hence $f$ is contra $\alpha$-continuous.

Theorem 3.23: The following are equivalent for a function $f: (X, \tau) \rightarrow (Y, \sigma)$:

1. For every closed subset $F$ of $Y$, $f^{-1}(F) \in \beta wg O(X)$;
2. $f(\beta wg Cl(A)) \subseteq ker(f(A))$ for every subset $A$ of $X$;
3. $\beta wg Cl(f^{-1}(B)) \subseteq f^{-1}(ker(B))$ for every subset of $B$ of $Y$.

Proof: The implications (i) $\Rightarrow$ (ii) is obvious.

(ii) $\Rightarrow$ (iii): Let $A$ be any subset of $X$. Suppose that $y \notin ker(f(A))$. Then by Lemma 2.10, there exists $F \in C(X, Y)$ such that $f(A) \cap F = \varnothing$. Thus $A \cap f^{-1}(F) = \varnothing$ and $\beta wg-cl(A) \cap f^{-1}(F) = \varnothing$. Therefore, we obtain $f(\beta wg-cl(A)) \cap f^{-1}(F) = \varnothing$ and $y \notin f(\beta wg-cl(A))$. This implies that $f(\beta wg-cl(A)) \subseteq ker(f(A))$.

(iii) $\Rightarrow$ (iv): Let $B$ be any subset of $Y$. By (iv) and Lemma 2.10, we have $f(\beta wg Cl(f^{-1}(B))) \subseteq ker(f(\beta wg Cl(f^{-1}(B)))) \subseteq ker(B)$ and $\beta wg Cl(f^{-1}(B)) \subseteq f^{-1}(ker(B))$.

(iv) $\Rightarrow$ (i): Let $V$ be any open set of $Y$. Then by Lemma 2.10, we have $\beta wg Cl(f^{-1}(V)) \subseteq f^{-1}(ker(V))$ and $\beta wg-cl(f^{-1}(V)) = f^{-1}(V)$. This shows that $f^{-1}(V)$ is $\beta wg$-closed in $X$.

Remark 3.24: The Composition of two contra $\beta wg$-continuous maps need not be contra $\beta wg$-continuous map and this can be shown by the following example.

Example 3.25: Let $X = Y = \{a,b,c,d\} = Z$ with topologies, $\tau = \{X, \varnothing, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}, \{a,b,c,d\}\}$ and $\sigma = \{Y, \varnothing, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}\}$. Define a functions $f: X \rightarrow Y$ by $f(a) = b$, $f(b) = a$, $f(c) = c$ and $g: X \rightarrow Z$ by $g(a) = d$, $g(b) = c$, $g(c) = b$ and $g(d) = a$. Then both $f$ and $g$ are contra $\beta wg$-continuous functions. But $gof$ is not contra $\beta wg$-continuous functions. For $gof$ is contra $\beta wg$-continuous. But $gof$ is not contra $\beta wg$-continuous map, since $\{b,c,d\}$ is an open set in $Z$, then $(gof)^{-1}(\{b,c,d\}) = f^{-1}(g^{-1}(\{b,c,d\})) = f^{-1}(\{a,b,c\}) = \{a,b,d\}$ is not a $\beta wg$-closed set in $X$. 
Remark 3.26: The following two examples will show that the concept of $\beta wg$-continuity and contra $\beta wg$-continuity are independent from each other.

Example 3.27: Let $X = Y = \{a, b, c, d\}$, $\tau = \{X, \emptyset, \{a, b, c\}, \{a, b, c, d\}\}$ and $\sigma = \{Y, \emptyset, \{a\}, \{a, b\}, \{a, b, c\}, \{a, b, c, d\}\}$. Define $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = d$, $f(b) = a$, $f(c) = b$ and $f(d) = c$. Then $f$ is contra $\beta wg$-continuous but $f$ is not $\beta wg$-continuous, since $\{a, d\}$ is a closed set in $Y$, $f^{-1}(\{a, d\}) = \{a, b\}$ is not a $\beta wg$-closed set in $X$.

Example 3.28: Let $X = Y = \{a, b, c, d\}$, $\tau = \{X, \emptyset, \{a\}, \{a, b\}\}$ and $\sigma = \{Y, \emptyset, \{a\}, \{b\}, \{a, b, c\}, \{a, b, c, d\}\}$. Define a function $f: X \rightarrow Y$ by $f(a) = c$, $f(b) = a$, $f(c) = b$ and $f(d) = d$. Then $f$ is $\beta wg$-continuous but $f$ is not contra $\beta wg$-continuous, since $\{a, c, d\}$ is an open set in $Y$, $f^{-1}(\{a, c, d\}) = \{a, b, d\}$ is not a $\beta wg$-closed set in $X$.

Theorem 3.29: If $f: X \rightarrow Y$ is a contra $\beta wg$ -continuous function and $g: Y \rightarrow Z$ is a continuous function then $g \circ f: X \rightarrow Z$ is contra $\beta wg$-continuous.

Proof: Let $U$ be an open set in $Z$. Then $g^{-1}(U)$ is open in $Y$. Since $f$ is contra $\beta wg$-continuous, $f^{-1}(g^{-1}(U)) = (g \circ f)^{-1}(U)$ is $\beta wg$-closed in $X$. Therefore $g \circ f: X \rightarrow Z$ is contra $\beta wg$-continuous.

Theorem 3.30: If $f: X \rightarrow Y$ is a $\beta wg$-irresolute function and $g: Y \rightarrow Z$ is a contra $\beta wg$-continuous function then $g \circ f: X \rightarrow Z$ is contra $\beta wg$-continuous function.

Proof: Let $G$ be an open set in $Z$. Then $g^{-1}(G)$ is $\beta wg$-closed in $Y$. Since $f$ is $\beta wg$-irresolute, $f^{-1}(g^{-1}(G)) = (g \circ f)^{-1}(G)$ is $\beta wg$-closed in $X$. Therefore $g \circ f: X \rightarrow Z$ is contra $\beta wg$-continuous function.

Theorem 3.31: If $f: X \rightarrow Y$ is a $\beta wg$-irresolute function and $g: Y \rightarrow Z$ is a contra continuous function then $g \circ f: X \rightarrow Z$ is contra $\beta wg$-continuous.

4. Approximately $\beta wg$- Continuous Maps

Now, we define the following

Definition 4.1: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be approximately-$\beta wg$-continuous (briefly, ap-$\beta wg$-continuous) if $\beta cl (F) \subseteq f^{-1}(U)$ whenever $U$ is an open subset of $Y$ and $F$ is a $\beta wg$-closed subset of $X$ such that $F \subseteq f^{-1}(U)$.

Definition 4.2: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be approximately-$\beta wg$ - closed (briefly, ap-$\beta wg$-closed) function if $f(F) \subseteq \beta int(V)$ whenever $V$ is an open subset of $Y$ and $F$ is a $\beta wg$-closed subset of $X$ such that $F \subseteq f^{-1}(V)$.

Definition 4.3: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be approximately-$\beta wg$-open (briefly, ap-$\beta wg$-open) if $\beta cl (F) \subseteq f(U)$ whenever $U$ is an open subset of $X$, $F$ is a $\beta wg$ - closed subset of $Y$ and $F \subseteq f(U)$.

Definition 4.4: A function $f: X \rightarrow Y$ is said to be contra $\beta wg$-closed (resp. contra $\beta wg$-open) if $f(U)$ is $\beta wg$-open (resp $\beta wg$-closed) in $Y$ for each closed (resp. open) set $U$ of $X$.

Theorem 4.6: Let $f: X \rightarrow Y$ be a function then
1. If $f$ is contra precontinuous, then $f$ is an ap-$\beta wg$-continuous.
2. If $f$ is contra preclosed, then $f$ is ap-$\beta wg$-closed.
3. If $f$ is contra preopen, then $f$ is ap-$\beta wg$-open.

Proof: (i) Let $F \subseteq f^{-1}(U)$ where $U$ is a open subset in $Y$ and $F$ is a $\beta wg$-closed subset of $X$. Then $\beta cl (F) \subseteq pcl (f^{-1}(U))$. Since $f$ is contra precontinuous, $\beta cl (F) \subseteq pcl (f^{-1}(U)) = f^{-1}(U)$. This implies $f$ is ap-$\beta wg$-continuous.
(ii) Let $f(F) \subseteq V$, where $F$ is a closed subset of $X$ and $V$ is a $\beta wg$-open subset of $Y$. Therefore $f(F) = \beta int(f(F)) \subseteq \beta int(V)$. Hence $f$ is ap-$\beta wg$-closed.
(iii) Let $F \subseteq f(U)$ where $F$ is a $\beta wg$-closed subset of $Y$ and $U$ is an open subset of $X$. Since $f$ is contra preopen, $f(U)$ is preclosed in $Y$ for each open set $U$ of $X$. Thus $\beta cl (F) \subseteq pcl(f(U)) = f(U)$. Therefore $f$ is ap-$\beta wg$-open.

Theorem 4.7: If a function $f: X \rightarrow Y$ is ap-$\beta wg$-continuous and preclosed function, then the image of each $\beta wg$-closed set in $X$ is $\beta wg$-closed set in $Y$.

Proof: Let $F$ be a $\beta wg$-closed subset of $X$. Let $f(F) \subseteq V$ where $V$ is an open subset of $Y$. Then $F \subseteq f^{-1}(V)$. Since $F$ is ap-$\beta wg$-continuous, $\beta cl (F) \subseteq f^{-1}(V)$. Thus $f(\beta cl(F)) \subseteq V$. Therefore, we have $\beta cl (f(F)) \subseteq \beta cl(f(\beta cl(F))) = f(\beta cl(F)) \subseteq V$. Hence $f(F)$ is $\beta wg$-closed set in $Y$.

Definition 4.8: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called a contra $\beta wg$-irresolute if $f^{-1}(V)$ is $\beta wg$-closed in $X$ for each $\beta wg$-open set $V$ in $Y$.

Definition 4.9: A space $(X, \tau)$ is said to be $\beta wg$-Lindelof if every cover of $X$ by $\beta wg$-open sets has a countable sub cover.
Theorem 4.10: Let f: X → Y and g: Y → Z be two functions such that gof: X → Z.
1. If g is βwg-continuous and f is contra βwg-irresolute then gof is contra βwg-continuous.
2. If g is βwg-irresolute and f is contra βwg-irresolute, then gof is contra βwg-irresolute.

Proof: (i) Let V be closed set in Z. Then g⁻¹(V) is βwg-closed in Y. Since f is contra βwg-irresolute, f⁻¹(g⁻¹(V)) is βwg-open in X. Hence gof is contra βwg-continuous.
(ii) Let V be βwg-closed in Z. Then g⁻¹(V) is βwg-closed in Y. Since f is contra βwg-irresolute, f⁻¹(g⁻¹(V)) is βwg-open in X. Hence gof is contra βwg-irresolute.

Theorem 4.11: Let f: (X, τ) → (Y, σ) and g: (Y, η) → (Z, η) be two functions such that g o f: (X, τ) → (Z, η).
1. If f is closed and g is ap-βwg-closed then gof is ap-βwg-closed.
2. If f is βwg-closed and g is βwg-open and g⁻¹ preserves βwg-openclosed then gof is ap-βwg-closed.
3. If f is ap-βwg-continuous and g is continuous then gof is ap-βwg-continuous.

Proof: (i) Suppose B is an arbitrary closed subset in X and A is a βwg-open subset of Z for which (g o f)(B) ⊆ A. Then f(B) is closed in Y because f is closed. Since g is ap-βwg-closed, g(f(B)) ⊆ βint(A). This implies gof is ap-βwg-closed.
(ii) Suppose B is an arbitrary closed subset of X and A is a βwg-open subset of Z for which (gof)(B) ⊆ A. Hence f(B) ⊆ βint(g⁻¹(A)). Then F(B) ⊆ βint(g⁻¹(A)) because g⁻¹(A) is βwg-open and f is ap-βwg-continuous. Hence (gof)(B) = g(f(B)) ⊆ βint(g⁻¹(A)) ⊆ βint(g⁻¹(A)). This implies that gof is ap-βwg-continuous.
(iii) Suppose F is arbitrary βwg-closed subset of X and U is open in Z for which F ⊆ (g⁻¹)(U). Then g⁻¹(U) is open in Y, because g is continuous. Since f is ap-βwg-continuous then we have β Cl(F) ⊆ βint(g⁻¹(U)) = (gof)(U). This shows that gof is ap-βwg-continuous.

Next, we define almost contra βwg-continuous functions in the followings.

Definition 4.12: A function f: X → Y is called almost contra βwg-continuous if f⁻¹(U) is βwg-closed set in X for every regular open set U in Y.

Example 4.13: Let X = Y = \{a, b, c, d\}, τ = \{X, {a}, \varnothing\} and σ = \{\varnothing, \{a\}, \{b, c\}, \{a, b, c\}, Y\}. βwgC(X) = \{X, \varnothing, \{a\}, \{b\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\} and regular open (Y) = \{Y, \varnothing, \{a\}, \{b, c\}\}. Now, define a function f: X → Y by f(a) = a, f(b) = b, f(c) = d and f(d) = c. Then f is almost contra βwg-continuous function.

Theorem 4.14: Every contra βwg-continuous function is almost contra βwg-continuous but not conversely.

Proof: Let G be a regular open set in Y. Since every regular open set is open then G is an open set in Y. Since f is contra βwg-continuous function then f⁻¹(G) is βwg-closed set in X. Therefore f is almost contra βwg-continuous.

Example 4.15: In above Example 4.13, f is almost contra βwg-continuous but not contra βwg-continuous. Since \{a, b, c\} is an open set in Y, f⁻¹(\{a, b, c\}) = \{a, b, d\} is not a βwg-closed set in X.

Theorem 4.16: Let f: (X, τ) → (Y, σ) be for a function. Then the following statements are equivalent:
1. F is almost contra βwg-continuous.
2. F⁻¹(F) ∈ βwg O(X, τ) for every F ∈ RC(Y, σ).
3. F⁻¹(βint (G)) ∈ βwgC (X, τ) for every open subset G of Y.
4. F⁻¹(βint (F)) ∈ βwgO (X, τ) for every closed subset F of Y.

Proof: (i) ⇒ (ii) Let F ∈ RC(Y, σ). Then Y-F ∈ RO(Y, σ) by assumption.
Hence f⁻¹(Y-F) = X-f⁻¹(F) ∈ βwgC(X, τ). This implies that f⁻¹(F) ∈ βwgO(X, τ).
(ii) ⇒ (i) Let V ∈ RO(Y, σ). Then by assumption (Y-V) ∈ RC(Y, σ).
Hence f⁻¹(Y-V) = X-f⁻¹(F) ∈ βwgO(X, τ). This implies that f⁻¹(F) ∈ βwgC(X, τ).
(i) ⇒ (iii) Let G be a open subset of Y. Since int (βint (G)) is regular open then by (i), f⁻¹(int (βint (G))) ∈ βwg-C(X, τ).
(ii) ⇒ (i) Let V ∈ RO(Y, σ). Then V is open in Y. By (ii), f⁻¹(int (βint (G))) ∈ βwgC (X, τ).
This implies that f⁻¹(V) ∈ βwg C(X, τ)
(ii) ⇒ (iv) is similar as (i) ⇒ (iii).

Theorem 4.17: If f: X → Y is an almost contra βwg-continuous function and A is a open subset of X, then the restriction f/A : A→Y is almost contra βwg-continuous.

Proof: Let F ∈ RC(Y). Since f is almost contra βwg-continuous, f⁻¹(F) ∈ βwg -O(X).Since A is open, it follows that (f/A)⁻¹(F) = A ∩ f⁻¹(F) ∈ βwgO(A).Therefore f/A is an almost contra βwg-continuous.

Definition 4.18: A function f: (X, τ) → (Y, σ) is called regular set connected if f⁻¹(U) is clopen in X for every regular open set U in Y.

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Theorem 4.19: If a function \( f: (X, \tau) \rightarrow (Y, \sigma) \) is almost contra \( \beta_{wg} \)-continuous and almost continuous then \( f \) is regular set connected.

**Proof:** Let \( U \) be a regular open set in \( Y \). Since \( f \) is almost contra \( \beta_{wg} \)-continuous and almost continuous then \( f^{-1}(U) \) is \( \beta_{wg} \)-closed and open. Hence \( f^{-1}(U) \) is clopen. Therefore, \( f \) is regular set connected.

Theorem 4.20: Let \( f: X \rightarrow Y \) and \( g: Y \rightarrow Z \) be two functions. Then the following properties hold.
1. If \( f \) is almost contra \( \beta_{wg} \)-continuous and \( g \) is regular set connected, then \( gof: X \rightarrow Z \) is almost contra \( \beta_{wg} \)-continuous and almost \( \beta_{wg} \)-continuous.
2. If \( f \) is almost contra \( \beta_{wg} \)-continuous and \( g \) is perfectly continuous, then \( gof: X \rightarrow Z \) is \( \beta_{wg} \)-continuous and contra \( \beta_{wg} \)-continuous.

**Proof:**
(i) Let \( U \) be regular open in \( Z \). Since \( g \) is regular set connected, \( g^{-1}(U) \) is clopen in \( Y \). Since \( f \) is almost contra \( \beta_{wg} \)-continuous, \( f^{-1}(g^{-1}(U)) = (gof)^{-1}(U) \) is \( \beta_{wg} \)-open and \( \beta_{wg} \)-closed. Therefore \((g \circ f)\) is almost contra \( \beta_{wg} \)-continuous and almost \( \beta_{wg} \)-continuous.

(ii) Let \( G \) be open in \( Z \). Since \( g \) is perfectly continuous, \( g^{-1}(G) \) is clopen in \( Y \). Since \( f \) is almost contra \( \beta_{wg} \)-continuous, \( f^{-1}(g^{-1}(G)) = (gof)^{-1}(G) \) is \( \beta_{wg} \)-open and \( \beta_{wg} \)-closed. Hence \( gof \) is contra \( \beta_{wg} \)-continuous and \( \beta_{wg} \)-continuous function.

5. Conclusion
In this research article, we have focused on contra \( \beta_{wg} \)-continuity and its characteristics and contra \( \beta_{wg} \)-irresolute in topological spaces. Further with help these functions almost contra \( \beta_{wg} \)-continuous functions, contra \( \beta_{wg} \)-closed functions were studied.

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7. References
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