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On special D(17)-quadruple

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Abstract
This paper concerns with the study of constructing a special Dio quadruple (a,b,c,d) such that the product of any two elements of the set difference with their sum and increased by 17 is a perfect square.

Keywords: Diophantine Quadruples, Pell equation 2010 Mathematics subject classification 11D(99)

1. Introduction

The problem of constructing the sets with property that the product of any two of its distinct elements is one less than a square has a very long history and such sets were studied by Diophantus [1]. A set of m positive integers $\{a_1, a_2 .....a_m\}$ is said to have the property D(n), $n \in z - \{0\}$ if $a_i a_j + n$, a perfect square for all $1 \le i \le j \le m$ and such a set is called a Diophantine m-tuples with property D(n). Many mathematicians considered the construction of different formulations of Diophantine quadruples with the property D(n) for any arbitrary integer n and also for any linear polynomials in n. In this context, one may refer [2-15] for an extensive review of various problems on Diophantine quadruples. This paper aims at constructing special dio – quadruple where the special mention is provided because it differs from the earlier one and the special Dio – quadruple is constructed where the product of any two members of the quadruple with the difference of the sum of the same members and the addition of seventeen satisfies the required property.

Method of Analysis

Let $a(k) = 8k^2 + 8k + 1, b(k) = 8k^2 - 8k + 1, k \ge 1$ be two integers such that $a(k)b(k) - [a(k) + b(k)] + 17$ is a perfect square.

Let $c_N(k)$ be any non-zero integer such that

$$[a(k) - 1]c_N(k) - a(k) + 17 = p_n^2(k)$$

$$[b(k) - 1]c_N(k) - b(k) + 17 = q_n^2(k)$$

Which, on simplification, results in the equations

$$2(k^2 + k)c_N(k) - 2k^2 - 2k + 4 = P_N^2(k); p_n(k) = 2P_N(k) \tag{1}$$

$$2(k^2 - k)c_N(k) - 2k^2 + 2k + 4 = Q_n^2(k); q_n(k) = 2Q_N(k) \tag{2}$$

Eliminating $c_N(k)$ between (1) and (2), we obtain

$$(k-1)p_N^2(k) - (k+1)Q_N^2(k) = -8 \tag{3}$$

Taking $$P_N(k) = 2X_N(k) + (2k+2)T_N(k) \tag{4}$$

$$Q_N(k) = 2X_N(k) + (2k-2)T_N(k) \tag{5}$$

in (3), we have

$$X_N^2(k) = (k^2 - 1)T_N^2(k) + 1 \qquad (6)$$

which is satisfied by

$$X_N(k) = \frac{1}{2}\left[(k+\sqrt{k^2-1})^{N+1} + (k-\sqrt{k^2-1})^{N+1}\right]$$

$$T_N(k) = \frac{1}{2\sqrt{k^2-1}}\left[(k+\sqrt{k^2-1})^{N+1} - (k-\sqrt{k^2-1})^{N+1}\right] \qquad (7)$$

Substituting N=0,1,2,3,…… in (7) and using (4) & (1) in turn, we have

$$c_0(k) = 9$$

$$c_1(k) = 32k^2 - 7$$

$$c_2(k) = 128k^4 - 96k^2 + 17$$

$$c_3(k) = 512k^6 - 640k^4 + 224k^2 - 15$$

and so on.

It is seen that the quadruple $(a(k), b(k), c_{N-1}(k), c_N(k))$, N=1,2,3,…… is a special dio-quadruple with property D(17). A few illustrations are given below.

k	$(a(k),b(k),c_0(k),c_1(k))$	$(a(k),b(k),c_1(k),c_2(k))$	$(a(k),b(k),c_2(k),c_3(k))$
2	(49,17,9,121)	(49,17,121,1681)	(49,17,1681,23409)
3	(97,49,9,281)	(97,49,251,9521)	(97,49,9521,323409)
4	(161,97,9,505)	(161,97,505,31249)	(161,97,31249,1936881)

Conclusion

This paper concerns with the construction of special dio - quadruples with property D(17). One may search for special dio-quadruples consisting of special numbers with suitable property.

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