International Journal of Statistics and Applied Mathematics 2016; 1(1): 46-51

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2016; 1(1): 46-51
© 2016 Stats & Maths
www.mathsjournal.com
Received: 15-03-2016
Accepted: 18-04-2016

RS Wali
Department of Mathematics,
Bhandari Rathi College,
Guledagudd, Karnataka State,
India

Prabhavati S Mandalageri
Department of Mathematics,
K.L.E'S, S.K. Arts College &
H.S.K. Science Institute,
Hubballi, Karnataka State,
India

Correspondence:
RS Wali
Department of Mathematics,
Bhandari Rathi College,
Guledagudd, Karnataka State,
India

Some Propertıes of Rw-Locally Closed Sets In Topologıcal Spaces

RS Wali and Prabhavati S Mandalageri

Abstract
In this paper, we introduce three weaker forms of locally closed sets called RW-LC sets, RW-LC* set and RW-LC** sets each of which is weaker than locally closed set and study some of their properties in topologivcal spaces.

Keywords: Rw-closed sets, rw-open sets, locally closed sets, rw-locally closed sets

1. Introduction

Kuratowski and Sierpinski [11] introduced the notion of locally closed sets in topological spaces. According to Bourbaki [6], a subset of a topological space (X, τ) is locally closed in (X, τ) if it is the intersection of an open set and a closed set in (X, τ). Stone [2] has used the term FG for locally closed set. Ganster and Reilly [7] have introduced locally closed sets, which are weaker forms of both closed and open sets. After that Balachandran *et al.* [2, 3], Gnanambal [10], Arockiarani *et al.* [1], Pusphalatha [12] and Sheik John [13] have introduced α-locally closed, generalized α-locally closed, semi locally closed, semi generalized locally closed, regular generalized locally closed, strongly locally closed and w-locally closed sets and their continuous maps in topological space respectively. Recently as a generalization of closed sets rw-closed sets and continuous maps were introduced and studied by Benchalli *et al.* [12]. A subset A of a topological space (X, T) is said to be rw-closed set if Cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi regular-open.

2. Preliminaries: A subset A of topological space (X, τ) is called a

1. Locally closed (briefly LC) set [7] if A=U$\cap$F,where U is open and F is closed in X.
2. rw-closed set [14] if Cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi regular-open.
3. αrω-closed set [11] if αCl(A) $\subseteq$ U whenever A $\subseteq$ U and U is rw-open.
4. θg-lc set [4] if A=U$\cap$F,where U is θg-open and F is θg-closed in X.
5. θg-lc* set [4] if A=U$\cap$F,where U is θg-open and F is closed in X.
6. θg-lc set** [4] if A=U$\cap$F,where U isopen and F θg-closed in X.
7. g-lc set if A=U$\cap$F [6] where U is g-open and F is g-closed in X.
8. g-lc* set if A=U$\cap$F [6] where U is g-open and F is closed in X.
9. g-lc set** if A=U$\cap$F [6] where U is open and F is g-closed in X.
10. w-lc set if A=U$\cap$F [8] where U is w-open and F is w-closed in X.
11. w-lc* set if A=U$\cap$F [8] where U isw-open and F isclosed in X.
12. w-lc** set if A=U$\cap$F [8] where U isopen and F is w-closed in X.
13. rg-lc set if A=U$\cap$F [5] where U is rg-open and F rg-closed in X.
14. rg-lc* set if A=U$\cap$F [5] where U is rg-open and F is closed in X.
15. rg-lc** set if A=U$\cap$F [5] where U is open and F is rg-closed in X.

16. δg-lc set if A=U$\cap$F [3] where U is $l\delta$g-open and F is lδg-closed in X.
17. lδg-lc* set if A=U$\cap$F [3] where U is $l\delta$g-open and F isclosed in X.
18. lδg-lc** set if A=U$\cap$F [3] where U is open and F is lδg-closed in X.
19. αrω-lc set if A=U$\cap$F [14] where U is αrω-open and F is αrω-closed in X.
20. αrω-lc* set if A=U$\cap$F [14] where U is αrω-open and F is closed in X.
21. αrω-lc** set if A=U$\cap$F [14] where U is open and F is αrω-closed in X.

2.1 Lemma: [14]
(i) Every closed set is rw-closed set.
(ii) Every w-closed set is rw-closed set.
(iii) Every θ-closed set is rw-closed set.
(iv) Every δ-closed set is rw-closed set.
(v) Every rw-closed set is rg-closed set.

2.2 Lemma: The space (X, τ) is T_{rw}-space if every rw-closed set is closed set.

3. rw-locally closed sets in topological spaces

3.1 Definition: A Subset A of t.s (X, τ) is called rw-locally closed (briefly RW-LC) if A=U$\cap$F where U is rw-open in (X, τ) and F is rw-closed in (X, τ).
The set of all rw-locally closed sets of (X, τ) is denoted by RW-LC(X, τ).

3.2 Example: Let X={a, b, c, d} and τ = {X, ϕ ,{a},{b},{a,b},{a,b,c}}
RWC(X, τ) = { X, ϕ ,{d},{a,b},{c,d},{a,b,c},{a,c,d},{b,c,d},{a,b,d}}.
RW-LC-Setτ = {X, ϕ ,{a},{b},{c},{d},{a,b}{a,c},{b,c},{c,d},{a,b,c},{a,b,d},{b,c,d},{a,c,d}}.

3.3 Remark: The following are well known
(i) A Subset A of (X, τ) is RW-LC set iff it's complement X-A is the union of a rw-open set and a rw-closed set.
(ii) Every rw-open (resp. Rw-closed) subset of (X, τ) is a RW-LC set.
(iii) The Complement of a RW-LC set need not be a RW-LC set.
(In Example 3.2 the set {b,c} is RW-LC set, but complement of {b,c} is {a,d}, which is not RW-LC set.)

3.4 Theorem: Every locally closed set is a RW-LC set but not conversely.
Proof: The proof follows from the two defintions [follows from Lemma 2.1] and fact that every closed (resp.open) set is rw-closed (rw-open).

3.5 Example: Let X={a,b,c} and τ ={X, ϕ ,{a},{b,c}} then {a,b} is RW-LC set but not a locally closed set in (X, τ).
3.6 Theorem: Every w-lc set is a RW-LC set but not conversely.
Proof: The proof follows from the two defintions [follows from Lemma 2.1] and fact that every w-closed (resp.w-open) set is rw-closed (rw-open).

3.7 Example: Let X={a,b,c} and τ ={X, ϕ ,{a}} then {a,c} is RW-LC set but not a w-locally closed set in (X, τ).
3.8 Theorem: Every θ-lc set is a RW-LC set but not conversely.
Proof: The proof follows from the two defintions [follows from Lemma 2.1] and fact that every θ-closed (resp. θ-open) set is rw-closed (rw-open).

3.9 Example: Let X={a,b,c,d} and τ ={X, ϕ ,{a},{b},{a,b},{a,b,c}} then {c} is RW-LC set but not a θ-locally closed set in (X, τ).
3.10 Theorem: Every $l\delta$c set is a RW-LC set but not conversely.
Proof: The proof follows from the two defintions [follows from Lemma 2.1] and fact that every δ-closed (resp. δ-open) set is rw-closed (rw-open).

3.11 Example: Let X={a,b,c} and τ ={X, ϕ ,{a},{c},{a,c},{b,c}} then {a,b} is RW-LC set but not a $l\delta$c set in (X, τ).
3.12 Theorem:.Every RW-LC set is rg-lc set but not conversely.
Proof: The proof follows from the two defintions [follows from Lemma 2.1] and fact that every rw-closed (resp. rw-open) set is rg-closed (rg-open).

3.13 Example: Let X={a,b,c,d} and τ ={X, ϕ , {a},{b},{a,b},{a,b,c}} then {a,d} is rg-lc set but not RW-LC set in (X, τ).
3.14 Remark: lδgc-sets and RW-LC sets are independent of each other as seen from the following example
3.15 Example: (i) Let X={a, b, c, d} and τ ={X, ϕ , {a}, {b}, {a, b},{a, b, c}} then {a, d} is lδgc -lc but not RW-LC set in (X, τ).

(ii) Let X={a, b, c, d} and T ={X, Φ, {a}, {b}, {a, b},{a, b, c}} then {c} is RW-LC but not l$^{\delta}$gc-lc set in (X, τ).

3.16 Remark: θ-g lc sets and RW-LC sets are independent of each other as seen from the following example

3.17 Example: i) Let X={a,b,c,d} and τ ={X, Φ ,{a},{b},{a,b},{a,b,c}} then {b,d} is θ-g -lc but not RW-LC set in (X, τ).

(ii) Let X={a,b,c} and τ ={X, Φ ,{a},{a,b},{a,c}} then {a,b} is RW-LC but not θ-g-lc set in (X, τ).

3.18 Definition: A subset A of (X, τ) is called a RW-LC* set if there exists a rw-open set G and a closed F of (X, τ) s.t A=G$\cap$F the collection of all RW-LC* sets of (X, τ) will be denoted by RW-LC*(X, τ).

3.19 Definition: A subset B of (X, τ) is called a RW-LC** set if there exists an open set G and rw closed set F of (X, τ) s.t B=G$\cap$F the collection of all RW-LC** sets of (X, τ) will be denoted by RW-LC**(X, τ).

3.20 Theorem

1. Every locally closed set is a RW-LC* set.
2. Every locally closed set is a RW-LC** set.
3. Every RW-LC* set is RW-LC set.
4. Every RW-LC** set is RW-LC set.
5. Every W-LC* set is RW-LC* set.
6. Every W-LC**set is RW-LC** set.
7. Every RW-LC* set is rg-lc* set.
8. Every RW-LC** set is rg rg-lc** set
9. Every RW-LC* set is rg-lc set.
10. Every RW-LC** set is rg-lc set.
11. Every $l\delta$c set is a RW-LC* set.
12. Every $l\delta$c set is a RW-LC** set.
13. Every θ-lc* set is a RW-LC* set.
14. Every θ-lc** set is a RW-LC** set.

Proof: The proof are obivious from the definitions and the relation between the sets.
However the converses of the above results are not true as seen from the following examples.

3.21 Example: Let X={a,b,c} and τ ={X, Φ ,{a}}
(i) The set {b} is RW-LC* set but not a locally closed set in (X, τ).
(ii) The set {a,b} is RW-LC** set but not a locally closed set in (X, τ).
(iii) The set {c} is RW-LC* set but not a w-lc* set in (X, τ).
(iv) The set {c} is RW-LC** set but not a w-lc** set in (X, τ).

3.22 Example: Let X={a,b,c,d} and τ ={ X, Φ ,{a},{b},{a,b},{a,b,c}}
(i) The set {a,b,d} is RW-LC set but not a RW-LC* set in (X, τ).
(ii) The set {a,d} is rg-lc* set but not a RW-LC* set in (X, τ).
(iii) The set {a,d} is rg-lc* set but not a RW-LC* set in (X, τ).
(iv) The set {b,d} is rg-lc set but not a RW-LC* set in (X, τ).
(v) The set {b,d} is rg-lc set but not a RW-LC** set in (X, τ).
(vi) The set {b,d} is αrω-lc set but not a RW-LC set in (X, τ).

3.23 Example: Let X={a,b,c,d} and τ ={ X, Φ ,{a},{b},{a,d},{a,b,d}}
(i) The set {b} is RW-LC* set but not a θ-lc* set in (X, τ).
(ii) The set {d} is RW-LC** set but not a θ-lc** set in (X, τ).

3.24 Example: Let X={a,b,c} and τ ={ X, Φ ,{a},{c},{a,c},{b,c}}
(i) The set {c} is RW-LC* set but not a $l\delta$c set in (X, τ).
(ii) The set {c} is RW-LC** set but not a $l\delta$c set in (X, τ).

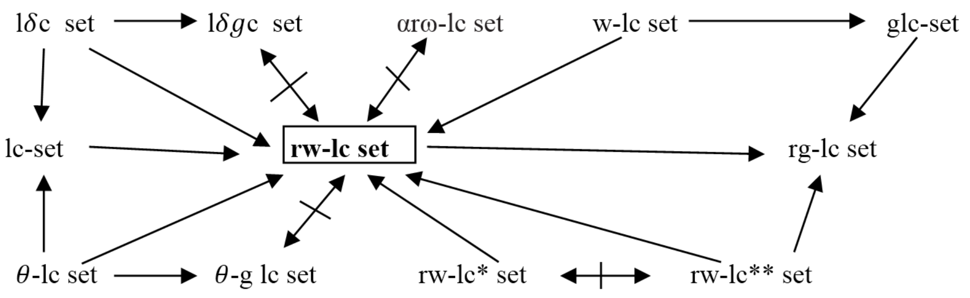
3.25 Example: i) Let X={a,b,c,d} and τ ={ X, Φ ,{a},{b},{a,b}} then the set {c} is RW-LC set but not a RW-LC** set in (X, τ)
ii) Let X={a,b,c} and τ ={ X, Φ ,{a},{b,c}} then the set {c} is RW-LC set but not a αrω-lc set in (X, τ).

3.26 Remark: RW-LC* sets and RW-LC** sets are independent of each other as seen from the examples.

3.27 Example: (i) Let X={a,b,c,d} and τ ={ X, ϕ ,{a},{b},{a,b},{a,b,c}} then set {a,b,d} is RW-LC** set but not a RW-LC* set in (X, τ).

(ii) Let X={a,b,c,d} and τ ={ X, ϕ ,{a},{b},{a,b}} then set {c} is RW-LC* set but not a RW-LC** set in (X, τ).

3.38 Remark: From the above discussion and known results we have the following implications in the diagram.



A ←┼→ B means A & B are independent of each other

A → B means A implies B but not conversely

3.39 Theorem: If RWO(X, τ) = τ then

(i) RW-LC(X, τ) = LC (X, τ).

(ii) RW-LC(X, τ) = W-LC (X, τ).

(iii)RW-LC(X, τ) $\subseteq$ GLC (X, τ).

Proof: (i) For any space (X, τ), W.k.t LC(X, τ) $\subseteq$ RW-LC(X, τ). Since RWO(X, τ) = τ,
that is every rw-open set is open and every rw-closed set is closed in (X, τ), RW-LC(X, τ) $\subseteq$LC (X, τ); hence RW-LC(X, τ) = LC (X, τ).

(ii) For any space (X, τ), LC(X, τ) $\subseteq$W-LC(X, τ) $\subseteq$RW-LC(X, τ) From (i) it follows that RW-LC(X, τ) = W-LC(X, τ).

(iii) For any space (X, τ), LC(X, τ) $\subseteq$GLC (X, τ) from (i) RW-LC(X, τ) = LC (X, τ) and hence
RW-LC(X, τ) $\subseteq$GLC (X, τ).

3.40 Theorem: If RWO(X, τ) = τ, then RW-LC*(X, τ) = RW-LC** (X, τ) = RW-LC(X, τ).

Proof: For any space,(X, τ) LC (X, τ) $\subseteq$ RW-LC*(X, τ) $\subseteq$ RW-LC(X, τ) and LC (X, τ) $\subseteq$ RW-LC**(X, τ) $\subseteq$ RW-LC(X, τ) since RWO(X, τ) = τ. RW-LC(X, τ) =LC(X, τ) by theorem 3.39,it follows that LC(X, τ) =RW-LC*(X, τ) = RW-LC**(X, τ) = RW-LC(X, τ).

3.41 Remark: The converse of the theorem 3.40 need not be true in general as seen from the following example.

3.42 Example: Let X={a, b, c} with the topology τ= { X, ϕ, {a}, {b}, {a,b}} then RW-LC*(X, τ) = RW-LC**(X, τ) = RW-LC(X, τ) =P(X). However RWO(X, τ) ={ X, ϕ , {a}, {b}, {c}, {a,b} }$\neq$ τ.

3.43 Theorem: If GO(X, τ) = τ, then GLC(X, τ) $\subseteq$ RW-LC(X, τ)

Proof: For any space (X, τ) w.k.t LC(X, τ) $\subseteq$ GLC(X, τ) and LC (X, τ) $\subseteq$RW-LC(X, τ).....(i) GO(X, τ) =T, that is every g-open set is open and every g-closed set is closed in (X, τ) and so GLC(X, τ) $\subseteq$LC(X, τ) that is GLC(X, τ) =LC(X, τ)......(ii) from (i) and (ii) we have GLC(X, τ) $\subseteq$ RW-LC(X, τ).

3.44 Theorem: If RWC(X, τ) $\subseteq$ LC(X, τ) then RW-LC(X, τ) = RW-LC*(X, τ)

Proof: Let RWC(X, τ) $\subseteq$ LC(X, τ), For any space (X, τ), w.k.t RW-LC*(X, τ) $\subseteq$ RW-LC(X, τ)...(i) Let A$\in$RWC(X, τ), then A= U$\cap$F, where U is rw-open and F is a rw-closed ın (X, τ). Now, F$\in$RW-LC(X, τ) by hypothesis F is locally closed set in (X, τ), then F= G$\cap$E, where G is an open set and E is a closed set in (X, τ).

Now, A= U$\cap$F=U $\cap$ (G$\cap$E)= (U$\cap$G) $\cap$ E, where U$\cap$G is rw-open as the intersection of rw-open sets is rw-open and E is a closed set in (X, τ). It follows that A is RW-LC*(X, τ). That is A$\in$ RW-LC* (X, τ) and so, RWC(X, τ) $\subseteq$ RW-LC*(X, τ)......(ii).
From (i) and (ii) we have RW-LC(X, τ) = RW-LC*(X, τ).

3.45 Remark: The converse of the theorem 3.44 need not be true in general as seen from the following example.

3.46 Example: Consider X= {a,b,c,d} and τ = { X, ϕ ,{a,b},{c,d}},then RW-LC(X, τ) = RW-LC*(X, τ) =P(X). But RWC(X, τ) =P(X) and LC(X, τ) = { X, ϕ ,{a,b},{c,d}} That is RWC(X, τ) $\nsubseteq$ LC(X, τ).

3.47 Theorem: For a subset A of (X, τ) if A $\in$ RW-LC(X, τ) then A = U $\cap$ (rw-cl(A)) for some open set U.

Proof : Let, A$\in$ RW-LC(X, τ) then there exist a rw-open U and a rw-closed set F s.t. A= U$\cap$F.Since A$\subseteq$ F, Rw-cl(A)= rw-cl(F) =F. Now U $\cap$ (rw-cl(A)) $\subseteq$U$\cap$F= A, that is U$\cap$ (rw-cl(A))= A.
Conversely A$\subseteq$ U and A $\subseteq$ Rw-cl(A) implies A $\subseteq$ U $\cap$ (rw-cl(A)) and therefore A= U $\cap$ (RW-cl(A)) for some rw-open set U.

3.48 Remark: The converse of the theorem 3.47 neeed not be true in general as seen from the following example.
3.49 Example: Consider X= {a,b,c,d} with the topology τ= { X, ϕ ,{a},{b},{a,b},{a,b,c}} then
RWC(X, τ) = { X, ϕ , {d}, {a,b}, {c,d}, {a,b,c}, {a,b,d}, {a,c,d}, {b,c,d}} then
RW-LC(X, τ) ={ X, ϕ , {a}, {b},{c}, {d}, {a,b}, {c,d}, {b,c}, {a,c}, {a,b,c}, {a,b,d}, {a,c,d}, {b,c,d}}
Take A= {b,d}, rw-cl(A)= {b,d} now, A = X $\cap$ (rw-cl(A)) for some rw-open set X but {b,d} $\notin$ RW-LC (X, τ).

3.50 Theorem: For a subset A of (X, τ), the following are equivalent.
(i) A $\in$ RW-LC*(X, τ).
(ii) A= U $\cap$ (cl(A) for some rw-open set U.
(iii) cl(A)-A is rw-closed.
(iv) A $\cup$ (cl(A)^c is rw-open.

Proof : (i)implies (ii) Let A$\in$ RW-LC*(X, τ) then there exists a rw-open set U and a closed set F s.t A = U$\cap$F. Since A $\subseteq$ F, cl(A) $\subseteq$ cl(F)= F. Now U$\cap$cl(A) $\subseteq$ U$\cap$F = A that is U$\cap$ cl(A)= A. Conversely A $\subseteq$ U, and
A $\subseteq$ cl(A) implies A $\subseteq$ U$\cap$ cl(A) and therefore A = U$\cap$cl(A) for some rw-open set U.
(ii) implies (i) since U is a rw-open set and cl(A) is a closed set, A= U $\cap$ (cl(A) $\in$ RW-LC*(X, τ).
(iii) implies (iv) let F = cl(A)-A, then F is rw-closed by the assumption and X-F=X-[cl(A)-A] = X $\cap$ [cl(A)-A]^c = A $\cup$ (X- cl(A)) = A $\cup$ (cl(A))^c. But X- F is rw-open. This shows that A $\cup$ (cl(A)) ^{c} is rw-open.
(iv) implies (iii) Let U = A $\cup$ (cl(A)^c then U is rw-open, this implies X-U is rw-closed and X-U = X – (A$\cup$cl(A)^c) = cl(A) $\cap$ (X-A)= cl(A)-A is rw-closed.
(iv) impies (ii) Let U = A $\cup$ (cl(A)^c then U is rw-open. hence we prove that A =U$\cap$ (cl(A)c for some rw-open set U.
Now A = U$\cap$ (cl(A) = [A$\cup$(cl(A)^c] $\cap$ cl(A) = A$\cap$ [cl(A)] $\cup$ cl(A)^c$\cap$ cl (A) = A $\cup$ ϕ = A.
Therefore A = U$\cap$ (cl(A) for some rw-open set U.
(ii) implies (iv) Let A = U $\cap$ (cl(A) for some rw-open set then we p.t A $\cup$ (cl(A)^c is rw-open. Now A $\cup$ (cl(A)^c = (U$\cap$ (cl(A)) $\cup$ [cl(A))^c = U$\cap$ (cl(A)) $\cup$ [cl(A))^c =U $\cap$ X = U, which is rw-open.Thus A= (cl(A))^c is rw-open.

3.51 Theorem: For a subset A of (X, τ) if A$\in$RW-LC**(X, τ), then there exists an open set U s.t A= U $\cap$ rw-cl(A).
Proof: Let A$\in$RW-LC**(X, τ), then there exist an open set U and a rw-closed set s.t A = U $\cap$ F Since A $\subseteq$ U and A $\subseteq$rw-cl(A) we have A $\subseteq$ rw-cl(A).
Conversely, Since A $\subseteq$ F and rw-cl(A) $\subseteq$ rw-cl(F) = F, as F is rw-closed. Thus U $\cap$ rw-cl(A) $\subseteq$ U $\cap$ F= A. That is U $\cap$ rw-cl(A) $\subseteq$ A; hence A= U $\cap$ rw-cl(A). For some open set U.

3.52 Remark: The converse of the theorem 3.27 need not be true in general as seen from the following example.
3.53 Example: Let X= {a,b,c,d} with the topology τ= { X, ϕ ,{a},{b},{a,b},{a,b,c}} take A= {a,d}. Then rw-cl(A) = rw-cl{a,d}={a,d}; also A= X $\cap$ rw-cl(A)= {a,b,c,d}$\cap${a,d}={a,d} for some open set X but {a,d}$\in$ RW-LC**(X, τ).

3.54 Theorem: For A and B in (X, τ) the following are true.
(i) if A$\in$RW-LC*(X, τ) and B$\in$RW-LC*(X, τ), then A$\cap$B$\in$RW-LC*(X, τ).
(ii) if A$\in$RW-LC**(X, τ) and B is open,then A$\cap$B$\in$RW-LC**(X, τ).
(iii) if A$\in$RW-LC(X, τ) and B is rw-open,then A$\cap$B$\in$RW-LC(X, τ).
(iv)) if A$\in$RW-LC*(X, τ) and B is rw-open,then A$\cap$B$\in$RW-LC*(X, τ).
(v)) if A$\in$RW-LC*(X, τ) and B is closed,then A$\cap$B$\in$RW-LC*(X, τ).

Proof: (i) Let A, B $\in$RW-LC*(X, τ), it follows from theorem 3.--- that there exist rw-open sets P and Q s.t A = P$\cap$ cl(A) and B = Q $\cap$ cl(B). Therefore A$\cap$B = P$\cap$ cl(A) $\cap$ Q $\cap$ cl(B) =P $\cap$ Q $\cap$[cl(A) $\cap$ cl(B)] where P $\cap$ Q is rw-open and cl(A) $\cap$ cl(B) is closed. This shows that A$\cap$B $\in$ RW-LC*(X, τ).
(ii)Let A$\in$RW-LC**(X, τ) and B is open. Then there exist an open set P and rw-closed set F s.t A = P $\cap$ F. Now, A$\cap$B = P$\cap$ F $\cap$ B = (P$\cap$B) $\cap$ F, Where (P$\cap$B) is open and F is rw-closed. This implies A$\cap$B$\in$RW-LC**(X, τ).

(iii) Let A $\in$ RW-LC(X, τ) and B is rw-open then there exists a rw-open set P and Q rw-closed set F s.t A = P $\cap$ F. Now, A $\cap$ B = P $\cap$ F $\cap$ B = (P $\cap$ B) $\cap$ F, Where (P $\cap$ B) is rw-open and F is rw-closed. This shows that A $\cap$ B $\in$ RW-LC(X, τ).

(iv) Let A $\in$ RW-LC*(X, τ) and B is rw-open then there exists a rw-open set p and Q rw-closed set F s.t A = P $\cap$ F. Now, A $\cap$ B = (P $\cap$ F) $\cap$ B = (P $\cap$ B) $\cap$ F, Where (P $\cap$ B) is rw-open and F is closed. This implies that A $\cap$ B $\in$ RW-LC*(X, τ).

(v) A $\in$ RW-LC*(X, τ) and B is closed. Then there exist an rw-open set P and a closed set F s.t A = P $\cap$ F. Now, A $\cap$ B = (P $\cap$ F) $\cap$ B = P $\cap$ (F $\cap$ B), Where (F $\cap$ B) is closed and p is rw-open. This implies A $\cap$ B $\in$ RW-LC*(X, τ).

3.55 Definition
A topological space (X, τ) is called RW-submaximal if every dense set in it is RW-open.

3.56 Theorem: If (X, τ) is submaximal space then it is RW-submaximal space but converse need not be true in general.
Proof: Let (X, τ) be submaximal space and A be a dense subset of (X, τ). Then A is open. But every open set is rw-open and so A is rw-open. Therefore (X, τ) is a RW-submaximal space.

3.57 Example: Let X = {a, b, c} and τ = { ϕ, {a}, {b,c}, X}. Then Topological space (X, τ) is RW-submaximal but set A={a,b} is dense in (X, τ) but not open therefore (X, τ) is not submaximal.

3.58 Theorem: A topological space (X, τ) RW-submaximal if and only if P(X) = RW-LC*(X, τ).
Proof
Necessity: Let A $\in$ P(X) and U = A $\cup$ (X-cl(A)). Then it follows cl(U)= cl(A $\cup$ (X-cl(A))) =cl(A) $\cup$ (X-cl(A) = X. Since (X, τ) is rw-sub maximal, U is rw-open, so A $\in$ rwLC*(X, τ) from the Theorem 3.47 Hence P(X) = rwLC*(X, τ).

Sufficiency: Let A be dense sub set of (X, τ).Then by assumption and Theorem 3.50 (iv) that A $\cup$ (X-cl(A)) = A holds, A $\in$ rwLC*(X, τ) and A is rw-open. Hence (X, τ) rw-sub maximal.

3.59 Theorem: If (X, τ) T_{rw}-space then RW-LC(X, τ) = LC(X, τ).
Proof: Straight Forward.

3.60 Theorem: Let (X, τ) and (Y, σ) be toplogical spaces.

i) If A $\in$ RW-LC(X, τ) and B $\in$ RW-LC(Y, σ) then A × B $\in$ RW-LC (X × Y, τ ×σ).
ii) If A $\in$ RW-LC*(X, τ) and B $\in$ RW-LC*(Y, σ) then A × B $\in$ RW-LC* (X × Y, τ ×σ).
iii) If A $\in$ RW-LC**(X, τ) and B $\in$ RW-LC**(Y, σ) then A × B $\in$ RW-LC** (X × Y, τ ×σ).

Proof : i) If A $\in$ RW-LC(X, τ) and B $\in$ RW-LC(Y, σ). Then there exist rw-open sets U and V of (X, τ) and (Y,σ) and rw-clos d sets G and F of X and Y respectively such that A= U ∩ G and B= V ∩ F. Then A × B = (U × V) ∩ (G × F) holds. Hence A × B $\in$ RW-LC (X × Y, τ ×σ).
ii) and iii) Similarly the follow from the definition.

4. References

1. Pushpalatha A. Studies on generalizations of mappings in topological spaces, Ph.D., thesis, Bharathiar University, Coimbatore, 2000.
2. Kuratowski C, Sierpinski W. Sur les differences deux ensembles fermes, Tohoku Math. Jl. 1921; 20:22-25.
3. Perk HP, Perk JK. on locally δg *closed sets*, $l\delta g$ c-continuous functions,solitions and fractals 2004; 19:995-1002.
4. Arockiarani I, Balachandran K. on θ-g locally closed set (pre print).
5. Arockiarani I, Balachandran K, Ganster M. Regular generalized locally closed sets and rgl-continuous functions, Indian J Pure Appl Math. 1997; 28:661-669.
6. Balachandran K, sundaram P, Maki H. g-lc and glc-continuous function, indian j pure appp Math. 1966; 27:235-244.
7. Ganster M, Reilly IL. locally closed sets ad lc-contınuous function internret, j math sci. 1989; 12:417-424.
8. Sheik john M. a study on generalization of closed sets on continuous maps in topological & bitopological spaces, Ph.D thesis, Bharathiar university, Coimbatore, 2002.
9. Stone M. Application of the theorey of boolean rings to general topology, trans, amer, math, soc, al, 1937, 374-481.
10. Bourkbaki N. General topology, part I adison-wesley, reading mass., 1966.
11. Wali RS, Mandalgeri PS. On α Regular ω-closed sets in topological spaces, Int. J ofMath Archive. 2014; 5(10):68-76.
12. Wali RS, Mandalgeri PS. On α Regular ω-open sets in topological spaces, J of comp & Math Sci. 2014; 5(6):490-499.
13. Wali RS, Mandalgeri PS. On α Regular-locally closed sets in Topological spaces, Submitted.
14. Benchalli SS, Wali RS. on rw-Closed sets is Topological Spaces, Bull, Malays, Math, sci, soc. 2007; 30:99-110.
15. Gnanambal Y. On generalized pre regular closed sets in topological spaces, Indian J Pure Appl Math. 1997; 28(3):351-360.
16. Gnanambal Y. On generalized pre-regular Closed sets and generalized of locally closed sets in Topological Spaces, Ph.D thesis, Bharathiar university, Coimbatore, 1998.