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On Pgprw-closed maps and pgprw-open maps in topological spaces

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Abstract
In this paper, we introduce pgprw-closed map from a topological space X to a topological space Y as the image of every closed set is pgprw-closed and also we prove that the composition of two pgprw-closed maps need not be pgprw-closed map. We also obtain some properties of pgprw-closed maps.

Keywords: Pgprw-closed maps, pgprw-open maps, pgprw-closed set, pgprw-open set.

1. Introduction

Generalized closed mappings were introduce and studied by Malghan [12]. Rg-closed maps by Arockiaran [13]. In this paper, a new class of maps called pre generalized pre regular weakly closed maps (briefly, pgprw-closed) maps have been introduced and studied their relations with various generalized closed maps. We prove that the composition of two pgprw -closed maps need not be pgprw-closed map. We also obtain some properties of pgprw-closed maps. Wali and Vivekananda Dembre [14] introduced new class of sets called pre generalized pre regular weakly - closed (briefly pgprw - closed) sets in topological spaces which lies between the class of all p - closed sets and the class of all gpr - closed sets

2. Preliminaries

Throughout this paper space (X, τ) and (Y, σ) (or simply X and Y) always denote topological space on which no separation axioms are assumed unless explicitly stated. For a subset A of a space X, Cl(A), Int(A), A^c, P-Cl(A) and P-int(A) denote the Closure of A, Interior of A, Compliment of A, pre-closure of A and pre-interior of (A) in X respectively.

2.1 Definition: A subset A of a topological space (X,τ) is called

1. Semi-open set [1] if $A \subseteq cl(int(A))$ and semi-closed set if $int(cl(A)) \subseteq A$..
2. Semi-pre open set [2] (= β-open[1] if $A\subseteq cl(int(cl(A)))$and a semi-pre closed set(=β-closed) if $int(cl(int(A)))\subseteq A$.
3. Pre-open set [3] if $A \subseteq int(cl(A))$ and pre-closed set if $cl(int(A)) \subseteq A$.
4. Generalized pre closed(briefly gp-closed)set [4] if$pcl(A)\subseteq U$ whenever$A\subseteq U$ and U is open in X.
5. Generalized pre regular closed set(briefly gpr-closed) [5] if $pcl(A)\subseteq U$ whenever $A\subseteq U$ and U is regular open in X.
6. Generalized semi pre regular closed (briefly gspr-closed) set [6] if $spcl(A)\subseteq U$ whenever $A\subseteq U$ and U is regular open in X.
7. Pre-generalized-pre-regular closed(briefly pgpr-closed)set [7] if $pcl(A)\subseteq U$ whenever $A\subseteq U$ and U is rg- open in X.
8. W-closed set [8] if $cl(A) \subseteq U$ whenever $A\subseteq U$ and U is semi-open in X.
9. Regular generalized closed set(briefly rg-closed) [9] if $cl(A)\subseteq U$ whenever $A\subseteq U$ and U is regular open in X.
10. Generalized closed set(briefly g-closed) [10] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
11. Pre generalized pre regular ωeakly closed set[14](briefly pgprω-closed set) if $pCl(A)\subseteq U$ whenever $A\subseteq U$ and U is rgα open in (X, τ).
12. pre generalized pre-regular ωeakly open(briefly pgprω-open) [15] set in X if A^c is pgprω-closed in X.

2.2 Definition: A map f: (X, τ) → (Y, σ) is called
1. [12] g-closed map if f (F) is g-closed in (Y, σ) for every closed set of (X, τ)
2. [8] w-closed map if f (F) is w-closed in (Y, σ) for every closed set of (X, τ).
3. [13] rg-closed map if f (F) is rg-closed in(Y, σ) for every closed set of (X, τ) .
4. [5] gpr-closed map if f (F) is gpr-closed in(Y, σ) for every closed set of(X, τ) .
5. [23] gspr-closed map if f (F) is gspr-closed in (Y, σ) for every closed set of (X, τ).
6. [19] β-closed map if f (F) is β-closed in (Y, σ) for every closed set of (X, τ).
7. [22] semi-closed map if f (F) is semi-closed in(Y, σ) for every closed set of (X, τ).
8. [20] p-closed map if f (F) is p-closed in (Y, σ) for every closed set of (X, τ).
9. [21] gp-closed map if f (F) is gp-closed in (Y, σ) for every closed set of (X, τ)
10. [17] Irresolute map if $f^{-1}(V)$ is semi- closed in X for every semi-closed subset V of Y.
11. [16] pgprw-irresolute (pgprw–irresolute) map if $f^{-1}(V)$ is pgprw–Closed set in X for every pgprw–closed set V in Y.
12. pgprw continuos function [18] if $f^{-1}(V)$ is a pgprw-closed of (X,τ) for every closed set of (Y,σ).

2.3 Definition: [10] A topological space X is said to be $T_{1/2}$ space if every g-closed set is closed.

2.4 Theorem: [14]
1. Every closed set is pgprw-closed set.
2. Every pre closed set is pgprw-closed set.
3. Every pgprw closed set is gp closed set.
4. Every pgprw closed set is gpr closed set.
5. Every pgprw closed set is gspr closed set.

3. On Pgprw-Closed Maps

3.1 Definition: A map f: (X, τ) → (Y, σ) is said to be Pre generalized pre regular weakly closed map (briefly pgprw-closed) if the image of every closed set (X, τ) is pgprw-closed in (Y, σ).

3.2 Theorem: Every closed map is pgprw-closed map but not conversely.

Proof: The proof follows from the definitions and fact that every closed set is pgprw-closed set from Theorem 2.4 [14].

3.3 Remark: The converse of the above theorem need not be true as seen from the following example.

3.4 Example: Consider X=Y= {a,b,c} with topologies τ = {X, ϕ,{a,c}} and σ = { Y, ϕ, {a}}. Let f: (X, τ) → (Y, σ) be the identity map. Then this function is pgprw-closed but not closed as the image of closed set {b} in X is {b} which is not closed set in Y.

3.5 Theorem: Every p-closed map is pgprw-closed map but not conversely.

Proof: The proof follows from the definitions and fact that every p-closed set is pgprw-closed set from Theorem 2.4 [14].

3.6 Remark: The converse of the above theorem need not be true as seen from the following example.

3.7 Example: Consider X=Y={a,b,c,d} with topologies τ = {X, ϕ,{a},{b},{a,b},{a,b,c}} and σ ={Y, ϕ,{a},{b},{a,b},{a,b,c}}. Let map f: (X, τ) →(Y, σ) defined by f(a)=c,f(b)=c,f(c)=b,f(d)=d. Then f is pgprw closed map but not closed map and not p-closed map as the image of closed set F={c,d}in X then f(F)={b,d} in Y which is not p-closed set in Y.

3.8 Theorem: If a map f: (X, τ)→(Y, σ) is closed then the following holds. If f is pgprw closed map then f is gp, gpr, gspr closed map but not conversely.

Proof: The proof follows from the defintions and fact that every pgprw closed set is gp, gpr, gspr closed set from Theorem 2.4 [14].

3.9 Remark: The converse of the above theorem need not be true as seen from the following example.

3.10 Example: Consider X=Y={a,b,c} with topologies τ = {X, ϕ,{a},{b,c}} and σ ={Y, ϕ,{a}.Let map f: X, τ) (Y, σ) →defined by f (a)=b, f (b)=a, f (c)=c. Then f is gp, gpr, gspr, closed map but not pgprw closed map as the image of closed set
F= {b, c} in X then f (F)={a, c} in Y which is not pgprw-closed set in Y.

3.11 Remark: The following example show that the β-closed maps and pgprw closed maps are independent and similarly semi-closed maps and pgprw-closed maps.

3.12 Example: Consider X=Y={a,b,c} with topologies τ = {X, ϕ,{a}}and σ ={Y, ϕ,{a}}.Let map f: (X, τ) (Y, σ) →defined by f(a)=b,f(b)=a,f(c)=c.. Then f is pgprw closed map but f is not semi closed map, β-closed map as closed set F= {b, c} in X then f(F)={a,c} in Y which is not semi-closed, β-closed set in Y.

3.13 Example: Consider X= {a, b, c},Y={a, b, c, d} with topologies τ = {X, ϕ,{a}}and σ ={Y, ϕ,{a},{b},{a, b},{a, b, c}}.Let map f: (X, τ) → (Y, σ)defined by f(a)=d, f(b)=b, f(c)=c.. Then f is semi-closed map, β-closed map, but f is not pgprw closed map as closed set F= {b, c} in X then f (F)={b, c} in Y which is not pgprw-closed set in Y.

3.14 Theorem: A map f: (X, τ)→(Y, σ) is pgprw-closed map if and only if for each subset S of (Y, σ) and each open set U containing $f^{-1}(S) \subseteq U$, there is a pgprw-open set of (Y, such that $S \subseteq V$ and $f^{-1}(V) \subseteq U$.

Proof: Suppose f is pgprw-closed map. Let $S \subseteq Y$ and U be an open set of (X, τ) such that $f^{-1}(S) \subseteq U$. Now X-U is closed set in (X, τ),. Since f is pgprw-closed map, f (X-U) is pgprw closed set in (Y, σ) . Then V=Y- f (X-U) is a pgprw-open set in(Y, σ). Note that $f^{-1}(S) \subseteq U$ implies $S \subseteq V$ and $f^{-1}(V) = X- f^{-1}(f(X-U)) \subseteq X-(X-U) = U$. That is $f^{-1}(V) \subseteq U$. For the converse, let F be a closed set of (X, τ) Then $f^{-1}(f(F)^c) \subseteq F^c$ and F^c is an open set in (X, τ) By hypothesis, there exists a pgprw-open set V in (Y, σ) such that $f(F)^c \subseteq V$ and $f^{-1}(V) \subseteq F^c$ and so $F \subseteq (f^{-1}(V))^c$. Hence $V^c \subseteq f(F) \subseteq f(((f^{-1}(V))^c)$ which implies $f(V) \subseteq V^c$. Since V^c is pgprw-closed in (Y, σ) and therefore f is pgprw-closed map.

3.15 Remark: The composition of two pgprw-closed maps need not be pgprw-closed map in general and this is shown by the following example.

3.16 Example: Consider X=Y=Z= {a,b,c} with topologies τ = {X, ϕ,{a},{a,b}} and σ ={Y, ϕ,{a}},

μ= { Z,ϕ,{a},{c},{a,c}}.Define f: (X, τ)→ (Y, σ) and g: (X, τ)→ (Y, σ) be the identity map. Then f and g are pgprw-closed maps but their composition g of: (X, τ) →(Z, μ) is not pgprw-closed map because F={c} is closed in (X, τ) but g of({c}) ={c} which is not pgprw-closed in (Z, μ).

3.17 Theorem: If f: (X, τ)⟶ (Y, σ) is closed map and g: (Y, σ)→ (Z, μ) is pgprw-closed map, then the g of: (X, τ)→ (Z, μ) composition is pgprw-closed map.

Proof: Let F be any closed set in (X, τ) Since f is closed map, f (F) is closed set in (Y, σ) .Since g is pgprw-closed map, g (f (F)) is pgprw-closed set in (Z,μ). That is g of (F) = g(f(F)) is pgprw-closed and hence g of is pgprw-closed map.

3.18 Remark: If f: (X, τ) → (Y, σ) is pgprw-closed map and g: (Y, σ) →(Z, μ).)is closed map, then the composition need not be pgprw-closed map as seen from the following example.

3.19 Example: Consider X=Y=Z= {a,b,c} with topologies τ = {X, ϕ,{a},{a,b}} and σ ={Y, ϕ,{a}}& μ ={ Z,ϕ,{a},{c},{a,c}}.Define f: X, τ)→ (Y, σ) and g:(Y, σ) → (Z, μ)be the identity map. Then f is pgprw closed map and g is a closed map but their composition g of: (X, , τ)→(Z, μ) is not pgprw-closed map since for the closed set {c}in (X, τ) but g of({c})= {c} which is not pgprw-closed in (Z, μ)

3.20 Theorem: Let (X, τ) and (Z, μ) be topological spaces and (Y, σ) be topological spac where every pgprw-closed subset is closed. Then the composition g of:(X, τ)⟶ (Z, μ) of the pgprw-closed maps f: (X, τ)→(Y, σ) and g: (Y, σ)→ (Z, μ) is pgprw-closed map.

Proof: Let A be a closed set of (X, τ). Since f is pgprw-closed map, f (A) is pgprw-closed map in. Then by hypothesis f (A) is closed. Since g is pgprw-closed map, g (f (A)) is pgprw closed map in (Z, μ) and g (f (A)) = g of (A). Therefore g of is pgprw-closed map.

3.21 Theorem: If f: (X, τ)→(Y, σ) is g-closed map, g: (Y, σ) → (Z, μ) be pgprw-closed map and (Y, σ) is $T_{1/2}$ – space then their composition is g of: (X, τ)→ (Z, μ) is pgprw closed map.

Proof: Let A be a closed set of (X, τ) Since f is g-closed map,f (A) is g-closed in (Y, σ),. Since g is pgprw-closed map,g (f (A)) is pgprw-closed in (Z, μ) and g (f (A)) = g of (A).Therefore g of is pgprw-closed map.

4. pgprw-open maps

Definition 4.1: A map f: (X, τ)→(Y, σ) is called a pgprw-open map if the image f (A) is pgprw-open in (Y, σ) for each open set A in (X, τ)

4.2 Theorem: For any bijection map f: (X, τ)→(Y, σ), the following statements are equivalent.
1. f^{-1}: (Y, σ)⟶(X, τ) is pgprw-continuous map.
2. f is pgprw-open map and
3. f is pgprw-closed map.

Proof

(i) implies (ii**)** Let U be an open set of(X, τ)By assumption, $(f^{-1})^{-1}$ (U) = f (U) is pgprw-open in (Y, σ) and so f is pgprw-open map.

(ii) implies (iii) Let F be a closed set of (X, τ), Then F^c is open set in (X, τ). By assumption
f (F^c) is pgprw-open in (Y, σ) . That is f (F^c) = f $(F)^c$ is pgprw-open in (Y, σ) and therefore f(F) is pgprw-closed in (Y, σ) .Hence F is pgprw-closed map.

(iii) implies (i) Let F be a closed set of (X, τ), By assumption, f (F) is pgprw-closed in (Y, σ)
But f (F) = $(f^{-1})^{-1}$ (F) and therefore f^{-1} is pgprw continuous map.

4.3 Theorem: A map f:(X, τ) →(Y, σ) is pgprw-open if and only if for any subset S of (Y, σ) and any closed set of containing f^{-1}(S), there exists a pgprw-closed set K of (Y, σ) containing S such that f^{-1}(K) ⊆F.

Proof: Suppose f is pgprw-open map. Let S ⊆ Y and F be a closed set of (X, τ), such that f^{-1}(S) ⊆ F. Now X-F is an open set in (X, τ).Since f is pgprw-open map, f (X-F) is pgprw-open set in (Y, σ). Then K=Y- f (X-F) is a pgprw closed set in (Y, σ).
Note that f^{-1}(S) ⊆ F implies S ⊆K and f^{-1}(K) = X- f^{-1}(X-F) ⊆X-(X-F) = F. That is f^{-1}(K) ⊆ F. For the converse let U be an open set of (X, τ), Then $f^{-1}((f(U))^c)$ ⊆ U^c and U^c is a closed set in (X, τ). By hypothesis, there exists a pgprw-closed set K of (Y, σ) such that $(f (U))^c$ ⊆K and f^{-1}(K) ⊆U^c and so U $(f^{-1}(K))^c$. Hence K^c⊆ f (U) ⊆f $((f -1(K))^c$ which implies f(U) = K^c. Since K^c is a pgprw-open, f(U) is pgprw open in (Y, σ) and therefore f is pgprw-open map.

4.4 Theorem: Let f: (X, τ)→(Y, σ) and g: (Y, σ)→ (Z, μ) be two mappings such that
Their composition g of: (X, τ) →(Z, μ) is composition be pgprw-closed mapping. Then the following statements are true.

1. If f is continuous map and surjective, then g is pgprw-closed map
2. If g is pgprw-irresolute map and injective, then f is pgprw-closed map.
3. If f is g-continuous map, surjective and (X, τ) is a $T_{1/2}$ – space, then g is pgprw-closed map.

Proof

1. Let A be a closed set of (Y, σ) Since f is continuous map, f^{-1}(A) is closed in g of (f $^{-1}$(A)) is pgprw-closed in (Z, μ). That is g (A) is pgprw closed in (Z, μ) since f is surjective. Therefore g is pgprw closed map.
2. Let B be a closed set of (X, τ).Since g of is pgprw-closed map, g of (B) is pgprw-closed map in (Z, μ) Since g is pgprw-irresolute map, g^{-1}(g of (B)) is pgprw-closed set in (Y, σ) That is f (B) is pgprw-closed map in (Y, σ) since f is injective. Therefore f is pgprw-closed map.
3. Let C be a closed set of (Y, σ) Since f is g-continuous map,f^{-1}(c) is g-closed set in (X, τ). Since (X, τ) is a $T_{1/2}$ -space, f^{-1}(c) is closed set in (X, τ)Since g of is pgprw-closed map (g of) (f $^{-1(}$c)) is pgprw-closed map in (Z, μ), That is g(c) is pgprw-closed map in (Z, μ) since f is subjective. Therefore g is pgprw-closed map.

5. Conclusion: In this paper, a new class of maps called pgprw-closed maps and pgprw-open maps in topological spaces are introduced and investigated and we observed that the composition of two pgprw-closed maps need not be pgprw-closed maps. In future the same process will be analyzed for pgprw-properties.

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