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On semı-mınımal weakly open and semı-maxımal weakly closed sets in topologıcal spaces

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Abstract
In this paper new class of sets called semi- minimal weakly open sets and semi-maximal weakly closed sets are introduced in topological spaces. We show that the complement of semi-minimal weakly open set is a semi-maximal weakly closed set and some properties of the new concepts have been studied.

Keywords: Minimal open set, Maximal closed set, Minimal weakly open set, maximal weakly closed set, Semi- Minimal weakly open set, Semi- Maximal weakly closed set.

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1. Introduction

In the year 2001 and 2003, F.Nakaoka and N.oda, [1-3] introduced and studied minimal open [resp. minimal closed] sets which are subclass of open [resp.closed sets]. The family of all minimal open [minimal closed] in a topological space X is denoted by $m_io(X)$ [$m_ic(X)$]. Similarly the family of all maximal open [maximal closed] sets in a topological space X is denoted by $M_aO(X)[M_aC(X)]$. The complements of minimal open sets and maximal open sets are called maximal closed sets and minimal closed sets respectively. In the year 1963, N.Levine [4] introduced and studied semi-open sets. A subset A of a topological space X is said to be semi-open set if there exist some open set U such that $U \subset A \subset Cl(U)$. The family of all semi-open sets of X is denoted by SO(X). The Complement [5] of semi-open set is called semi-closed set in X. The family of all semi-closed sets are denoted by SC(X). In the year 2014, R.S.Wali and Vivekananda Dembre [6] introduced and studied semi-minimal open and semi-maximal closed sets in topological spaces. In the year 2000, M.Sheik John [7] introduced and studied weakly closed sets and weakly open sets in topological spaces. In the year 2014 R.S.Wali and Vivekananda Dembre [8] introduced and studied minimal weakly open sets and maximal weakly closed sets in topological spaces.

1.1 Definition [1]: A proper non-empty open subset U of a topological space X is said to be minimal open set if any open set which is contained in U is $\emptyset$ or U.

1.2 Definition [2]: A proper non-empty open subset U of a topological space X is said to be maximal open set if any open set which is contained in U is X or U.

1.3 Definition [3]: A proper non-empty closed subset F of a topological space X is said to be minimal closed set if any closed set which is contained in F is $\emptyset$ or F.

1.4 Definition [3]: A proper non-empty closed subset F of a topological space X is said to be maximal closed set if any closed set which is contained in F is X or F.

1.5 Definition [4]: A subset A of a topological spaces X is said to be semi-open set if there exist some open set U such that $U \subset A \subset Cl(U)$.

1.6 Definition [5]: The complement of semi-open set is called semi-closed set in X.

1.7 Definition [6]**:** A set A in a topological space X is said to be semi-minimal open set if there exists a minimal open set M such that M⊂A⊂Cl(M).

1.8 Definition [6]**:** A subset N of a topological space X is said to be semi-maximal closed set if X-N is semi-minimal open set.

1.9 Definition [7]**:** A subset A of (X, τ) is called weakly closed set if Cl(A)⊆ U whenever A ⊆ U and U is semi-open in X.

1.10 Definition [7]**:** A subset A in (X,τ) is called weakly open set in X if A^c is weakly closed set in X.

1.11 Definition [8]**:** A proper non-empty weakly open subset U of X is said to be minimal weakly open set if any weakly open set which is contained in U is ∅ or U.

1.12 Definition [8]**:** A proper non-empty weakly closed subset F of X is said to be maximal weakly closed set if any weakly closed set which is contained in F is X or F.

2. Semı-Mınımal Weakly Open Sets

2.1 Definition: A set A in a topological space X is said to be semi-minimal weakly open set if there exists a minimal weakly open set M Such that M⊂A⊂S-Cl(M).

The family of all semi-minimal weakly open sets in a topological space X is denoted by $Sm_i wo(X)$.

2.2 Example: Let X={a,b,c}, τ ={X,∅,{a}} be a topological space.

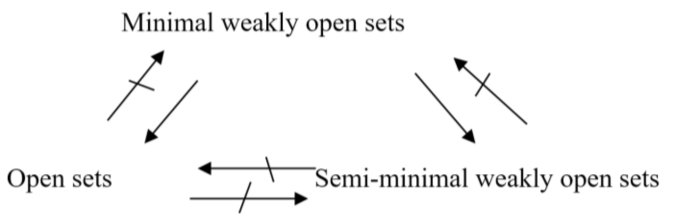
Weakly open sets are: {X, ∅,{a},{a,b}}

Minimal weakly open sets are : {{a}}

Semi-minimal-weakly-open-sets: {X, {a},{a,b},{a,c}}

$m_i wo(x) \subset Sm_i wo(x)$.

The above results are given in below implication diagram.



2.3 Theorem: If M is a semi- minimal weakly open set in a topological space X and M⊂N⊂S-Cl(M) then N is also semi-minimal weakly open in X.

Proof: Let M be a semi-minimal weakly open in X. Then by definition 2.1 there exists a minimal-weakly open set U in X such that U⊂M⊂S-Cl(U). Since M⊂S-Cl(U) it follows that S-Cl(M)□Cl(S-Cl(U))=S-Cl(U). But from hypothesis N⊂ S-Cl(M) therefore it follows that U⊂N⊂S-Cl(U). Therefore by definition 2.1 it follows that N is semi-minimal weakly open in X.

2.4 Theorem: Let X be a topological space and $m_i wo(x)$ be the class of all minimal weakly open sets in X the following results hold good.

(i) $m_i wo(x) \subset Sm_i wo(X)$

(ii) If $M \in Sm_i wo(X)$ and M⊂N⊂S-Cl(M) then $N \subset Sm_i wo(X)$.

Proof: This follows from theorem 2.3.

2.5 Theorem: Let X be a topological space. Y be subspace of X and M be a subset of Y. If M is semi-minimal weakly open in X then M is semi-minimal weakly open in Y.

Proof: Suppose M is semi-minimal weakly open in X. By definition 2.1 there exists a minimal weakly open set N in X such that N⊂M⊂ S-Cl(N). Now N⊂M⊂Y. Hence Y∩N=N. Since N is minimal weakly open in X. Y ∩ N =N is minimal weakly open in Y. Now we have N⊂M⊂S-Cl(N).Therefore Y∩N⊂Y∩M⊂Y∩S-Cl(N), which implies N⊂M⊂S-Cl γ(N). Thus there exists a minimal weakly open set N in Y Such that N⊂M⊂ S -Clγ(N). Therefore by definition 2.1 it follows that M is semi-minimal weakly open in Y.

2.6 Theorem: Let X be a topological space. Let M,N be minimal weakly open sets in X and U⊂X such that N⊂U⊂S-Cl(N) if M∩N=∅ then U∩W= ∅.

Proof: Since M∩N=∅, it follows that N⊂X–M τtherefore S-Cl(N)⊂Cl(X-M)=X-M. Since X-M is maximal weakly closed set and every maximal weakly closed set is closed set. Also we have N⊂U⊂S-Cl(N). Therefore U⊂S -Cl(N)⊂X-M .Thus U⊂X-M which means U∩W= ∅.

2.7 Theorem: Intersection of two semi-minimal weakly open sets need not be semi-minimal weakly open. It can be Shown by the following example

Let X={a,b,c}, τ ={X,∅,{a,b}} be a topological space.

Semi-minimal-weakly-open-sets:

{X, {a},{b},{a,b},{b,c},{a,c}}

take any two semi-minimal open sets {b,c} ∩ {a,c}={c} Which is not a semi-minimal weakly open set.

3. Semı-Maxımal-Weakly Closed Sets

3.1 Definition: A subset N of a topological space X is said to be semi-maximal weakly closed set if X-N is semi-minimal weakly open set.

The family of all semi-maximal weakly closed sets in a topological space X is denoted by $SM_i WC(X)$.

3.2 Example: Let X= {a,b,c}, τ = {X, ∅,{a}} be a topological space.

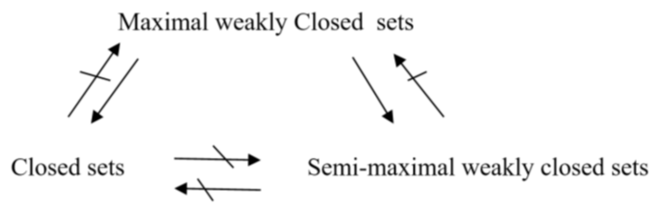
Closed sets are: {X, ∅,{b,c}}

Weakly closed-sets:{X, ∅,{c},{b,c}}

Maximal Weakly Closed sets: {{b,c}}

Semi-maximal-weakly-closed-sets: { ∅, {b,c},{c},{b}}

The above results are given in below implication diagram.



3.3 Theorem: A subset W of a topological space X is semi-maximal-weakly closed iff there exists a maximal weakly closed set N in X such that int(N)⊂W⊂N.

Proof: Suppose W is a semi-maximal weakly closed in X then by definition 3.1 X-W is semi-minimal weakly open in X. Therefore by definition 2.1 there exists a minimal weakly open set M such that M ⊂X-W⊂S-Cl(M) which implies that X-[S-Cl(M)] ⊂X-[X-W]⊂X-M which implies X-[S-Cl(M)] ⊂W⊂X-M. But it is know that X-[S-Cl(M)]=int(X-M) take X-M=N so,that N is a maximal weakly closed set such that int(N) ⊂W ⊂N.

Conversly, suppose that there exist a maximal weakly closed set N in X such that int(N) ⊂W ⊂N, therefore it follows that X-N ⊂[X-W]⊂X-int(N).But it is know that X-int(N))=Cl(X-N). Therefore there exists a minimal weakly open set X-N such that X-N ⊂X-W⊂[S-Cl(X-N)]. Thus by definition 2.1 it follows that X-W is semi-minimal weakly open in X. Hence by definition 3.1 it follows that W is semi-maximal weakly closed set.

3.4 Theorem: If N is semi-maximal weakly closed in X and int(N)⊂W⊂N then W is semi-maximal weakly closed in X.

Proof: Let N be semi-maximal weakly closed in X then by definition of semi-maximal weakly closed sets there exists a maximal weakly closed set F such that int(F) ⊂N ⊂F. Now int(F) ⊂N which implies int(F)=int(int(F))⊂int(N). But int(N) ⊂W, we have int(F) ⊂W. Further since int(F) ⊂ int(N) ⊂W ⊂ N ⊂F. It follows that int(F) ⊂W ⊂ F. Thus there exists a maximal weakly closed set F such that int(F) ⊂W ⊂F therefore W is semi-maximal weakly closed in X.

3.5 Theorem: The following three properties of a subset N of a topological space X are equivalent

(i) N is Semi-maximal weakly closed set in X.
(ii) int(cl(N)) ⊂N
(iii) (X-N) is semi-minimal weakly open set.

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