MHD flow on a moving infinite vertical porous plate in the presence of thermal radiation

E Raghunandana Sai and V Ramana Murthy

Abstract
An attempt has been made in this paper to study the MHD Flow on a Moving Infinite Vertical Porous Plate In The Presence Of Thermal Radiation. The effects of various parameters on the flow entities has been examined. It is noticed that as Pr increases, the velocity in general increases. Through, there is a marginal change in the value of t, not much of significant change in the velocity profiles is observed. Further, it is observed that as Grash of number increases, the flow rate decreases. Also, as the Magnetic field increases, the flow rate decreases. This in agreement with the real life situation that the Magnetic field suppress the fluid motion.

Keywords: Impulsively started vertical plate, MHD flow, heat and mass, radiation transfer, heat flux

Introduction
Nomenclature

<table>
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<th>Symbol</th>
<th>Definition</th>
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<tr>
<td>$c_p$</td>
<td>Specific heat at constant pressure</td>
</tr>
<tr>
<td>$g$</td>
<td>Acceleration due to gravity</td>
</tr>
<tr>
<td>$k$</td>
<td>Thermal conductivity of the fluid</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Prandtl number</td>
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<tr>
<td>$\rho$</td>
<td>Pressure</td>
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<tr>
<td>$q_r$</td>
<td>Radiative heat flux in the y-direction</td>
</tr>
<tr>
<td>$k$</td>
<td>Thermal diffusivity</td>
</tr>
<tr>
<td>$K$</td>
<td>Porosity</td>
</tr>
<tr>
<td>$M$</td>
<td>Magnetic field</td>
</tr>
<tr>
<td>$N$</td>
<td>Radiation parameter</td>
</tr>
<tr>
<td>$t$</td>
<td>Time</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature of the fluid near the plate</td>
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<tr>
<td>$T_w$</td>
<td>Temperature of the plate</td>
</tr>
<tr>
<td>$T_\infty$</td>
<td>Temperature of the fluid far away from the plate</td>
</tr>
<tr>
<td>$u$</td>
<td>Velocity of the fluid</td>
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<tr>
<td>$u_0$</td>
<td>Velocity of the fluid plate</td>
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<tr>
<td>$v$</td>
<td>Dimensionless velocity</td>
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<tr>
<td>$y$</td>
<td>Coordinate axis normal to the plate</td>
</tr>
<tr>
<td>$y'$</td>
<td>Dimensionless coordinate axis normal to the plate</td>
</tr>
<tr>
<td>$k'$</td>
<td>Mean absorption coefficient</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Thermal diffusivity</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Volumetric coefficient of thermal expansion</td>
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<tr>
<td>$\mu$</td>
<td>Coefficient of viscosity</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Kinematic viscosity</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Stefan-Boltzmann constant</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Dimensionless skin-friction</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Dimensionless temperature</td>
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</table>

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In several industrial and environmental situations, the radiative convective flow occurs flow place a very important role. Several applications are noticed in combustion and cooling chambers. The applications are more seen astrophysical flows, solar power technology and space vehicles. In the design high precision equipment, the radiative heat transfer plays a very significant role. The radiative transfer is more found in nuclear power plants, gas turbines, missiles and in air craft engines.

The problem of viscous incompressible fluid over an infinite horizontal plate which moves in its own plane once was studied by stokes [1]. Thereafter, the viscous force imparted by flowing fluid in a dense swarm of particles was studied by Brinkman [2]. Later on, Stewartson [3] obtained the analytical solution for viscous flow past and impulsively started semi-infinite horizontal plate. Thereafter, Berman [4], studied the case of two dimensional study flow of an incompressible fluid with parallel rigid porous walls where the flow is influenced by uniform section and injection. Next, the flow between two vertical plates where the plates are electrically non-conducting with the assumption that the wall temperature influences linearly in the direction of the flow was studied by Macy [5] and Mori [6].

Hall [7] studied the same problem with the help of finite differences method of a mixed explicit implicit time for the stability of the solution. Change et al. [8] examined the effects of radioactive heat transfer of free convection in an enclosed with specialized applications. Thereafter, Mahajan et al. [9] examined the influence of viscous heat dissipating effect in natural convective flows. Subsequently, Soundalgekar and Thaker [10]. Investigated radiation effects of an optionally thin gray gas bounded by a stationary vertical plate. By applying Rossland’s approximation, Hossain et al. [11] studied the radiation effects and mixed convection along a vertical plate with uniform free surface temperature by applying Rossland’s approximation. Subsequently, the effects of thermal radiation and convective flow past a moving infinite vertical plate was analyzed, discussed and presented by Raptis and Perdikis [12]. The effects of thermal radiation and flow past semi-infinite vertical isothermal plate with uniform heat flux in the presence of applied magnetic field was examined by Antony Raj et al. [13].

The Influence of velocity with reference to the critical parameters which appear in the equation of motion were neither commented nor examined in detail by any of the above investigators is a point of great concern academic interest. Therefore, an attempt has been made to study the effects of such parameters in this paper. Also the effect of critical parameter on the flow entities has been examined.

**Mathematical Formulation**

Flow of an incompressible viscous radiating fluid past an impulsively started infinite vertical plate with uniform heat flux is considered. The x-axis is taken along the plate in the vertical direction and the y-axis is taken normal to the plate. The flow geometry is as shown below.

![Schematic representation of the problem](image)

Initially, the plate and fluid are at the same temperature in a stationary condition. The plate is given an impulsive motion in the vertical direction against the gravitational field with constant velocity $u_0$ when $t > 0$. At the same time, the heat is supplied from the plate to the fluid at uniform rate. The fluid exhibits the properties of grey, absorbing-emitting radiation but a non-scattering medium. Then by usual Bossiness’s approximation, the unsteady flow is governed by the following equations.

\[
\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + \nu \frac{\partial^2 u}{\partial y^2} \quad (1)
\]

\[
\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_v}{\partial y} \quad (2)
\]
In view of Rossland approximation $q_r$ is given by:

$$q_r = -\frac{4\sigma}{3k^*} \frac{\partial T^4}{\partial y}$$  \hspace{1cm} (3)

While, the initial and boundary conditions are:

$$
t' \leq 0: u = 0, T = T_\infty \text{ For all } y' \\
t' > 0: u = u_0, \frac{\partial T}{\partial y} = -\frac{q}{k} \text{ at } y' = 0 \\
u = 0, T \to T_\infty \text{ as } y' \to \infty
$$

Under the assumption that, the temperature differences within the flow are sufficiently small such that $T^4$ may be expressed as a linear function of the temperature. This is accomplished by expanding $T^4$ in a Taylor series about $T_\infty$ and neglecting higher order terms, thus

$$T^4 \equiv 4T_\infty^4 T - 3T_\infty^4$$  \hspace{1cm} (5)

By using equations (4) and (5), equation (2) reduces to

$$\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} + \frac{16\sigma T^3}{3k^*} \frac{\partial^2 T}{\partial y^2}$$  \hspace{1cm} (6)

On introducing the following non-dimensional quantities

$$U = \frac{u}{u_0}, t = \frac{t'u_0^2}{\nu}, y = \frac{y'u_0}{\nu}, \theta = \frac{T - T_\infty}{T_w - T_\infty},$$

$$Gr = \frac{g\beta(T_w - T_\infty)}{u_0^3}, Pr = \frac{\mu C_p}{\kappa^* k}, N = \frac{\kappa^* k}{4\sigma T_\infty^3}$$ \hspace{1cm} (7)

By considering the magnetic intensity as $M$ and the permeability of the boundary as $K$ Eqs. (1) to (6), can be re defined as:

$$\frac{\partial U}{\partial t} - (1 + \phi u) \frac{\partial U}{\partial y} = \frac{\partial^2 U}{\partial y^2} + \frac{1}{K} U$$  \hspace{1cm} (8)

$$3N Pr \frac{\partial \theta}{\partial t} = (3N + Pr) \frac{\partial^2 \theta}{\partial y^2}$$  \hspace{1cm} (9)

The initial and boundary conditions in non-dimensionless form are

$$u = 0, \theta = 0, \text{ For all } y, t \leq 0$$

$$t > 0: u = 1, \frac{\partial \theta}{\partial y} = -1 \text{ At } y = 0$$

$$u = 0, \theta \to 0, \text{ As } y \to \infty$$  \hspace{1cm} (10)

**Methodology for solution**

We assume that the solutions for Eqn (8) and Eqn (9) in form of:

$$u(x,t) = u_0(y)e^{iat}$$ \hspace{1cm} (11)

$$\theta(y,t) = \theta_0(y)e^{iat}$$ \hspace{1cm} (12)
Under the modified initial and boundary conditions:

\[ u_0 = 0, \theta_0 = 0, \text{ for all } y, t \leq 0 \]

\[ t > 0: u_0 = e^{-i\sigma}, \frac{d\theta_0}{dy} = e^{-i\sigma} \text{ at } y = 0 \]

\[ u_0 = 0, \theta_0 \to 0 \text{ as } y \to \infty \]  

(13)

Using Eqns (11), (12) and (13) in Eqns (8) and (9)

\[ u(y, t) = \frac{Gr}{R1} (\exp(-m_2 y) - \exp(-m_1 y)) + \exp(-m_2 y) \]  

(14)

\[ \theta(y, t) = \frac{\exp(-m_1 y)}{m_1} \]  

(15)

The expression for the skin friction is:

\[ \left[ \frac{\partial u}{\partial y} \right]_{y=0} = \frac{Gr}{R_1} [m_1 - m_2] - m_2 \]  

(16)

Where

\[ m_1 = \sqrt{\frac{3N Pr i\omega}{3N + Pr}}, m_2 = \sqrt{(i\omega - M - \frac{1}{K})}, R_1 = m_1 (m_1^2 - (i\omega - M - \frac{1}{K})) \]

Results and Discussion

Figures 1, 2, 3 and 4 illustrates the effect of time on the velocity profiles for Pr=0.05, 0.01 0.04 and 0.03 respectively. In all these illustrations, it is observed that their relationship between Y (plate width) and velocity is always linear. In some cases, their relationship is positive and while in some cases it is negative.
Fig 2: Effect of time on velocity for Pr = 0.01

Fig 3: Influence of time on velocity for Pr = 0.04

Fig 4: Influence of time on velocity for Pr = 0.03
Fig 5 – 10 shows the nature of the velocity profiles for Prandtl numbers 0.01, 0.02, 0.03, 0.04 and 0.05 respectively for $t = 0.6, 0.5, 0.4, 0.3, 0.2$ and 0.1 respectively. In each of these figures, their relationship is found to be perfectly linear. The relationship is positive while sometimes is negative as Pr increase. Further, it is noticed that as Pr increases, the velocity in general increases. Through, there is a marginal change in the value of $t$, not much of significant change in the velocity profile is observed.

**Fig 5**: The nature of velocity for $t=0.6$

**Fig 6**: The nature of velocity for $t=0.5$
Fig 7: The nature of velocity for t=0.4

Fig 8: The nature of velocity for t=0.3
Fig 9: The nature of velocity for $t=0.2$

Fig 10: The nature of velocity for $t=0.1$

Fig 11 and 12 illustrates the influence of Grashoff number (Gr) on flow rate for a fixed Magnetic Parameter $M=0.04$ and $M=0.02$ respectively. It is observed that as Grashoff number increases, the flow rate decreases in both the cases.
Fig 11: The nature of velocity for $M=0.04$

Fig 12: The nature of velocity for $M=0.02$

Fig 13 and 14 illustrates the effect of magnetic field on the flow rate for $Gr=0.04$ and $Gr=0.02$ respectively. It is seen as the Magnetic field increases, the flow rate decreases. This in agreement with the real life situation that the Magnetic field suppress the fluid motion.
1. References

5. Mori Y. on combined free and forced convective laminar MHD flow and heat transfer in channels with transverse magnetic field, international developments in heat transfer, ASME. 1961; 124:1031-1037.


