A view on rb-convergence in multiset topological spaces

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Abstract
This paper deals with the concepts of regular b-closed M-sets and regular b-continuous M-set functions are studied with necessary examples. Also the concepts of rb-convergence to a point, rb-accumulates to a point, rb-S-closed M-spaces are studied and some interesting properties are discussed. Finally, sequence of rb-convergence and rb-Hausdorff M-spaces are discussed.

Keywords: regular b-closed M-sets, regular b-continuous M-set functions, rb-convergence, rb-accumulates, rb-S-closed M-spaces

2010 AMS Subject Classification: 54A05, 54A10, 54A20

1. Introduction

The notion of M-topological space and the concept of open M-sets are introduced by Girish and sunil Jacob John [4]. Moreover, topologies on multisets can be associated to IC-bags or nk-bags introduced by Chakrabarty [3] (Chakrabarty, 2000; Chakrabarty and Despi, 2007) with the help of rough set theory. The association of rough set theory and topologies on multisets through bags with interval counts (Chakrabarty and Despi, 2007) can be used to develop theoretical study of covering based rough sets with respect to universe as multisets. Nagaveni and Narmadha [9, 10] introduced the notions of regular b-closed (rb-closed) sets in topological space. Regular open sets and b-open sets have been introduced and investigated by Stone [12] and Andrijević [11] Generalized α-closed sets have been introduced and investigated Maki, Devi and Balachandran [8].

2. Preliminaries

Definition 2.1 [4]: Let $M \in [X]$, and $\tau \subseteq P^*_M(M)$. Then $\tau$ is called a Multiset topology of $M$ if $\tau$ satisfies the following properties.

1. The M-set M and the empty M-set $\phi$ are in $\tau$.
2. The M-set union of the elements of any sub collection of $\tau$ is in $\tau$.
3. The M-set intersection of the elements of any finite sub collection of $\tau$ is in $\tau$.

Definition 2.2 [4]: Given a subM-set $A$ of an M-topological space $M$ in $[X]$, the interior of $A$ is denoted by $\text{Int}(A)$ and it is is defined as follows: $\text{Int}(A) = \bigcup \{ G \subseteq M : G$ is an open M-set and $G \subseteq A \}$. and $C_{\text{Int}}(A)(x) = \max \{ C_G(x) : G \subseteq A, G \in \tau \}$.

Definition 2.3 [4]: Given a subM-set $A$ of an M-topological space $M$ in $[X]$, the closure of $A$ is denoted by $\text{Cl}(A)$ and is defined as follows: $\text{Cl}(A) = \bigcap \{ K \subseteq M : K$ is a closed M-set and $A \subseteq K \}$ and $C_{\text{Cl}}(A)(x) = \min \{ C_K(x) : A \subseteq K, K \in \tau^c \}$.

3. On rb-Closed M-Sets And rb-Continuous M-Set Functions

Definition 3.1: Let $(M, \tau)$ be an M-topological space in $[X]^\text{rb}$. Any subM-set $A$ of $M$ is called a regular open M-set if $A = \text{Int}(\text{Cl}(A))$ with $C_A(x) = C_{\text{Int}(\text{Cl}(A))}(x)$, for all $x \in X$. 

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Definition 3.2: Let \((M, \tau)\) be an \(M\)-topological space in \([X]^{W}\). Any sub\(M\)-set \(A\) of \(M\) is called a regular closed \(M\)-set if \(A = \text{cl}(\text{int}(A))\) with \(C_A(x) = C_{\text{cl}(\text{int}(A))}(x)\), for all \(x \in X\).

Definition 3.3: Let \((M, \tau)\) be an \(M\)-topological space in \([X]^{W}\) and \(A\) be any sub\(M\)-set of \(M\). The \(M\)-set union of all regular open \(M\)-sets of \((M, \tau)\) contained in \(A\) is called the regular closure of \(A\) and is denoted by \(rcl(A)\). i.e. \(rcl(A) = \{ G \subseteq M : \text{each } G \text{ is a regular open } M\text{-set and } G \subseteq A \} \) with \(C_{C_{\text{rcl}(A)}}(x) = \text{Min} \{ C_G(x) : G \subseteq A, \text{each } G \text{ is a regular open } M\text{-set} \}\), for all \(x \in X\).

Definition 3.4: Let \((M, \tau)\) be an \(M\)-topological space in \([X]^{W}\) and \(A\) be any sub\(M\)-set of \(M\). The \(M\)-set intersection of all regular closed \(M\)-sets of \((M, \tau)\) containing \(A\) is called the \(M\)-topology and the ordered pair \(\langle M, \tau \rangle\) is an \(M\)-topological space. Also, the collection of the regular closed \(M\)-sets of \((M, \tau)\) is \(\{ M, \{ \phi, \{ 1/a, 1/c \}, \{ 1/c \}, \{ 1/c, 1/b \}, \{ 1/b, 1/c \} \} \} \) and the collection of the \(M\)-topology and the ordered pair \(\langle M, \tau \rangle\) is an \(M\)-topological space. Hence, \(A\) is a regular closed \(M\)-set.

Definition 3.5: Let \((M, \tau)\) be an \(M\)-topological space in \([X]^{W}\). Any sub\(M\)-set \(A\) of \(M\) is called a \(b\)-open \(M\)-set if \(A \subseteq \text{int}(\text{cl}(A)) \cup \text{cl}(\text{int}(A))\) with \(C_{\text{int}(A)}(x) = \text{Max} \{ C_{\text{cl}(\text{int}(A))}(x), C_{\text{cl}(\text{int}(A))}(x) \}\), for all \(x \in X\).

Definition 3.6: Let \((M, \tau)\) be an \(M\)-topological space in \([X]^{W}\). Any sub\(M\)-set \(A\) of \(M\) is called a \(b\)-closed \(M\)-set if \(rcl(A) \subseteq \text{cl}(\text{int}(A)) \cup \text{int}(\text{cl}(A))\) with \(C_{\text{cl}(\text{int}(A))}(x) = \text{Max} \{ C_{\text{cl}(\text{int}(A))}(x), C_{\text{cl}(\text{int}(A))}(x) \}\), for all \(x \in X\).

Definition 3.7: Let \((M, \tau)\) be an \(M\)-topological space in \([X]^{W}\). For any sub\(M\)-set \(A\) of \(M\), the \(M\)-interior of \(A\) is defined as the \(M\)-set union of all \(M\)-open \(M\)-sets of \((M, \tau)\) contained in \(A\) and is denoted by \(\text{int}(A)\). i.e. \(\text{int}(A) = \{ U \subseteq M : \text{each } U \text{ is a } M\text{-open } M\text{-set and } U \subseteq A \} \) with \(C_{\text{int}(A)}(x) = \text{Max} \{ C_U(x) : U \subseteq A, \text{each } U \text{ is a } M\text{-open } M\text{-set} \}\), for all \(x \in X\).

Definition 3.8: Let \((M, \tau)\) be an \(M\)-topological space in \([X]^{W}\). For any sub\(M\)-set \(A\) of \(M\), the \(M\)-closure of \(A\) is defined as the \(M\)-set intersection of all \(M\)-closed \(M\)-sets of \((M, \tau)\) containing \(A\) and is denoted by \(\text{cl}(A)\). i.e. \(\text{cl}(A) = \{ K \subseteq M : \text{each } K \text{ is a } M\text{-closed } M\text{-set and } K \subseteq A \} \) with \(C_{\text{cl}(A)}(x) = \text{Min} \{ C_K(x) : K \subseteq A, \text{each } K \text{ is a } M\text{-closed } M\text{-set} \}\), for all \(x \in X\).

Definition 3.9: Let \((M, \tau)\) be an \(M\)-topological space in \([X]^{W}\). Any sub\(M\)-set \(A\) of \(M\) is called an \(\alpha\)-open \(M\)-set if \(A \subseteq \text{int}(\text{cl}(A))\) with \(C_A(x) \leq C_{\text{int}(\text{cl}(A))}(x)\), for all \(x \in X\). The complement of an \(\alpha\)-open \(M\)-set is called an \(\alpha\)-closed \(M\)-set.

Definition 3.10: Let \((M, \tau)\) be an \(M\)-topological space in \([X]^{W}\) and \(A\) be any sub \(M\)-set of \(M\). The \(M\)-set intersection of all \(\alpha\)-closed \(M\)-sets of \((M, \tau)\) contained in \(A\) is called the \(\alpha\)-closure of \(A\) and is denoted by \(\text{cl}(A)\). Similarly, the \(M\)-set union of all \(\alpha\)-open \(M\)-sets of \((M, \tau)\) contained in \(A\) is called the \(\alpha\)-interior of \(A\) and is denoted by \(\text{int}(A)\). i.e. \(\text{int}(A) = \{ K \subseteq M : \text{each } K \text{ is an } \alpha\text{-open } M\text{-set and } K \subseteq A \} \) with \(C_{\text{int}(A)}(x) = \text{Min} \{ C_K(x) : K \subseteq A, \text{each } K \text{ is an } \alpha\text{-open } M\text{-set} \}\), for all \(x \in X\).

Definition 3.11: Let \((M, \tau)\) be an \(M\)-topological space in \([X]^{W}\). Any sub\(M\)-set \(A\) of \(M\) is called a generalized \(\alpha\)-closed (\(g\)-closed) \(M\)-set if \(\text{cl}(A) \subseteq U \cap C_{\text{cl}(\text{int}(A))}(x) \subseteq C_U(x)\), whenever \(A \subseteq U \cap C_A(x) \subseteq C_U(x)\), and \(U\) is an \(\alpha\)-open \(M\)-set in \((M, \tau)\), for all \(x \in X\).

Proposition 3.1.1: Every regular closed \(M\)-set is a \(b\)-closed \(M\)-set.

Remark 3.1.2: the converse of the above Proposition 3.1.1 need not be true as shown in the Example 3.1.12.

Example 3.1.3: Let \(X = \{a, b, c\}, W = 2\). Let \(M = \{1/a, 2/b, 1/c\}\). Let \(\tau = \{ M, \phi, \{ 1/a \}, \{ 2/b \}, \{ 1/a, 2/b \}\}\). Then \(\tau^c = \{ M, \phi, \{ 2/b, 1/c \}, \{ 1/a, 1/c \}, \{ 1/c, 1/e \}\}\). Clearly, \(M\) is an \(M\)-topology and the ordered pair \((M, \tau)\) is an \(M\)-topological space.

Problem 3.1.4: Every regular closed \(M\)-set is a \(g\alpha\)-closed \(M\)-set.

Example 3.1.6: Let \(X = \{a, b, c\}, W = 2\). Let \(M = \{1/a, 2/b, 1/c\}\). Let \(\tau = \{ M, \phi, \{ 1/a \}, \{ 1/a, 2/b \}, \{ 2/b \}\}\). Then \(\tau^c = \{ M, \phi, \{ 2/b, 1/c \}, \{ 1/a, 1/c \}\}\)). Clearly, \(M\) is an \(M\)-topology and the ordered pair \((M, \tau)\) is an \(M\)-topological space. Also, the collection of the regular closed \(M\)-sets of \((M, \tau)\) is \(\{ M, \phi, \{ 2/b, 1/c \}, \{ 1/a, 1/c \}\}\).
3.2 Regular b-continuous M-set functions

Definition 3.2.1 Let (M, τ) and (N, σ) be any two M-topological spaces. Any M-set function f: (M, τ) → (N, σ) is said to be a continuous M-set function if f⁻¹(A) is closed (resp. open) M-set in (M, τ), for every closed (resp. open) M-set A in (N, σ).

Definition 3.2.2 Let (M, τ) and (N, σ) be any two M-topological spaces. Any M-set function f: (M, τ) → (N, σ) is said to be a regular continuous M-set function if f⁻¹(A) is regular closed (resp. regular open) M-set in (M, τ), for every closed (resp. open) M-set A in (N, σ).

Definition 3.2.3 Let (M, τ) and (N, σ) be any two M-topological spaces. Any M-set function f: (M, τ) → (N, σ) is said to be a regular b-continuous (rb-continuous) M-set function if f⁻¹(A) is rb-closed (resp. rb-open) M-set in (M, τ), for every closed (resp. open) M-set A in (N, σ).

Definition 3.2.4 Let (M, τ) and (N, σ) be any two M-topological spaces. Any M-set function f: (M, τ) → (N, σ) is said to be a generalized α-continuous (ga-continuous) M-set function if f⁻¹(A) is ga-closed (resp. ga-open) M-set in (M, τ), for every closed (resp. open) M-set A in (N, σ).

Proposition 3.2.5 Let (M, τ) and (N, σ) be any two M-topological spaces in [X] \(^w\). For any M-set function f: (M, τ) → (N, σ), the following statements are equivalent:

i. f is a rb-continuous M-set function.

ii. f(rb-cl(A)) ⊆ cl(rb(A)), for each A ⊆ M with C_{rb-cl(A)}(x) ≤ C_{rb}(x), for all x ∈ X

iii. f(cl(B)) ⊆ f⁻¹(cl(B)), for each B ⊆ N with C_{rb-clf}(f⁻¹(B)) ≤ C_{rb}(f⁻¹(B)), for all x ∈ X

iv. f⁻¹(int(B)) ⊆ rb-int(f⁻¹(B)), for each B ⊆ N with C_{rb-int}(x) ≤ C_{rb-int}(x), for all x ∈ X.

Proposition 3.2.6: Every regular continuous M-set function is a continuous M-set function.

Remark 3.2.7: The converse of the above Proposition need not be true as shown in the Example 2.2.5

Example 3.2.8: Let X = [a, b, c], W₁ = 2, Y = {x, y, z}, and W₂ = 1. Let M = {1/a, 2/b, 1/c} and N = {1/x, 1/y, 1/z} be two M-sets. Let τ = { M, φ, { 1/a }, { 2/b }, { 1/a, 2/b } } and σ = { N, φ, { 1/x, 1/y, 1/z } } be the M-topologies on M and N respectively. Then (M, τ) and (N, σ) are M-topological spaces. Now the collection of the closed M-sets of (N, σ) is { φ, N, { 1/x, 1/y, 1/z } } and the collection of the regular closed M-sets of (M, τ) is { M, φ, { 2/b, 1/c }, { 1/a, 1/c } } and the collection of the closed M-sets of (M, τ) is { φ, M, { 2/b, 1/c }, { 1/a, 1/c, { 1/c } } }

Let M-set function f: (M, τ) → (N, σ) be defined by f = { (1/a, 1/x)/1, (2/b, 1/y)/2, (1/c, 1/z)/1 }. For the closed M-set A = {1/z} in (N, σ), f⁻¹(A) = {1/c} is an closed M-set in (M, τ). Trivially f⁻¹ (φ) = φ and f⁻¹(N) = M are closed M-sets in (M, τ). But {1/c} is not a regular closed M-set in (M, τ). Hence, every continuous M-set function need not be a regular continuous M-set function.

Proposition 3.2.9: Every regular continuous M-set function is a rb-continuous M-set function.

Remark 3.2.10: The converse of the Proposition 2.2.3 need not be true as shown in following Example.

Example 3.2.11: Let X = [a, b, c], W₁ = 2, Y = {x, y, z}, and W₂ = 1. Let M = {1/a, 2/b, 1/c} and N = {1/x, 1/y, 2/z} be two M-sets. Let τ = { M, φ, { 1/a }, { 2/b }, { 1/a, 2/b } } and σ = { N, φ, { 1/x, 1/y, 1/z } } be two M-topologies on M and N respectively. Then (M, τ) and (N, σ) are M-topological spaces. Now the collection of the regular closed M-sets of (M, τ) is { M, φ, { 2/b, 1/c }, { 1/a, 1/c } } and the collection of the rb-closed M-sets of (M, τ) is { M, φ, { 1/c, 2/b }, { 1/a, 1/c, { 1/c } } } and the collection of the closed M-sets of (N, σ) is { φ, N, { 2/c } }.

Let M-set function f: (M, τ) → (N, σ) be defined by f = { (1/a, 1/x)/1, (2/b, 1/y)/2, (1/c, 2/z)/2 }. For the closed M-set A = {2/z} in (N, σ), f⁻¹(A) = {1/c} is an rb-closed M-set in (M, τ). Trivially f⁻¹ (φ) = φ and f⁻¹(N) = M are rb-closed M-sets in (M, τ). But {1/c} is not a regular closed M-set in (M, τ). Hence, every rb-continuous M-set function need not be a regular continuous M-set function.

Proposition 3.2.12: Every rb-continuous M-set function is a ga-continuous M-set function.

Remark 3.2.13: The converse of the Proposition 2.2.4 need not be true as shown in the Example 2.2.7

Example 3.2.14: Let X = [a, b, c], W₁ = 2, Y = {x, y, z}, and W₂ = 2. Let M = {1/a, 2/b, 1/c} and N = {2/x, 1/y, 1/z} be two M-sets. Let τ = { φ, M, { 1/a }, { 2/b }, { 1/a, 2/b } } and σ = { N, φ, { 2/x, 1/y, 1/z } } be the M-topologies on M and N respectively. Then (M, τ) and (N, σ) are M-topological spaces. Now the collection of the closed M-sets of (N, σ) is { φ, N, { 1/y, 1/z } } and the collection of the rb-closed M-sets of (M, τ) is { M, φ, { 1/c }, { 2/b, 1/c }, { 1/a, 1/c } } and the collection of the ga-closed M-sets of (M, τ) is { M, φ, { 1/c }, { 1/b, 1/c }, { 1/a, 1/b, 1/c } }.

Let M-set function f: (M, τ) → (N, σ) be defined by f = { (1/a, 2/x)/2, (1/b, 1/y)/2, (1/c, 1/z)/1 }. For the closed M-set A = {1/y, 1/z} in (N, σ), f⁻¹(A) = {1/b, 1/c} is a ga-closed M-set in (M, τ). Trivially f⁻¹ (φ) = φ and f⁻¹(N) = M are ga-
closed M-sets in \((M, \tau)\), but \(\{1/b, 1/c\}\) is not a rb-closed in \((M, \tau)\). Hence every go-continuous M-set function need not be a rb-continuous M-set function.

**Proposition 3.2.15:** Every regular continuous M-set function is go-continuous M-set function.

**Remark 3.2.16:** The converse of the Proposition 2.2.5 need not be true as shown in the Example 2.2.6

**Example 3.2.17:** Let \(X = \{a, b, c\}\), \(W_1 = 2\), \(Y = \{x, y, z\}\) and \(W_2 = 2\). Let \(M = \{1/a, 2/b, 1/c\}\) and \(N = \{2/x, 1/y, 1/z\}\) be two M-sets. Let \(\tau = \{\phi, M, \{1/a\}, \{2/b\}, \{1/a, 2/b\}\}\) and \(\sigma = \{N, \phi, \{2/x, 1/y\}\}\) be the M-topologies on \(M\) and \(N\) respectively. Then \((M, \tau)\) and \((N, \sigma)\) are M-topological spaces. Now the collection of the closed M-sets of \((N, \sigma)\) is \(\{\phi, N, \{1/z\}\}\) and the collection of the regular closed M-sets of \((M, \tau)\) is \(\{M, \phi, \{2/b, 1/c\}, \{1/c, 1/a\}\}\). Let \(M\)-set function \(f: (M, \tau) \rightarrow (N, \sigma)\) be defined by \(f = (1/a, 2/x)/2, (2/b, 1/y)/2, (1/c, 1/z)/1\). For the closed M-set \(A = \{1/z\}\) in \((N, \sigma)\), \(f^{-1}(A) = \{1/c\}\) is an go-closed M-set in \((M, \tau)\). Trivially \(f^{-1}(\phi) = \phi\) and \(f^{-1}(N) = M\) are go-closed M-sets in \((M, \tau)\), but \(\{1/c\}\) is not a regular closed in \((M, \tau)\). Hence, every go-continuous M-set function need not be a regular continuous M-set function.

**Remark 3.2.18:** From the above discussions, the following implications hold: In the following diagram by A means A implies B

4. **On rb-Convergence in Multiset Topological Spaces**

In this chapter, the concepts of rb-convergence to a point, rb-accumulates to a point, rb-S-closed M-spaces are studied and some interesting properties are discussed. Finally, sequence of rb-convergence and rb-Hausdorff M-spaces are discussed.

**Definition 4.1:** A filter is a non-empty collection \(F\) of subM-sets of an M-topological space \((M, \tau)\) such that
1. \(\phi \notin F\);
2. If \(A \in F\) and \(B \supseteq A\) with \(C_B(x) \geq C_A(x)\), for all \(x \in X\), then \(B \in F\) and
3. If \(A \in F\) and \(B \in F\), then \(A \cap B \in F\) with \(C_A \cap B(x) \leq C_F(x)\), for all \(x \in X\).

**Definition 4.2:** Let \((M, \tau)\) be an M-topological space. A filter base of a filter \(F\) is a sub collection \(B\) of \(M\) such that for every \(F \in F\) there exists \(B \in B\) with \(B \subseteq F\) with \(C_B(x) \leq C_F(x)\), for all \(x \in X\). Such a subcollection \(B\) satisfies the following conditions:
1. \(\phi \notin B\)
2. For every \(B_1, B_2 \in B\) there exists \(B_3 \in B\) with \(B_1 \subseteq B_1 \cap B_2 \) with \(C_{B_3}(x) \leq C_B(x) \cap CB_2(x)\), for all \(x \in X\).

**Definition 4.3:** Let \(F\) be a filter on \(B\). Then \(F\) is a maximal filter if every filter including \(F\) coincides with \(F\).

**Definition 4.4:** Let \((M, \tau)\) be an M-topological space. A filter base \(B = \{A_\alpha\}_{\alpha \in \alpha}\), where \(J\) is an indexed set and \(A_\alpha \in M\) rb-converges to a point \(x_0 \in M\) if for each rb-open M-set \(V\) containing \(x_0\), there exists \(A_\alpha \in B\) such that \(A_\alpha \subseteq \text{rb-cl}(V)\) with \(C_{A_\alpha}(x) \leq C_{\text{rb-cl}(V)}(x)\), for all \(x \in X\).

**Definition 4.5:** A filter base \(B = \{A_\alpha\}_{\alpha \in \alpha}\), where \(J\) is an indexed set and \(A_\alpha \in M\) rb-accumulates to a point \(x_0 \in M\) if for each rb-open M-set \(V\) containing \(x_0\), \(A_\alpha \in B\) and \(\text{rbcl}(V) \neq \phi\) with \(\text{Min}\{C_{A_\alpha}(x), C_{\text{rb-cl}(V)}(x)\} \neq 0\), for all \(x \in X\).

**Definition 4.6:** Let \((M, \tau)\) be an M-topological space. A collection \(\mathcal{J}\) of sub M-sets of a space \(M\) is said to cover \(M\), or to be a covering of \(M\), if the union of the elements of \(\mathcal{J}\) is equal to \(M\). It is called an open covering of \(M\) if its elements are open sub M-sets of \(M\).

**Definition 4.7:** Let \((M, \tau)\) be an M-topological space. An M-topological space \((M, \bigcup)\) is said to be rb-S-closed if for every cover \(\{V_α\}_{α \in I}\) I is an indexed set \(\{\text{of } (M, \tau)\}\) rb-closed relative to \(M\) if for any cover \(\{U_α\}_{α \in I}\) \(I\) is an indexed set \(\{\text{of } (M, \tau)\}\), \(K \subseteq \bigcup \{\text{rb-cl}(U_α)\}_{α \in I}\) with \(\text{rb-cl}(K) \leq \text{Max}\{C_{\text{rb-cl}(V_α)}(x)\}_{α \in \alpha} \leq C_M(x)\), for all \(x \in X\).

**Definition 4.8:** Let \((M, \tau)\) be an M-topological space. A subM-set \(K\) of an M-topological space \((M, \bigcup)\) is said to be rb-closed relative to \(M\) if for any cover \(\{U_α\}_{α \in I}\) \(I\) is an indexed set \(\{\text{of } (M, \tau)\}\), \(K \subseteq \bigcup \{\text{rb-cl}(U_α)\}_{α \in I}\) with \(\text{rb-cl}(K) \leq \text{Max}\{C_{\text{rb-cl}(V_α)}(x)\}_{α \in \alpha} \leq C_M(x)\), for all \(x \in X\).

**Remark 4.9:** The family of all rb-open (res, rb-closed) M-sets of \((M, \tau)\) denoted by \(\text{RBO}(X)\) (res, \(\text{RBC}(X)\)). The family of all rb-open M-sets of \((M, \tau)\) containing a point \(x_0 \in M\) is denoted by \(\text{RBO}(M, m/x)\).

**Remark 4.10:** A filter base \(B\) is rb-convergent to a point \(x_0 \in M\) if and only if \(B\) contains the collection \(\{\text{rbcl}(U)\}_{U \in \text{RBO}(M, m/x)}\).

**Proposition 4.11:** For an M-topological space \((M, \tau)\), the following statements are equivalent:
1. \(M\) is an rb-S-closed M-space.
2. For each family of rb-closed M-sets \(\{F_α\}_{α \in I}\) \(I\) is an indexed set, such that \(\bigcap(F_α) = \phi\) with \(\text{Min}\{C_{F_α}(x)\}_{α \in I} = 0\), for all \(x \in X\), there exists a finite subfamily \(\{F_{α_i}\}_{i=1}^n\) such that \(\bigcap_{i=1}^n \text{rb-int}(F_{α_i}) = \phi\) with \(\text{Min}\{C_{\text{rb-int}(F_{α_i})}(x)\}_{i=1}^n = 0\), for all \(x \in X\).
3. Each filter base \(B = \{A_α\}_{α \in I}\) with \(C_B(x) = C_{\{A_α\}_{α \in I}}(x)\), for all \(x \in X\), rb-accumulates to some point \(x_0 \in M\).
4. Each maximum filter base \(B\) rb-converges.
Proposition 4.12: A subM-set A of a M-topological space (M, τ) is rb-closed relative to M if and only if every filter base B on M with A ∈ B rb-converges to a point in A.

Definition 4.13: Let (M, τ) be an M-topological space. A sequence in a M-set M is a function f: N → M. Then m/x = f(n) for all n ∈ N, where N is positive integer and denoted the sequence as (m/x), n ∈ N.

Definition 4.14: Let (M, τ) be an M-topological space, let x ∈ M and N ≤ M. Then N is said to be a rb-neighborhood of m/ x if there is an open M-set V in τ such that x ∈ M V and C(x) = C(x), for all y ≠ x. i.e. a rb-neighborhood of m/ x in M means any rb-open M-set containing m/ x. Here m/ x is said to be an interior point of N.

Definition 4.15: Let (M, τ) be an M-topological space. A sequence m/x₁, m/x₂, m/x₃, ..., of points in M is said to be rb-converges to a point x ∈ M if for each rb-neighborhood U of M/x, there is a positive integer N such that xₙ ∈ M U, for all n ≥ N. It is denoted by (m/ xₙ) → m/ x.

Definition 4.16: An M-topological space (M, τ) is said to be a rb-Hausdorff M-space, if each pair of distinct points m/ x and m/ y in M, there exist rb-open M-sets U and V containing m/ x and m/ y respectively such that U ∩ V = φ with C(U ∩ V(x)) = 0, for all x ∈ X.

Definition 4.17: Let (M, τ) be an M-topological space. A rb-countable basis at a point m/ x is a countable M-set { Uₙ | n ∈ N } of rb-neighborhoods of m/ x, such that for any rb-neighborhood V of m/ x there is an n ∈ N such that Uₙ ⊆ V with C(Uₙ) ≤ C(V(x)), for all x ∈ X. An M-topological space (M, τ) is said to be first rb-countable if every point has a rb-countable basis.

Proposition 4.3: Let (M, τ) be an M-topological space. Let M be a rb-Hausdorff space and xₙ ∈ M a rb-convergent sequence. Then the limit limₙ n→∞ m/ xₙ is unique.

5. Acknowledgement
The authors would like to express their sincere gratitude to the referees for their valuable suggestions and comments for the betterment of this paper.

6. References