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Mathematical analysis of MHD flow of an optically thin viscous fluid through a channel

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Abstract
In this paper, the influence of radiation on MHD flow of an optically thermal conductivity viscous incompressible fluid a through channel with a sliding wall and irregular temperatures are presented. The approximated analytical expressions of the dimensionless temperature and the dimensionless velocity are derived by using the New Homotopy analysis method. Further the analytical expression of the Nusselt number is also derived. The graphical representations of temperature and velocity profiles are depicted with the help of thermal conductivity variation parameters, thermal radiation parameter, axial pressure gradient parameter, magnetic parameter, Grashof number. We also present the qualitative of the critical relationship between the parameters. The New Homotopy analysis method can be further extended to solve some strongly non-linear boundary value problems in other MHD fluid flow in engineering problems.

Keywords: Thermal radiation; Variable thermal conductivity; Non-uniform temperature; Non-linear boundary value problem; New Homotopy analysis method

1. Introduction
Many authors Arpaci *et al* [1], Makinde [2], Sahin [3] has been studied about the flow and heat transfer between parallel plates channel and is applied in bio-medical engineering, petro-chemical industries etc. To design the pertinent equipment, intensive knowledge of radiative heat transfer and high temperature technological process is required by Makinde and Mhone [4]. In the study of Kay [5], the thermal conductivity of the fluid is kept constant and we know that the physical property may be change accordingly with temperature.
The thermal boundary layer equation for variable conductivity fluid in the presence of thermal radiation composes a nonlinear problem. Thus, the solution of theory of non-linear differential equation is ideal for practical relevance in science and engineering. We have referred the variable viscosity and thermal radiation effects on entropy generation rate, the problem of irreversibility in the flow of a temperature dependent variable viscosity optically small size fluid through a channel with isothermal walls by Makinde [6]. The effect of variable thermal conductivity and viscosity on single phase convective heat transfer in slip flow was analyzed by Sadik *et al* [7]. The heat transfer to MHD oscillatory flow in a channel filled with porous medium was analyzed by Cogely [8]. The thermal radiation effects on steady boundary-layer flow with variable thermal conductivity over a non-isothermal stretching sheet placed at the bottom of a saturated porous medium was studied by Paresh and Archana [9]. The effect of combined radiation and convection in thermally developing poiseuille flow with scattering was studied by Chawla and Chan [10].
By the above analysis, the aim of this paper is to study the effect of thermal radiation on various temperature guidance flows through a channel with sliding wall applied homogenous magnetic field. It can be assumed that thermal conductivity different linearly temperature. It has, therefore prompted us to investigate heat transfer with thermal radiative and variable conductivity MHD flows through channel with a sliding wall by using New Homotopyanalysis method.

2. Mathematical formulation of the problem
We follows that a steady two-dimensional laminar incompressible flow of conducting optically

thin viscous fluid through a channel with the lower sliding wall and irregular wall temperatures under guidance of an externally applied homogeneous magnetic field and radiative heat transfer. Consider the fluid with negligible absorption has small electrical conductivity and the electromagnetic force produced is very small. A Cartesian coordinate system is used and the flow is chosen along the x-direction under constant pressure-gradient which is driven solely by uniform velocity at the lower wall. That is the velocity linear with zero at the upper fixed wall and maximum value at the lower moving wall.

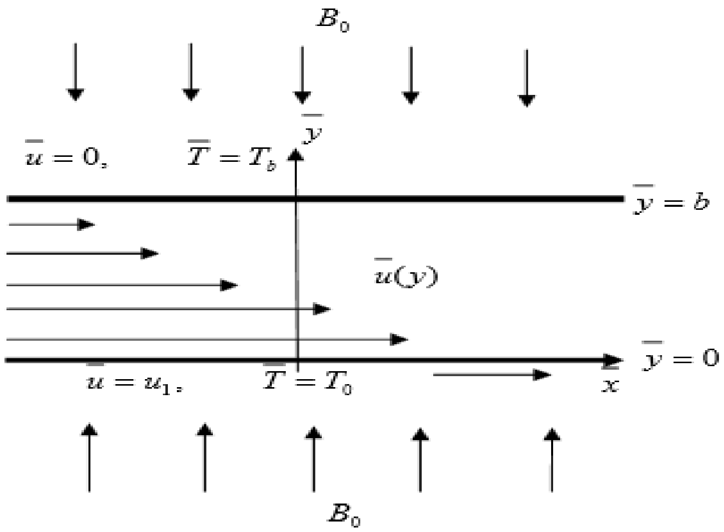

Fig 1: Physical model of the problem

Assume that all the physical properties of the fluid constant except the thermal conductivity which varies linearly with temperature, assuming Boussinesq approximation for radiative heat flux and neglecting the viscous dissipation in the energy equation, the boundary-layer equations are given below

$$v\frac{d^2\bar{u}}{d\bar{y}^2}-\frac{1}{\rho}\frac{dp}{dx}-\frac{\sigma_e B_0^{\ 2}}{\rho}\bar{u}+g\beta(\bar{T}-T_0)=0 \tag{1}$$

$$\frac{\bar{k}}{\rho c_p}\frac{d^2\bar{T}}{d\bar{y}^2}-\frac{1}{\rho c_p}\frac{dq}{dy}=0 \tag{2}$$

Here $\bar{u}$ denotes flow velocity the x-direction, v denotes kinematic velocity, $\bar{T}$ denotes temperature, $\bar{k}$ denotes thermal conductivity, ρ denotes density of the fluid, p represent the constant pressure, c_p denotes specific heat at constant pressure, g denotes gravitational force, q denotes radiative heat flux, β coefficient of volume expansion due to temperature, $B_0=(\mu_e H_0)$ represents electromagnetic induction, μ_e denotes magnetic permeability, H_0 denotes intensity of magnetic field, σ_e denotes conductivity of the fluid.

The boundary conditions are given below

$$\bar{u}=u_1\ ,\bar{T}=T_0 \qquad \text{at} \qquad \bar{y}=0 \tag{3}$$

$$\bar{u}=0\ ,\bar{T}=T_b \qquad \text{at} \qquad \bar{y}=b \tag{4}$$

Following that the medium is optically thin and with relatively low density. The radiative heat flux is $\frac{dq}{dy}=4\gamma^2(T_0-T_b)$ (5)

Here γ is the mean radiation absorption coefficient. The dimensionless temperature represented as $T=\frac{\bar{T}-T_0}{T_b-T_0}$ (6)

The variable thermal conductivity is $\bar{k}=k_\infty\left[1+\varepsilon T\right]$ (7)

Here k_∞ represents the thermal conductivity at the fluid ambient temperature T_0 and ε denotes $\varepsilon=\frac{1}{k}\left(\frac{\partial\bar{k}}{\partial T}\right)$. $N=\frac{1}{u}\frac{dp}{d\bar{x}}$ represents the constant axial pressure gradient with constant viscosity.

Dimensionless quantities and parameters are given below

$$\eta = \frac{\overline{y}}{b}\ ,\ \tau = \frac{\overline{x}}{b}\ ,\ u = \frac{\overline{u}}{u_1 N}\ ,\ k = \frac{\overline{k}}{k_\infty}\ ,\ P = \frac{b\overline{P}}{\rho \nu u_1},\ \alpha = \varepsilon T$$

$$g\beta(T_b - T$$

$$\overline{R} = \frac{4\gamma^2 b^2}{k_\infty}\ ,\ \overline{H} = \sqrt{\frac{b^2 \sigma_e {B_0}^2}{\mu}}\ ,\ Gr = \frac{g\beta(T_b - T_0) b^2}{\nu u_1} \tag{8}$$

Here α denotes thermal conductivity variation parameter, $\overline{R}$ denotes radiation parameter $\overline{H}$ denotes Hartmann number and Gr Grashof number respectively.

The equations (1) and (2) together with boundary conditions (3) and (4) reduced to the form

$$\frac{d^2u}{d\eta^2} - \overline{H}^2 u + GrT = N, \qquad \frac{d^2T}{d\eta^2} + \alpha T \frac{d^2T}{d\eta^2} - \overline{R}T = 0 \tag{9}$$

$$\frac{d^2u}{d\eta^2} - \alpha H^2 u + GrT = N \tag{10}$$

$$\frac{d^2T}{d\eta^2} + \alpha\left[T \frac{d^2T}{d\eta^2} - RT\right] = 0 \tag{11}$$

Here $k = 1 + \alpha T$, $R = \frac{\overline{R}}{\alpha}, H^2 = \frac{\overline{H}^2}{\alpha}$,

With $u = 0,\quad T = 1,\ at \quad \eta = 1$ (12)

$u = 1,\quad T = 0,\ at\ \ \eta = 0$ (13)

The velocity u and the temperature T as well as the wall heat transfer rate,

$$N_u = -\frac{dT}{d\eta} \text{ at } \eta = 1 \tag{14}$$

In terms of $\alpha\, R, H, Gr, N.$

3. Solution of the problems using the New Homotopy analysis method

HAM is a non perturbative analytical method for obtaining series solutions to nonlinear equations and has been successfully applied to numerous problems in science and engineering [7-22]. In comparison with other perturbative and non perturbative analytical methods, HAM offers the ability to adjust and control the convergence of a solution via the so-called convergence-control parameter. Because of this, HAM has proved to be the most effective method for obtaining analytical solutions to highly non-linear differential equations. Previous applications of HAM have mainly focused on non-linear differential equations in which the non-linearity is a polynomial in terms of the unknown function and its derivatives. As seen in (1), the non-linearity present in electro hydrodynamic flow takes the form of a rational function, and thus, poses a greater challenge with respect to finding approximate solutions analytically. Our results show that even in this case, HAM yields excellent results.

Liao [7-15] proposed a powerful analytical method for non-linear problems, namely the Homotopy analysis method. This method provides an analytical solution in terms of an infinite power series. However, there is a practical need to evaluate this solution and to obtain numerical values from the infinite power series. In order to investigate the accuracy of the Homotopy analysis method (HAM) solution with a finite number of terms, the system of differential equations were solved. The Homotopy analysis method is a good technique comparing to another perturbation method. The Homotopy analysis method contains the auxiliary parameter h , which provides us with a simple way to adjust and control the convergence region of solution series. Using this method, we can obtain the following solution to (1) and (2) (see Appendix B).

The approximate analytical solution of the equations (1) and (2) using HAM is given by

$$T(\eta) = \frac{\sinh(k\eta)}{\sinh(k)} - h\left(\begin{array}{l}\left(\dfrac{2}{3}\dfrac{a}{\sinh(k)^2} + \dfrac{1}{24}\dfrac{a\,(e^{(2k)} + e^{(-2k)} + 6 - 8e^{(k)}}{\sinh(k)^3}\right)e^{(k\eta)} \\ -\left(\dfrac{1}{24}\dfrac{a\,(e^{(2k)} + e^{(-2k)} + 6 - 8e^{(k)}}{\sinh(k)^3}\right)e^{(-k\eta)} - \dfrac{1}{12}\dfrac{a\left(e^{(2k\eta)} + e^{(-2k\eta)} + 6\right)}{\sinh(k)^2}\end{array}\right) \tag{15}$$

$$u(\eta)=\left(-\frac{1}{2}\frac{e^{(-m)}}{\sinh(m)}+\frac{1}{2}\frac{N}{m^2\sinh(m)}-\frac{1}{2}\frac{Ne^{(-m)}}{m^2\sinh(m)}\right)e^{(m\eta)}+$$
$$\left(1+\frac{N}{m^2}+\frac{1}{2}\frac{e^{(-m)}}{\sinh(m)}-\frac{1}{2}\frac{N}{m^2\sin(m)}+\frac{1}{2}\frac{Ne^{(-m)}}{m^2\sinh(m)}\right)e^{(-m\eta)}-\frac{N}{m^2}- \qquad (16)$$
$$h\left(\frac{1}{2}\frac{Gr\,e^{(m\eta)}}{\left(k^2-m^2\right)\sinh(m)}-\frac{1}{2}\frac{Gr\,e^{(-m\eta)}}{(k^2-m^2)\sinh(m)}-\frac{Gr\sinh(k\eta)}{(k^2-m^2)\sinh(k)}\right)$$

$$Nu=-\frac{\cos(k\eta)k}{\sinh(k)}+h\left(\begin{array}{l}\left(\frac{2}{3}\frac{a}{\sinh(k)^2}+\frac{1}{24}\left(\frac{a\,(e^{(2k)}+e^{(-2k)}+6-8e^{(k)}}{\sinh(k)^3}\right)\right)ke^{(k\eta)}\\ -\left(\frac{1}{24}\frac{a\,(e^{(2k)}+e^{(-2k)}+6-8e^{(k)}}{\sinh(k)^3}\right)ke^{(-k\eta)}-\frac{1}{12}\frac{a\left(2e^{(2k\eta)}-2e^{(-2k\eta)}\right)}{\sinh(k)^2}\end{array}\right) \qquad (17)$$

Where,

$$k=\sqrt{aR} \qquad (18)$$

$$m=\sqrt{a\,H^2} \qquad (19)$$

4. Results and discussion

In this section the effects of physical parameters including thermal conductivity variation parameter α, thermal radiation parameter R, axial pressure gradient parameter N, magnetic parameter H, and Grashof number Gr will be presented. Fig.1 shows that the physical model of the problem. Fig.2 and 3 shows the dimensionless temperature $(T(\eta))$ versus the dimensionless similarity variable (η). From Fig. 2, we inferred that increasing the thermal conductivity variation parameter α, decreases the dimensionless temperature in some fixed values of R. From Fig. 3, we depicts that the dimensionless temperature decreases, when the thermal radiation parameter R increases in some fixed values of α.

Fig.4 to 8 shows that the dimensionless velocity $(u(\eta))$ versus the dimensionless similarity variable (η). From Fig.4, we infer that the dimensionless velocity decreases, when the thermal conductivity variation parameter (α) increases in some values of the fixed parameter. From Fig.5, we illustrates that increasing the thermal radiation parameter R, decreases the dimensionless velocity in some fixed values of parameter. From Fig.6, we observe that increasing the axial pressure gradient parameter N, decreases the dimensionless velocity in some fixed parameters. From Fig.7, it is noted that when the Hartmann number H increases, the dimensionless velocity decreases in some fixed parameter. From Fig.8 it is evident that when the Grash of number increases, the corresponding dimensionless velocity increases in some fixed values of parameter.

Fig.9 describes the Nusselt number (Nu) versus thermal conductivity variation parameter (α).It is observed that Nusselt number Nu is improved with increasing (R) at the upper wall region, while a decrease in the Nusselt number at the lower wall region. Fig. 10, represent that Nusselt number (Nu) versus thermal radiation parameter (R). We demonstrate that increasing the thermal conductivity variation parameter α, decreases the Nusselt number Nu with the fixed parameter R.

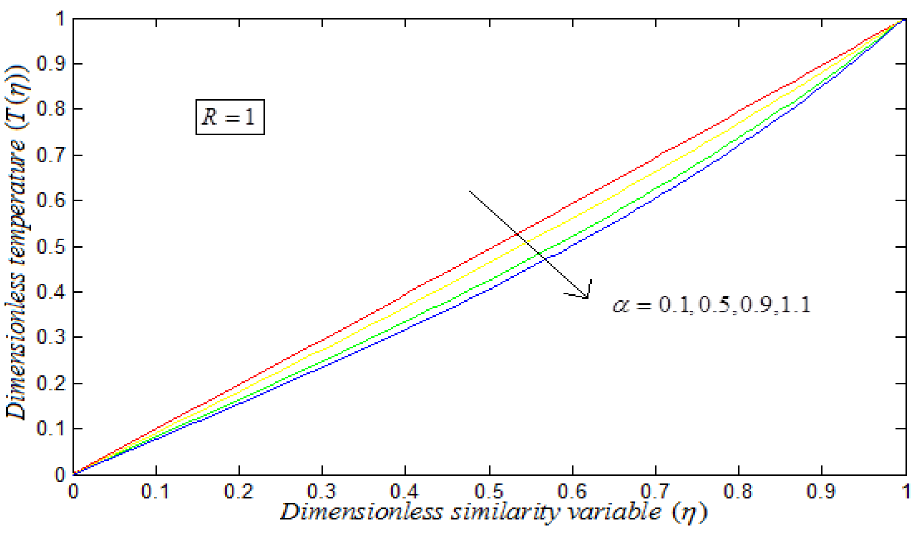

Fig 2: Dimensionless temperature $(T(\eta))$ versus the dimensionless similarity variable (η) for various values of α and some fixed values of the other parameter R using the eqn. (14) with $h=0.98$

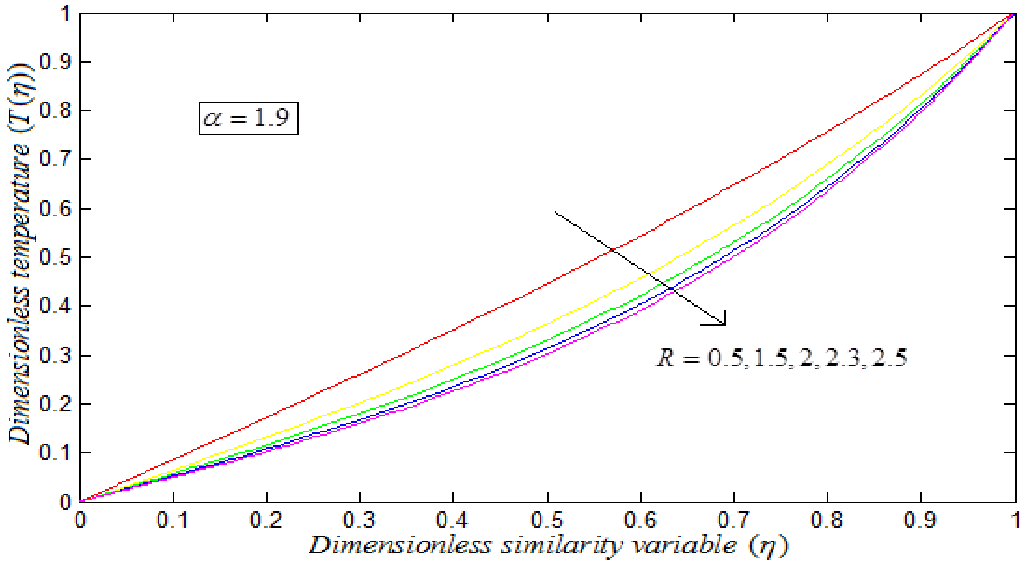

Fig 3: Dimensionless temperature $(T(\eta))$ versus the dimensionless similarity variable (η) for various values of R and some fixed values of the other parameter α using the eqn. (14) with $h = -0.00196$

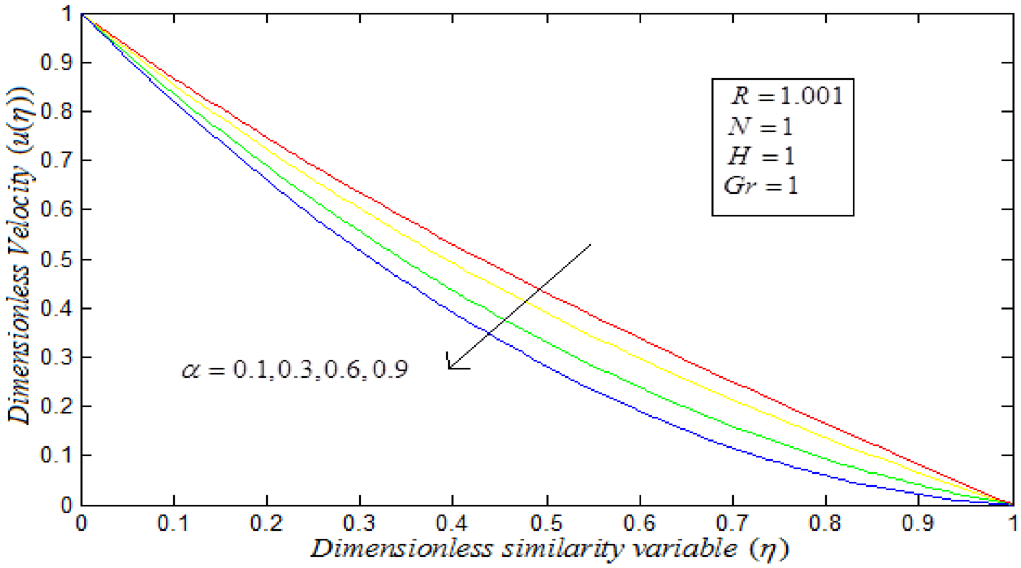

Fig 4: Dimensionless velocity $(u\,(\eta))$ versus the dimensionless similarity variable (η) for various values of α and some fixed values of the other parameter R, N, H, Gr using the eqn. (15) with $h = -0.05$

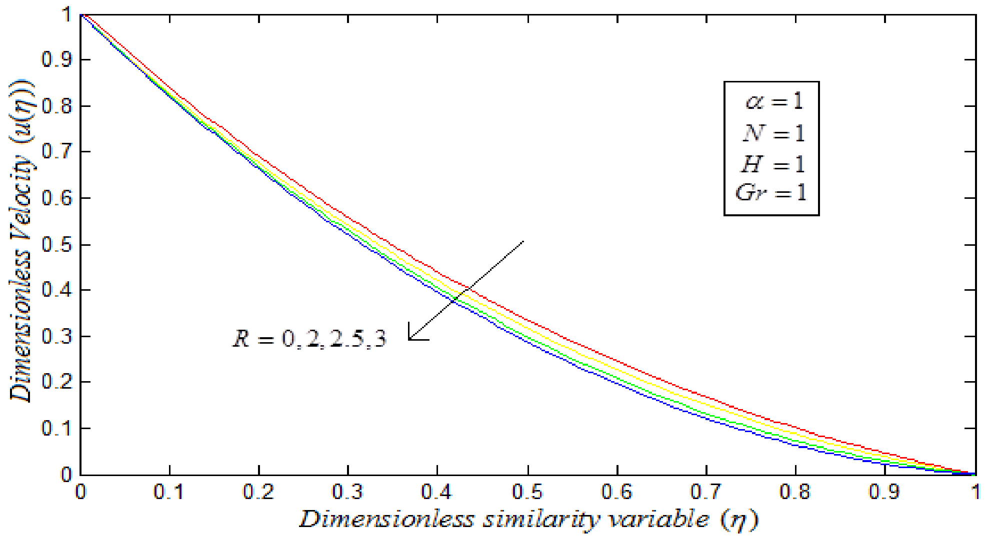

Fig 5: Dimensionless velocity $(u\,(\eta))$ versus the dimensionless similarity variable (η) for various values of R and some fixed values of the other parameter α, N, H, Gr using the eqn. (15) with $h = 0.47$

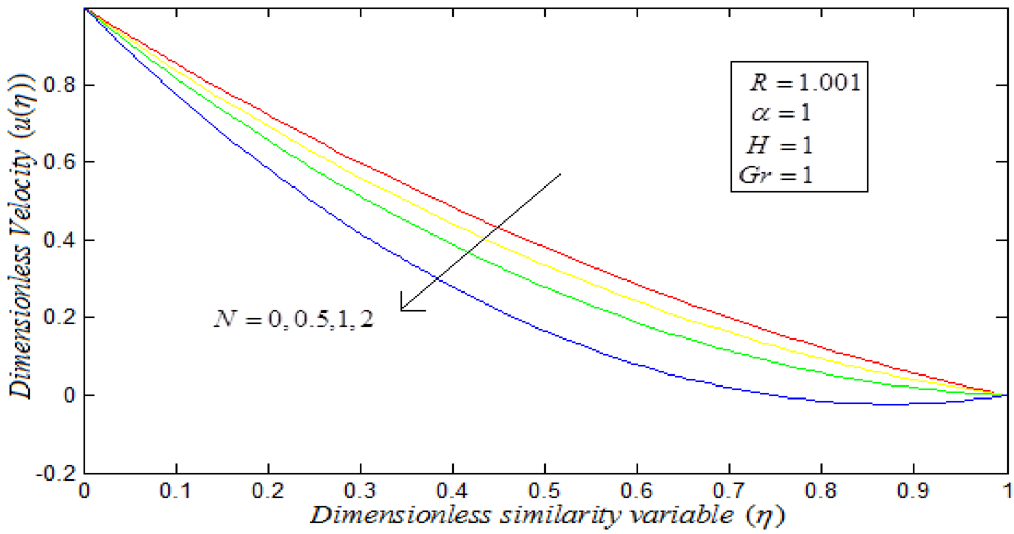

Fig 6: Dimensionless velocity $(u(\eta))$ versus the dimensionless similarity variable (η) for various values of N and some fixed values of the other parameter α, R, H, Gr using the eqn. (15) with $h = -0.825$

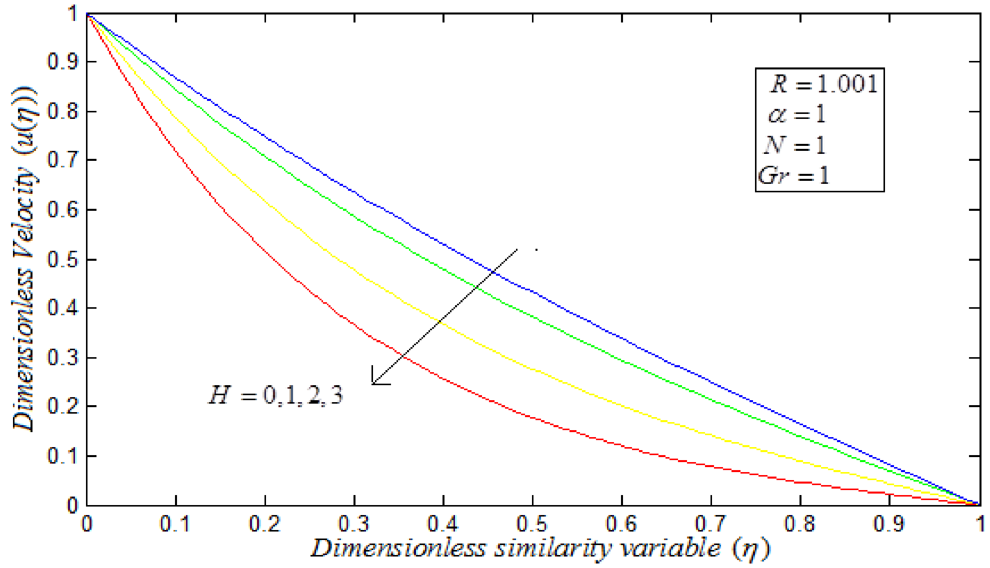

Fig 7: Dimensionless velocity $(u(\eta))$ versus the dimensionless similarity variable (η) for various values of H and some fixed values of the other parameter α, R, N, Gr using the eqn. (15) with $h = -0.9$

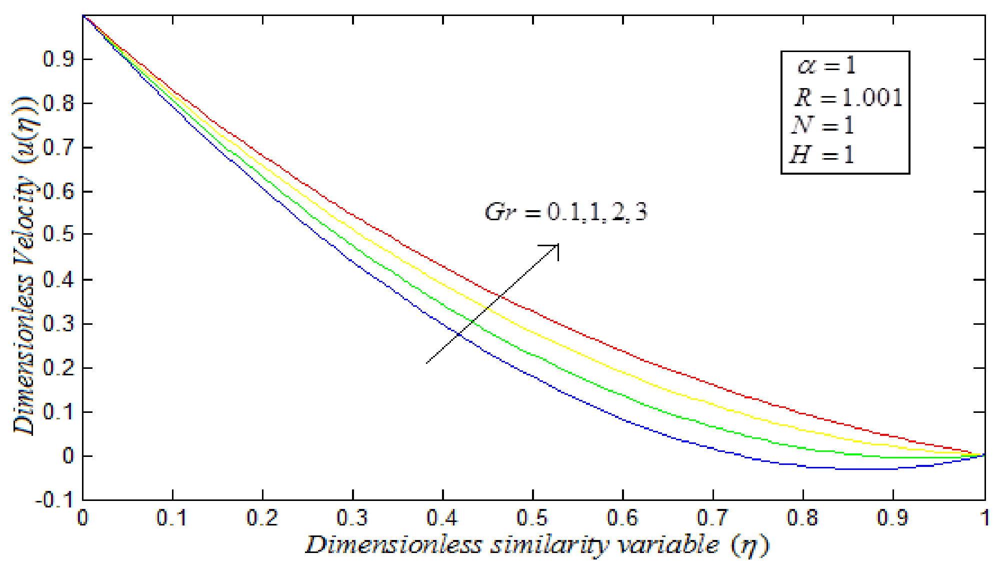

Fig 8: Dimensionless velocity $(u(\eta))$ versus the dimensionless similarity variable (η) for various values of Gr and some fixed values of the other parameter α, R, N, H using the eqn. (15) with $h = 0.725$

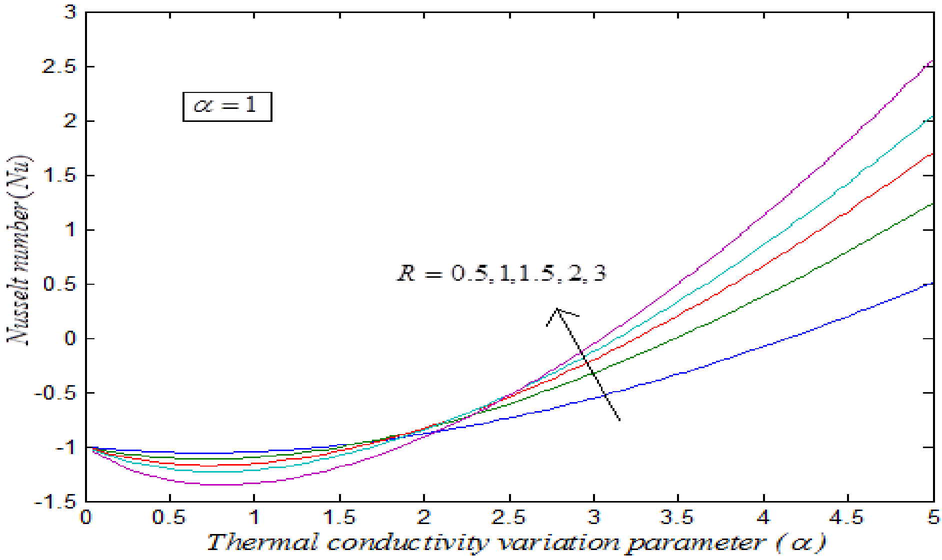

Fig 9: Nusselt number (Nu) versus the thermal conductivity variation parameter (α) for various values of R using the eqn. (16) with $h = -1$

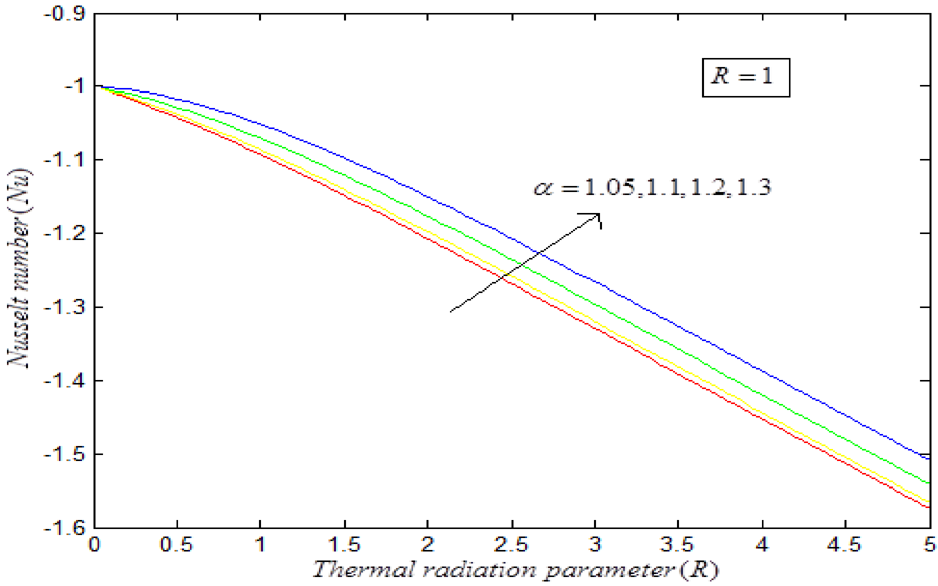

Fig 10: Nusselt number (Nu) versus the thermal radiation parameter (R) for various values of α using the eqn. (16) with $h = -1$

Appendix A

Basic Concept of HAM

Consider the following differential equation:

$$N[u(t)] = 0 \tag{A.1}$$

Where N is a nonlinear operator, t denotes an independent variable, $u(t)$ is an unknown function. For simplicity, we ignore all boundary or initial conditions, which can be treated in the similar way. By means of generalizing the conventional Homotopy method, Liao (2012) constructed the so-called zero-order deformation equation as:

$$(1-p)L[\varphi(t;p) - u_0(t)] = phH(t)N[\varphi(t;p)] \tag{A.2}$$

where $p \in [0,1]$ is the embedding parameter, $h \neq 0$ is a nonzero auxiliary parameter, $H(t) \neq 0$ is an auxiliary function, L an auxiliary linear operator, u_0 (t) is an initial guess of $u(t)$, $\varphi(t:p)$ is an unknown function. It is important to note that one has great freedom to choose auxiliary unknowns in HAM. Obviously, when $p = 0$ and $p = 1$, it holds:

$$\varphi(t;0) = u_0(t) \text{ and } \varphi(t;1) = u(t) \tag{A.3}$$

Respectively. Thus, as p increases from 0 to 1, the solution $\varphi(t;p)$ varies from the initial guess $u_0(t)$ to the solution u (t). Expanding $\varphi(t;p)$ in Taylor series with respect to p, we have:

$$\varphi(t;p) = u_0(t) + \sum_{m=1}^{+\infty} u_m(t)\, p^m \tag{A.4}$$

Where

$$u_m(t) = \frac{1}{m!}\frac{\partial^m \varphi(t;p)}{\partial p^m}\Big|_{p=0} \tag{A.5}$$

If the auxiliary linear operator, the initial guess, the auxiliary parameter *h*, and the auxiliary function are so properly chosen, the series (A.4) converges at *p* =1 then we have:

$$u(t) = u_0(t) + \sum_{m=1}^{+\infty} u_m(t)\,. \tag{A.6}$$

Differentiating (A.2) for *m* times with respect to the embedding parameter *p*, and then setting *p* = 0 and finally dividing them by *m*!, we will have the so-called *mth* -order deformation equation as:

$$L[u_m - \chi_m u_{m-1}] = hH(t)\Re_m(\vec{u}_{m-1}) \tag{A.7}$$

Where

$$\Re_m(\vec{u}_{m-1}) = \frac{1}{(m-1)!}\frac{\partial^{m-1} N[\varphi(t;p)]}{\partial p^{m-1}} \tag{A.8}$$

and

$$\chi_m = \begin{cases} 0, & m \le 1, \\ 1, & m > 1. \end{cases} \tag{A.9}$$

Applying L^{-1} on both side of eqn. (A7), we get

$$u_m(t) = \chi_m u_{m-1}(t) + hL^{-1}[H(t)\Re_m(\vec{u}_{m-1})] \tag{A10}$$

In this way, it is easily to obtain u_m for $m \ge 1$, at M^{th} order, we have

$$u(t) = \sum_{m=0}^{M} u_m(t) \tag{A.11}$$

When $M \to +\infty$, we get an accurate approximation of the original eqn. (A.1). For the convergence of the above method we refer the reader to Liao [20]. If an eqn. (A.1) admits unique solution, then this method will produce the unique solution.

Appendix B

Approximate analytical expressions of the non-linear differential eqns. (10)-(13) using the New Homotopy analysis method

In this appendix,

The eqns. (10) can be written in the following form

$$\frac{d^2T}{d\eta^2} + \alpha\left[T\frac{d^2T}{d\eta^2} - RT\right] = 0 \tag{B.1}$$

$$\frac{d^2u}{d\eta^2} - \alpha H^2 u + GrT = N \tag{B.2}$$

We construct the Homotopy for the eqn. (B.1) and (B.2) are as follows:

$$(1-p)\left[\frac{d^2T}{d\eta^2} + \alpha T(0)\frac{d^2T}{d\eta^2} - \alpha RT\right] - hp\left[\frac{d^2T}{d\eta^2} + \alpha\left[T\frac{d^2T}{d\eta^2} - RT\right]\right] = 0 \tag{B.3}$$

$$(1-p)\left[\frac{d^2u}{d\eta^2} - \alpha H^2 u + GrT(0) - N\right] - hp\left[\frac{d^2u}{d\eta^2} - \alpha H^2 u + GrT - N\right] = 0 \tag{B.4}$$

The approximate solution of the eqn. (B.3) and (B.4) are as follows:

$$T = T_0 + pT_1 + p^2T_2 + \ldots \tag{B.5}$$

$$u = u_0 + pu_1 + p^2T_2 + \ldots \tag{B.6}$$

The initial approximations are as follows:

$$T_0(0) = 0,\ T_0(1) = 1,\ u_0(0) = 1,\ u_1(1) = 0 \tag{B.7 (a)}$$

$$T_i(0) = 0,\ T_i(1) = 0,\ u_i(0) = 0,\ u_i(1) = 0, \quad i = 1,2,3,\ldots \tag{B.7 (b)}$$

Substituting the eqn. (B.5) into the eqn. (B.3) we get

$$(1-p)\left[\frac{d^2\left(T_0+pT_1+p^2T_2+\ldots\right)}{d\eta^2}+\alpha T(0)\frac{d^2\left(T_0+pT_1+p^2T_2+\ldots\right)}{d\eta^2}-\alpha R\left(T_0+pT_1+p^2T_2+\ldots\right)\right]$$
$$=hp\left[\frac{d^2\left(T_0+pT_1+p^2T_2+\ldots\right)}{d\eta^2}+\alpha\left(T_0+pT_1+p^2T_2+\ldots\right)\frac{d^2\left(T_0+pT_1+p^2T_2+\ldots\right)}{d\eta^2}-\alpha R\left(T_0+pT_1+p^2T_2+\ldots\right)\right] \quad \text{(B.8)}$$

Comparing the coefficients of like powers of p in eqn. (B.8), we get the following eqn.

$$p^0:\frac{d^2T_0}{d\eta^2}+\alpha T(0)\frac{d^2T_0}{d\eta^2}-\alpha RT_0 \quad \text{(B.9)}$$

$$p^1:\left[\frac{d^2T_1}{d\eta^2}+\alpha T(0)\frac{d^2T_1}{d\eta^2}-\alpha RT_1\right]-h\left[\frac{d^2T_0}{d\eta^2}+\alpha\left[T_0\frac{d^2T_0}{d\eta^2}-RT_0\right]\right] \quad \text{(B.10)}$$

Substituting the eqn. (B.6) into the eqn. (B.4) we get

$$(1-p)\left[\frac{d^2\left(u_0+pu_1+p^2T_2+\ldots\right)}{d\eta^2}-\alpha H^2\left(u_0+pu_1+p^2T_2+\ldots\right)+GrT(0)-N\right]$$
$$=hp\left[\frac{d^2\left(u_0+pu_1+p^2T_2+\ldots\right)}{d\eta^2}-\alpha H^2\left(u_0+pu_1+p^2T_2+\ldots\right)+GrT-N\right] \quad \text{(B.11)}$$

Comparing the coefficients of like powers of p in eqn. (B.11), we get the following eqn.

$$p^0:\frac{d^2u_0}{d\eta^2}-\alpha H^2u_0+GrT(0)-N \quad \text{(B.12)}$$

$$p^1:\left[\frac{d^2u_1}{d\eta^2}-\alpha H^2u_1+GrT(0)-N\right]-h\left[\frac{d^2u_0}{d\eta^2}-\alpha H^2u_0+GrT-N\right] \quad \text{(B.13)}$$

Solving the eqns. (B.9), (B.10) and using boundary conditions (B.7 (a)), we obtained the following results:

$$T_0(\eta)=\frac{\sinh(k\eta)}{\sinh(k)} \quad \text{(B.14)}$$

$$T_1(\eta)=\left(\begin{array}{l}\left(\frac{2}{3}\frac{a}{\sinh(k)^2}+\frac{1}{24}\frac{a(e^{(2k)}+e^{(-2k)}+6-8e^{(k)}}{\sinh(k)^3}\right)e^{(k\eta)}\\-\left(\frac{1}{24}\frac{a(e^{(2k)}+e^{(-2k)}+6-8e^{(k)}}{\sinh(k)^3}\right)e^{(-k\eta)}-\frac{1}{12}\frac{a\left(e^{(2k\eta)}+e^{(-2k\eta)}+6\right)}{\sinh(k)^2}\end{array}\right) \quad \text{(B.15)}$$

Solving the eqns. (B.12), (B.13) and using boundary conditions (B.7 (b)), we obtained the following results:

$$u_0(\eta)=\left(-\frac{1}{2}\frac{e^{(-m)}}{\sinh(m)}+\frac{1}{2}\frac{N}{m^2\sinh(m)}-\frac{1}{2}\frac{Ne^{(-m)}}{m^2\sinh(m)}\right)e^{(m\eta)}+$$
$$\left(1+\frac{N}{m^2}+\frac{1}{2}\frac{e^{(-m)}}{\sinh(m)}-\frac{1}{2}\frac{N}{m^2\sin(m)}+\frac{1}{2}\frac{Ne^{(-m)}}{m^2\sinh(m)}\right)e^{(-m\eta)}-\frac{N}{m^2} \quad \text{(B.16)}$$

$$u_1(\eta)=\frac{1}{2}\frac{Gr\,e^{(m\eta)}}{\left(k^2-m^2\right)\sinh(m)}-\frac{1}{2}\frac{Gr\,e^{(-m\eta)}}{(k^2-m^2)\sinh(m)}-\frac{Gr\sinh(k\eta)}{(k^2-m^2)\sinh(k)} \quad \text{(B.17)}$$

Where k *and* m are the constants defined in the text eqns. (17)-(18) respectively.

According to the Homotopy analysis method we have

$$T=\lim_{p\to 1}T(\eta)=T_0+T_1 \quad \text{(B.18)}$$

$$u=\lim_{p\to 1}u(\eta)=u_0+u_1 \quad \text{(B.19)}$$

Using the eqns. (B.14) - (B.17) in (B.18) – (B.19) respectively. We obtain the solution in the text eqns. (14)-(15).

Appendix C: Nomenclature

Symbol	Meaning
η	Dimensionless similarity variable
$T(\eta)$	Dimensionless Temperature
$u(\eta)$	Dimensionless velocity
v	Kinematic viscosity
$\overline{k}$	Thermal conductivity
ρ	Density of the fluid
c_p	Heat at constant pressure
μ_e	Magnetic permeability
H_0	Intensity of magnetic field
σ_e	Conductivity of the fluid
α	Conductivity variation parameter
$\overline{R}$	Variation parameter
$\overline{H}$	Hartmann number
Nu	Nusselt number
Gr	Grashof number
H	Magnetic parameter

5. Conclusion

This paper has investigated the thermal radiative and variable conductivity MHD flows through channel with a sliding wall. The approximate analytical expressions of the dimensionless velocity dimensionless temperatures are obtained by using the New Homotopy analysis method. The difference between Nusselt number and due to the effect of radiation parameter varies thermal conductivity radiation parameter. Also the difference between Nusselt number and due to the effect of thermal conductivity variation parameter varies number and radiation parameter. This method is can be easily extended to solve the non-linear boundary value problem for MHD fluid flow in physical sciences.

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