Optimization of special type double sampling plan using generalized Poisson distribution through minimum angle method

V Kaviyarasu and V Devika

Abstract
This paper presents a procedure for designing a Special Type Double Sampling (STDS) plan indexed through Minimum Angle Method along with Operating Characteristic (OC) curve. In sampling plan, whenever there is a declined angle $\theta$, the more nearly the OC curve approach an ideal form and it is achieved for the proposed plan using Generalized Poisson Distribution (GPD). Suitable tables are presented for the selection of the plan through Minimum Angle Method with declination in the OC curve. The designing procedures of the plan for various consumer’s and producer’s quality levels are developed using Minimum Angle Method, making an operating characteristic curve approaching to the ideal form. Theoretical and numerical evaluations are given with suitable illustrations.

Keywords: Special type double sampling plan, generalized poisson model, producer’s quality level, consumer’s quality level, minimum angle method

1. Introduction
In the present age of increasing globalization, it is a great challenge for industrialists to fulfill the requirements of consumers on customer related products, reliable lead time and finally customer satisfaction. Today, quality control is a statistical programme to prevent free from defects during and after productions with a complete guarantee for the quality product. Sampling inspection in product control is a statistical methodology which generally works with a basic assumption that the production process from which lots are formed is stable and the lot quality defined in terms of fraction non-conforming is a fixed constant. Generally, Poisson distribution is widely applied in acceptance sampling plans since it has the property that its mean and variance are equal, which is called equidispersion. However, in most of the real life situations, the production processes are not always stable and the lots coming from such processes will have quality variations which may occur due to random fluctuations. The most appropriate probability distribution to handle this kind of situation is the Generalized Poisson Distribution (GPD) developed by Consul and Jain (1973) [1]. This is a strong requirement in real life practice that, when the product quality exhibits a dispersed behaviour, there is a need for a distribution that can handle both over dispersed and under dispersed conditions, such as the aforementioned GPD. It can fulfill the property: the variance is greater than the mean (over-dispersion) or the variance is less than the mean (under-dispersion). The density function of GPD is given as follows:

$$p_x(\lambda_1, \lambda_2) = \lambda_1 \lambda_2^x e^{-(\lambda_1 + \lambda_2)} x!, \quad x = 0, 1, 2, \ldots$$

Where, $\lambda_1$ is the scale parameter and $\lambda_2$ is the shape parameter which determines the shape of the performance measures. Here we consider, the case $\lambda_2 > 0$, for establishing the tables. It ensures the protection for producer and consumer especially, the STDS plan is applicable in the area of safety related testing like medicine, army weapons, chemicals, etc. The single sampling plan under the conditions of GPD developed by Kaviyarasu and Devika (2018) [2] exaggerate the need and importance of GPD based sampling plan in the area of
Product control. According to Govindaraju (1984, 1991) [2,3], the Operating Characteristic (OC) curves of single sampling plan with smaller acceptance numbers results in conflicting interest between the producer and consumer. While single sampling plan with acceptance number Ac=0, which pays more attention towards the consumer but on the other hand acceptance number Ac=1 may favor to producer. This limitation could be overcome to a greater extent when a Special Type Double Sampling (STDS) plan is applied for sentencing the individual lots.

For any sampling plan is considered, the OC curve will determine the discriminating power existing between the producer risk and consumer risk. Here, the risk minimization could be achieved through the minimum angle method, making an OC curve approaching to the ideal form. An idealized OC curve has a greater slope; the greater is the discriminating power. Thus it emphasizes the quality manager to take decision making regarding the lot disposition and also make an alarm sense about risk minimization. Both these conditions can be achieved through the minimum angle method. Here an attempt is made to study the GPD through its twin properties of a distribution which strike out the dispersion of data according to \( \lambda_2 > 0 \) or \( \lambda_2 < 0 \) respectively. In this context, this paper is developed to find out its optimum parameters for the sampling plan for various producer’s and consumer’s quality levels through minimum angle method.

2. Conditions for Application
1. The production process is reasonably steady, so that results on current and preceding lots are broadly indicative of a continuing process.
2. Samples are taken from lot substantially in the order of production so that observed variations in quality of product reflect process performance.
3. The product comes from a source in which consumer has confidence.
4. Inspection is by attributes, with quality measured in terms of fraction defective \( p \).

3. Operating Procedure
According to Schilling and Neubauer (2009) [8], a SSP with zero acceptance number are very useful in the area of compliance and safety related testing. To overcome the pitfalls in SSP with smaller acceptance number, a lot by lot STDS plan is used as an alternative. In the first phase of the sampling acceptance number are considered as \( Ac = 0 \) and in second phase the acceptance number is relaxed for the large sample size which may have \( Ac = 1 \). The operating procedure of STDS plan under GPD is very simple as given below:
1. A random sample of size \( n_1 \) units are taken from a lot and observe the number of defectives \( d_1 \). If \( d_1 \geq 1 \), reject the entire lot.
2. If \( d_1 = 0 \), select a second random sample of size \( n_2 \) and observe the number of defectives \( d_2 \). If \( d_2 \leq 1 \), accept the lot. Otherwise reject the lot.

The OC function for the GPD based STDS plan is designed and expressed as follows:

\[
P_a(p) = P(d_1 = 0/n_1;p) \times P(d_2 = 1/n_2;p)
\]

Where \( d_1 \) is the number of defectives found in the first random sample of size \( n_1 \) and \( d_2 \) is the number of defectives found in the second random sample of size \( n_2 \) for the lot quality \( p \).

That is, \( P_a(p) = e^{-x}(1 + \phi e^{-\lambda_1}) \)

Here, \( x = np, \phi = n_2/n_1 + n_2 \) and \( \lambda_2 \) is the dispersion parameter.


4. Selection Procedure
Selection procedure for Special Type Double Sampling plan using Minimum Angle method under the condition of Generalized Poisson Distribution
Norman Bush et al. (1953) [6] used different techniques to describe the direction of the operating characteristic (OC) curve through Minimum Angle Method. When producer and consumer are negotiating for designing sampling plans, it is important especially to minimize the consumer risk. Further they have considered two points on the OC curve as (AQL, 1 – \( \alpha \)) and (IQL, 0.50) for minimizing the consumer risk. Peach and Littauer (1946) have taken two points on the OC curve as (AQL, 1 – \( \alpha \)) and (LQL, \( \beta \)) for making the ideal condition. Hence the optimization approach for ideal condition to minimize the consumers risks is carried through minimizing the angle \( \theta \) between the lines joining the points [(AQL, 1 – \( \alpha \), (AQL, \( \beta \))] and [(AQL, 1 – \( \alpha \), (LQL, \( \beta \))] as given in Figure 1. The formula for tan \( \theta \) is,

\[
tan \theta = \frac{opposite \ side}{adjacent \ side} = \frac{(p_2-p_1)}{(p_1\beta-p_2\beta)}
\]

Hence, for two given points on the OC curve, the minimum values of angle, \( \theta \) can be calculated and tabulated as in Table 1 for the proposed plan using \( \theta = tan^{-1}\{(ntan\theta/n)\} \).
4.1 Example

Given \( p_1=0.001 \) and \( p_2=0.021 \)

Then \( OR=210 \)

The associated sets of values corresponding to the computed \( OR \) form Table 1 are:

\[
\begin{align*}
\lambda_2 &= 0.7; \ \varphi = 0.75; \ \text{np}_1 = 0.01462; \ \text{ntan} \theta = 3.42509 \\
\lambda_2 &= 0.8; \ \varphi = 0.85; \ \text{np}_1 = 0.01462; \ \text{ntan} \theta = 3.44344 \\
\lambda_2 &= 0.9; \ \varphi = 0.65; \ \text{np}_1 = 0.01364; \ \text{ntan} \theta = 3.20474.
\end{align*}
\]

which implies that,

1. \( n = \text{np}_2/p_1 = 0.01462/0.0001 = 146 \)

From the above three choices, the minimum angle is \( \theta = 1.342 \). Hence the optimum plan parameters for the proposed sampling plan is \( c = 0.1 \) with \( n = 146, p_1 = 0.0001, p_2 = 0.021, \alpha = 0.01, \beta = 0.10, \text{n}_2=37, \text{n}_1=110 \) and \( \lambda_2 = 0.7 \) with minimum angle \( \theta = 1.342 \).

<table>
<thead>
<tr>
<th>( \lambda_2 )</th>
<th>( \varphi )</th>
<th>( \text{np}_1 )</th>
<th>( \text{np}_2 )</th>
<th>( \text{ntan} \theta )</th>
<th>( \text{OR} )</th>
<th>( \lambda_2 )</th>
<th>( \varphi )</th>
<th>( \text{np}_1 )</th>
<th>( \text{np}_2 )</th>
<th>( \text{ntan} \theta )</th>
<th>( \text{OR} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>0.50</td>
<td>0.0138</td>
<td>2.8852</td>
<td>3.2263</td>
<td>208.681</td>
<td>0.50</td>
<td>0.0129</td>
<td>2.7886</td>
<td>3.1187</td>
<td>215.436</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.55</td>
<td>0.0144</td>
<td>2.9366</td>
<td>3.2834</td>
<td>204.419</td>
<td></td>
<td>0.55</td>
<td>0.0133</td>
<td>2.8327</td>
<td>3.1679</td>
<td>212.543</td>
</tr>
<tr>
<td></td>
<td>0.60</td>
<td>0.0146</td>
<td>2.9868</td>
<td>3.3395</td>
<td>204.292</td>
<td></td>
<td>0.60</td>
<td>0.0137</td>
<td>2.8760</td>
<td>3.2161</td>
<td>209.390</td>
</tr>
<tr>
<td></td>
<td>0.65</td>
<td>0.0146</td>
<td>3.0356</td>
<td>3.3943</td>
<td>207.632</td>
<td></td>
<td>0.65</td>
<td>0.0142</td>
<td>2.9184</td>
<td>3.2632</td>
<td>205.981</td>
</tr>
<tr>
<td></td>
<td>0.70</td>
<td>0.0163</td>
<td>3.0831</td>
<td>3.4459</td>
<td>189.314</td>
<td></td>
<td>0.70</td>
<td>0.0146</td>
<td>2.9600</td>
<td>3.3094</td>
<td>202.460</td>
</tr>
<tr>
<td></td>
<td>0.75</td>
<td>0.0170</td>
<td>3.1294</td>
<td>3.4970</td>
<td>183.692</td>
<td></td>
<td>0.75</td>
<td>0.0146</td>
<td>3.0006</td>
<td>3.355</td>
<td>205.24</td>
</tr>
<tr>
<td></td>
<td>0.80</td>
<td>0.0179</td>
<td>3.1745</td>
<td>3.5468</td>
<td>177.757</td>
<td></td>
<td>0.80</td>
<td>0.0146</td>
<td>3.0404</td>
<td>3.3997</td>
<td>207.96</td>
</tr>
<tr>
<td></td>
<td>0.85</td>
<td>0.0188</td>
<td>3.2184</td>
<td>3.5951</td>
<td>171.521</td>
<td></td>
<td>0.85</td>
<td>0.0146</td>
<td>3.0793</td>
<td>3.4343</td>
<td>210.621</td>
</tr>
<tr>
<td></td>
<td>0.90</td>
<td>0.0198</td>
<td>3.2611</td>
<td>3.6420</td>
<td>164.996</td>
<td></td>
<td>0.90</td>
<td>0.0168</td>
<td>3.1174</td>
<td>3.4837</td>
<td>185.201</td>
</tr>
<tr>
<td></td>
<td>0.95</td>
<td>0.0209</td>
<td>3.3028</td>
<td>3.6875</td>
<td>158.192</td>
<td></td>
<td>0.95</td>
<td>0.0175</td>
<td>3.1546</td>
<td>3.5249</td>
<td>180.431</td>
</tr>
<tr>
<td>1</td>
<td>0.0221</td>
<td>3.3433</td>
<td>3.7317</td>
<td>151.122</td>
<td></td>
<td>1</td>
<td>0.0182</td>
<td>3.1911</td>
<td>3.565</td>
<td>175.456</td>
<td></td>
</tr>
</tbody>
</table>

5. Construction of tables

For STDs plan, the probability of acceptance, \( P_a(p) \) for a given lot proportion defective, \( p \), is given by,

\[
P_a(p) = e^{-\lambda}(1+\varphi e^{-\lambda_2})
\]

Where, \( \chi = np_1 \varphi = n_2/n_1 + n_2 \) and \( \lambda_2 \) is the dispersion parameter.

The minimum angle criteria for the proposed plan can be obtained as

\[
\begin{align*}
tan \theta &= \frac{(p_2-p_1)}{(e^{-np_1}+\varphi_1 e^{-\lambda_2})-(e^{-np_2}+\varphi_2 e^{-\lambda_2})} \text{ (or)} \\
tan \theta &= \frac{np_2-np_1}{1-a-\beta}
\end{align*}
\]

Based on the operating ratio value, the optimized plan parameters are obtained through the angle calculated as \( \theta = tan^{-1}\{(ntan\theta)/n\} \). The Goal seek search procedure has been used to obtain the optimum value of the tangent angle for specific values of \( np_1 \) and \( np_2 \) by keeping the producer’s risk below 1% and consumer’s risk below 10%.

6. Conclusion

This paper presents an optimization approach to support the design of Generalized Poisson Distribution (GPD) based Special Type Double Sampling (STDs) plan through Minimum Angle Method. In practice, it is desirable to design a sampling plan with the associated quality levels concerning producer and consumer. This method is useful in the area of testing related to safety and destructive cases. The general framework is studied which makes the OC curve approaching to the ideal form by minimizing the tangent angle joining the points \([AQL, 1-\alpha], (AQL, \beta)]\) and \([AQL, 1-\delta], (QL, \beta)]\). Thus it provides an alarm sense for quality controller towards the lot sentencing and the risks exists under this plan.

The work presented in this paper is mainly related to obtain optimum plan parameters for manufacturing industrialists for inspection procedure. Tables are provided to safeguard both producer and consumer simultaneously for obtaining good quality products in inspection. The proposed sampling plan through its economic aspects may be the future research work.

7. Acknowledgement

The authors are happy to acknowledge the unknown referees and parent University for providing necessary facilities in the Department through DST-FIST and UGC-SAP Programmes. We further acknowledge the DST-PURSE (II) for providing funding for the research work.

8. References


