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Concircular curvature tensor in Kenmotsu manifold

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Abstract
In this paper we study Concircular curvature tensor in Kenmotsu manifolds. We consider curvature tensor, scalar curvature in a Kenmotsu manifold admitting concircular curvature tensor and prove the condition for concircular curvature tensor to be shrinking, steady and expanding.

Keywords: Curvature tensor, scalar curvature, Ricci tensor, Einstein, Kenmotsu manifold

Introduction

The Concircular curvature $\tilde{C}$ on Kenmostu manifolds of dimensional n is defined by

$$\tilde{C}(X, Y)Z = R(X, Y)Z - \frac{r}{n(n-1)}[g(Y, Z)X - g(X, Z)Y] \tag{1.1}$$

for any vector fields X, Y, Z where R is the curvature tensor and r is the scalar curvature.
In an n- dimensional Kenmotsu manifold, we have

$$\eta(R(X,Y)Z) = g(X,Z)\,\eta(Y) - g(Y,Z)\,\eta(X), \tag{1.2}$$

$$R(X,Y)\xi = \eta(X)Y - \eta(Y)X, \tag{1.3}$$

$$R(\xi,X)Y = \eta(Y)X - g(X,Y)\xi, \tag{1.4}$$

$$R(\xi,X)\xi = X - \eta(X)\xi, \tag{1.5}$$

where R is the Riemannian curvature tensor.

Taking $Z = \xi$ in equation (1.1) and using equation (1.3), we get

$$\tilde{C}(X, Y)\xi = R(X, Y)\xi - \frac{r}{n(n-1)}[g(Y, \xi)X - g(X, \xi)Y]$$

$$\Rightarrow \tilde{C}(X,Y)\xi = [1 + \frac{r}{n(n-1)}[\eta(X)Y - \eta(Y)X]. \tag{1.6}$$

Taking inner product of equation (1.1) with ξ, we get

$$\eta(\tilde{C}(X, Y)Z) = \eta(R(X, Y)Z) - \frac{r}{n(n-1)}[g(Y, Z)\eta(X) - g(X, Z)\eta(Y)]. \tag{1.7}$$

Using equation (1.2) in above equation, we get

$$\eta(\tilde{C}(X, Y)Z) = [1 + \frac{r}{n(n-1)}][g(X, Z)\eta(Y) - g(Y, Z)\eta(X) \tag{1.8}$$

2.2 Preliminaries

An n- dimensional differential manifold M is said to be an almost contact metric manifold [3]. If it admits an almost contact metric structure (ϕ, ξ, η, g) consisting of a tensor field ϕ of type

(1, 1) a vector field ξ, a 1 - form η, and a Riemannian metric g compatible with (ϕ, ξ, η, g) satisfying.

$\phi^2 = -I + \otimes\xi, \eta\ (\xi) = 1,$ (2.1)

$\eta \circ \phi = 0, \phi\xi = 0$ (2.2)

$g(\phi X, \phi Y) = g(X,Y) - \eta\ (X)\ \eta\ (y),\ g(X,\xi) = \eta\ (X),$ (2.3)

for all vector fields X, Y on M.

An almost contact metric manifold M (ϕ, ξ, η, g) is said to be Kenmotsu manifold if

$(\nabla_X \phi)\ Y = g(\phi X, Y)\ \xi - \eta\ (Y)\ \phi X.$ (2.4)

From equation (2.4), we have

$\nabla_X\ \xi = X - \eta\ (X),$ (2.5)

where ∇ denotes the Riemannian connection of g.

Let (g, V, λ) be a Ricci soliton is an n- dimensional Kenmotsu manifold M.
From equation (2.5), we have

$(\mathcal{L}_\xi\ g)(X, Y) = 2[g(X, Y) - \eta(X)\eta(Y)],$ (2.6)

From equations (2.1) and (2.6), we have

$S(X, Y) = -(\lambda + 1)g(X, Y) + \eta(X)\eta(Y),$ (2.7)

$S(\phi X, \phi Y) = S(X, Y) + \lambda\ \eta(X)\eta(Y)$ (2.8)

From equation (2.8), we have

$QX = -(\lambda + 1)X + \eta(X)\xi,$ (2.9)

$S(X, \xi) = -\lambda\eta(X),$ (2.10)

$r = -\lambda n - (n - 1),$ (2.11)

where S is the Ricci tensor, Q is the Ricci operator and r is the scalar curvature on M.

Theorem 3.1: A Concircular curvature tensor $\tilde{C}$ in Kenmostu manifold satisfying $R(\xi, X).\ \tilde{C} = 0$ is expanding.

Proof: Let us suppose $(R(\xi, X).\ \tilde{C})\ (U, V)W = 0$

$\Rightarrow R(\xi, X)\ \tilde{C}\ (U, V)W - \tilde{C}(R(\xi, X)\ U, V)W - \tilde{C}(U, R(\xi, X)V)W - \tilde{C}(U,V)\ R(\xi, X)W = 0.$ (3.1)

Using equation (2.2.8) in above equation, we get

$\eta(\tilde{C}(U, V)W)X - g(X, \tilde{C}(U,V)W)\xi - \tilde{C}(\eta(U)X - g(X, U)\xi, V)W - \tilde{C}\ (U, \eta(V)X - g\ (X, V)\xi)W$
$- \tilde{C}\ (U, V)\ \{\eta\ (W)X - (X, W)\xi\ \} = 0,$
$\Rightarrow \eta(\tilde{C}(U, V)W)X - g(X, \tilde{C}\ (U, V)W)\xi - \eta(U)\ \tilde{C}\ (X, V)W + g(X, U)\ \tilde{C}\ (\xi, V)W - \eta(V)\tilde{C}\ (U, X)W$
$+ g(X, V)\ \tilde{C}\ (U, \xi)W - \eta(W)\tilde{C}\ (U, V)X + g(X, W)\ \tilde{C}\ (U, V)\xi = 0.$ (3.2)

Now taking an inner product of above equation with ξ, we get

$\Rightarrow \eta(\tilde{C}\ (U,V)W)\ \eta\ (X) - g(X, \tilde{C}\ (U, V)W - \eta(U)\ \eta(\tilde{C}(X, V)W) + g(X, U)\ \eta\ (\tilde{C}\ (\xi, V)W) - \eta(V)\tilde{C}\ (U, V)W) +$
$g(X, V)\ \eta\ (\tilde{C}(U, \xi)W) - \eta\ (w)\eta\ (\tilde{C}(U, V)X) + g\ (X, W)\ \eta\ (\tilde{C}(U, V)\ \xi) = 0.$ (3.3)

Using equation (1.6) and (1.7) in above equation, we get
$[1 + \frac{r}{n(n-1)}]\ [g(U, W)\ ]\eta(V) - g(V, W)\ \eta(U)]\eta(X) - g(X, \tilde{C}(U, V)W) - \eta(U)\left[1 + \frac{r}{n(n-1)}\right]\ [g(X, W)$
$\eta(V) - g(V, W)\eta(X)] + g(X, U)\left[1 + \frac{r}{n(n-1)}\right]\ [g(\xi, W)\ \eta(V) - g(V, W)\eta(\xi)] - \eta(V)\ \left[1 + \frac{r}{n(n-1)}\right]$

$[g(U, W)\ \eta(X) - g(X,W)\eta(U)] + g(X, V)\left[1 + \frac{r}{n(n-1)}\right] [g(U, W)\ \eta(\xi) - g(\xi,W)\eta(U)] - \eta(W)\left[1 + \frac{r}{n(n-1)}\right] [g(U, X)\ \eta(V) - g(V,X)\eta(U)] + g(X, W)\left[1 + \frac{r}{n(n-1)}\right] [g(U,\xi)\ \eta(V) - g(V,\xi)\eta(U)] = 0,$

$\Rightarrow \left[1 + \frac{r}{n(n-1)}\right] [g(U, W)\ \eta(X)\eta(V) - g(V,W)\eta(U)\eta(X) - g(X, W)\ \eta(V)\eta(U) + g(V,W)\eta(X)\eta(U) + g(X, U)\ \eta(W)\eta(V) - g(X, U)\ g(V, W) - g(U, W)\ \eta(V)\eta(X) + g(X, W)\ \eta(V)\eta(U) + g(X, V)\ g(U, W) - g(X, V)\ \eta(W)\eta(U) + g(U, X)\ \eta(W)\eta(V) + g(V, X)\ \eta(W)\eta(U) + g(X, W)\ \eta(U)\eta(V) - g(X, W)\ \eta(U)\eta(V)] - g(X, \tilde{C}(U, V)W) = 0,$

$\Rightarrow [1+ \frac{r}{n(n-1)}] [g(X, V)\ g(U,W) - g(X, U)\ g(V, W)] - g(X, \tilde{C}(U,V)W) = 0.$

The above equation can be written as

$$-g(X, \tilde{C}(U,V)W) + [1+ \frac{r}{n(n-1)}] [g(X, V)\ g(U, W) - g(X, U)\ g(V, W)] = 0. \tag{3.4}$$

In view of equation (1.1), above equation takes the form

$- g(X, R(U, V)W) - \frac{r}{n(n-1)} [g(V, W)U - g(U, W)V] + [1 + \frac{r}{n(n-1)}] [g(X, V)\ g(U, W) - g(X, U)\ g(V, W)] = 0$

$\Rightarrow - g(X, R(U, V)W) - \frac{r}{n(n-1)} [g(X, V)\ g(U, W) - g(X, U)\ g(V, W)] + [1 + \frac{r}{n(n-1)}] [g(X, V)\ g(U, W) - g(X, U)\ g(V, W)] = 0$

$$\Rightarrow - g(X, R(U, V)W) + [g(X, V)\ g(U, W) - g(X, U)\ g(V, W)] = 0 \tag{3.5}$$

Taking $X = U= e_i$ in above equation and summing over i, $1 \leq i \leq n$, we get

$- 'R(ei, V, W, ei) + [g(ei, v)\ g(ei, W) - g(ei, ei)\ g(V, W)] = 0$

$$\Rightarrow - S(V,W) + [g(V,W) - ng(V, W)] = 0. \tag{3.6}$$

Taking $V= W = \xi$ in above equation, we get

$- S(\xi,\xi) + [g(\xi,\xi) - ng(\xi,\xi)] = 0$

$\Rightarrow \lambda + (1 - n) = 0$

$\Rightarrow \lambda = n - 1.$

This shows that λ is positive, it means that the concircular curvative tensor $\tilde{C}$ in Kenmostu manifold satisfying $R(\xi,X).\tilde{C} = 0$ is expanding. This completes the proof.

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