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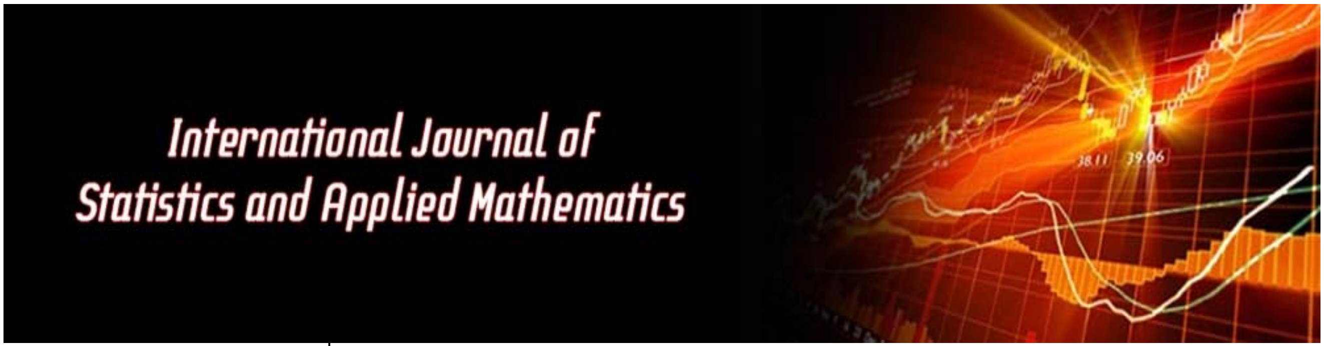

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Study on two Non-Archimedean Menger probability metric spaces

Dr. R Krishnakumar and Nagaral Pandit Sanatammappa

Abstract
The generalized common fixed point theorems in 2 Non Archimedean Menger PM-space by using the concepts of R- weakly commuting mappings, reciprocal continuity are established in this paper. The presented results extend some known existence results from the literature.

Keywords: Fixed point, 2 Non Archimedean Menger PM-space, R- weakly commuting mappings, reciprocally continuous

1. Introduction
In 2001, Renu Chugh and Sumitra [8] introduced 2 Non Archimedean Menger PM-space. The 2 Non Archimedean Menger PM-space is the generalization of 2-metric space in probabilistic setting. That is where instead of the distances between two or more points one can understand the probability of a possible value of the distance. The distance is represented by distribution function. In this paper, we will prove some generalized common fixed point theorems in 2 Non Archimedean Menger PM-space by using the concepts of R- weakly commuting mappings, reciprocal continuity. These results extend and generalize several results existing in the literature.

Definition 1.1
Let X be any non-empty set and let D be the set of all left continuous distribution functions. (X, G) which is an ordered pair, is said to be 2 Non Archimedean Menger Probability Metric Space (2 N.A. PM-space) if G is a mapping from $X \times X \times X$ *into* D satisfying the given following conditions, where the value of G at is $(x, y, z) \in X \times X \times X$ represented by $G_{x,y,z}$ *or* $G(x,y,z)$ *for each* $x,y,z \in X$ such that
i. G(x, y, z, t) = 1 t > 0 if and only if at least two of the three points are equal..
ii. G(x, y, z) = G(x, z, y) = G (z, x, y)
iii. G(x, y, z; 0) = 0
iv. If $G(x,y,s;t_1) = G(x,s,z;t_2) = G(s,y,s;t_3) = 1, then \; G(x,y,z;\max\{t_1,t_2,t_3\}) = 1$

Definition 1.2
A t-norm is a function $\delta : [0,1] \times [0,1] \times [0,1] \to [0,1]$ which is associative, commutative, non-decreasing in each coordinate and $\delta(l, 1, 1) = l$ for each $l \in [0,1]$.

Definition 1.3
A sequence $\{x_n\}$ in 2 N. A. Menger PM-space (X, G, δ) converges to x if and only if for each $\varepsilon > 0, \lambda > 0$ *there exists an integer* $Z(\varepsilon, \lambda)$
Such that $f(G(x_n, x, l\,;\, \varepsilon)) < f(1-\lambda), \forall\, n > Z$ *or* $\lim_n f(G(x_n, x, l; t)) = 0$, we write $x_n \to x$. we write $x_n \to x$.

Definition 1.4
A sequence $\{x_n\}$ in 2 N. A. Menger PM-space is Cauchy sequence if and only if for each $\varepsilon > 0$, $\lambda > 0$ there exists an integer $Z(\varepsilon, \lambda)$ such that
$f\left(G\left(x_n, x_{n+p}, l\,;\, \varepsilon\right)\right) < f(1-\lambda), \forall\, n, p \geq M \text{ and } p \geq 1 \text{ or if } \lim_{n,m} f\left(G(x_n, x_m, l; t)\right) = 0.$

Definition 1.5
Two self-maps A and S of a 2 N. A. Menger PM-space (X, G, δ) are called compatible if $\lim_n f\left(G(ASx_n, SAx_n, l\,;\, t)\right) = 0$ Whenever $\{x_n\}$ is a sequence is a sequence in X such that $\lim_n Ax_n = \lim_n Sx_n = p$ for some $p \in X$.

Definition 1.6
Two self-maps A and S of a 2 N. A. Menger PM-space (X, G, δ) are called weakly commuting if $f\left(G(ASx, SAx, l; t)\right) \leq f\left(G(Ax, Sx, l; t)\right)\ \forall\ x \in X \text{ and } t > 0.$

Definition 1.7
If two self-maps A and S of a 2 N.A. Menger PM-space (X, G, δ) are said to be R-weakly commuting, there exists R > 0 such that $\left(G(ASx, SAx, l; t)\right) \leq f\left(G\left(Ax, Sx, l; \frac{t}{R}\right)\right)\ \forall\ x \in X \text{ and } t > 0.$

Example 1.8 [5]
Let $X = R$ with 2-metric defined as $d(x, y, z) = \min\{\,|x-y|, |y-z|, |z-x|\,\}, \text{for all } x, y, z \in X,\ t > 0$ Define $G(x, y, z; t) = \frac{t}{t+d(x,y,z)}$, with $\delta(r, s, t) = \min(r, s, t)$ then (X, G, δ) is 2 Non Archimedean Menger Probability Metric Space.

Definition 4.2.11
Two self-maps A and S of a 2 N. A. Menger PM-space (X, G, δ) are said to be reciprocally continuous on X if $\lim_n ASx_n = Ax \text{ and } \lim_n SAx_n = Sx$ whenever $\{x_n\}$ is a sequence in X such that $\lim_n Ax_n = \lim_n Sx_n = x \text{ for some } x \in X.$

Lemma 1.1
Let $\{y_n\}$ be a sequence in 2 Non Archimedean Menger Probability Metric Space (X, G, δ) where δ is a continuous t-norm satisfying $\delta(l, l, l) \geq l$ for all $l \in [0,1]$. If there exists a positive number $h \in (0,1)$ such that $f\left(G\left(y_n, y_{n+1}, l, \frac{(1-h)t}{2h}\right)\right) \leq f\left(G(y_{n-1}, y_n, l, t)\right),\ n = 1,2,3, \ldots$ Then $\{y_n\}$ is a Cauchy sequence.

Lemma 1.2
Let (X, G, δ) be a complete 2 Non Archimedean Menger Probability metric space with $\delta(l, l, l) \geq l$ for all $l \in [0,1]$, and let $(M, P) \text{ and } (N, Q)$ be point wise R-weakly commuting pair of self-maps of X such that
(i) $MX \subset QX, NX \subset PX$.
(ii) There exists $k \in (0, 1)$ such that
$f(G(Mx, Nx, l, kt)) \leq f(G(Px, Qy, l, t))$ for all $x, y, l \in X, t > 0$.
Then the continuity of any one of the mappings in compatible pair $(M, P) \text{ or } (N, Q)$ on (X, G, δ) tends to its reciprocal continuity.

Theorem 1.1
Let (X, G, δ) be a complete 2 Non Archimedean Menger Probability Metric space with $\delta(l, l, l) \geq l$ for all $l \in [0,1]$, let $(M, F) \text{ and } (N, Q)$ be point wise R-weakly commuting pair of self-functions of X and such that
(i) $MX \subset QX, NX \subset PX$.
(ii) There exists $k \in (0,1)$ such that $R\left(f(G(Mx, Nx, l, kt))\right) \leq R\left(f(G(Px, Qy, l, t))\right)$
for all $x, y, l \in X, t > 0$. Where $R : [0,1] \to [0,1]$ is continuous function such that $R(s) > s$ for each $0 < s < 1$.
If one of the functions in compatible pair $(M, F) \text{ or } (N, Q)$ is continuous, then $M, N, P \text{ and } Q$ have a unique common fixed point in X.

Proof

By Lemma 1.2, if P is continuous and (M, P) are compatible then A and P are reciprocally continuous.
Let $x_0 \in X$. $\exists\ x_1, x_2 \in X$, Such that $Mx_0 = Qx_1 = y_0$ and $Nx_1 = Px_2 = y_1$.
Now, the sequences $\{x_n\}$ and $\{y_n\}$ in X are defined by $Mx_n = Qx_{n+1} = y_n$ and
$Nx_{n+1} = Qx_{n+2} = y_{n+1}$ for $n = 0,1,2,3, \dots$
Substitute $x = x_n,\ y = x_{n+1}$ in (ii) we have

$$R(f(G(Mx_n, Nx_{n+1}, l, kt))) \leq R(f(G(Px_n, Qx_{n+1}, l, t)))$$
$$\Rightarrow f(G(Mx_n, Nx_{n+1}, l, kt)) \leq f(G(Px_n, Qx_{n+1}, l, t))$$
$$\Rightarrow f(G(y_n, y_{n+1}, l, kt)) \leq f(G(y_{n-1}, y_n, l, t))$$

By Lemma 1.1, $\{y_n\}$ is a Cauchy sequence.
Since X is a complete 2 N.A. Menger PM space,
$\therefore$ There exists a point $p \in X$ such that $y_n \to p$ as $n \to \infty$.
Hence $y_n = Mx_n = Qx_{n+1} \to p$ and $y_{n+1} = Nx_{n+1} = Px_{n+2} \to p$
Since M and P are reciprocally continuous and compatible functions
$\therefore MPx_n \to Mp$ and $PMx_n \to Pp$
Compatibility of M and P gives,

$$\lim_n R\left(f\left(G(MPx_n, PMx_n, l, t)\right)\right) = 0$$
$$\Rightarrow \lim_n f\left(G(MPx_n, PMx_n, l, t)\right) = 0$$
$$\Rightarrow \lim_n f\left(G(Mp, Pp, l, t)\right) = 0$$

Thus $Mp = Pp$
Here $M(X) \subset Q(X)$, there exists a point $q \in X$ such that $Mp = Qq$,
By (ii) $R\left(f(G(Mp, Nq, l, kt))\right) \leq R(f(G(Pp, Qq, l, t)))$

$$\Rightarrow f(G(Mp, Nq, l, kt)) \leq f(G(Pp, Qq, l, t))$$
$$\Rightarrow f(G(Mx_n, Nx_{n+1}, l, kt)) \leq f(G(Mp, Pp, l, t)) \Rightarrow Mp = Nq$$

Thus $Mp = Nq = Pp = Qq$.
Since M and P are point wise R-weakly commuting functions.

$$\exists\ R > 0,\ f\left(G(MPp, PMp, l, t)\right) \leq f\left(G\left(Mp, Pq, l, \frac{t}{R}\right)\right) = 0$$

Thus $MPp = PMp$ and $MMp = MPp = PMp = PPp$.
Also, N and Q are point's wise R-weakly commuting functions.
We get $NNq = NQq = QNq = QQq$.
Now, by taking $x = Mp, y = q$ in (ii) we get

$$R\left(f\left(G(MMp, Nq, l, kt)\right)\right) \leq R\left(f\left(G(PMp, Qq, l, t)\right)\right)$$
$$\Rightarrow f\left(G(MMp, Nq, l, kt)\right) \leq f\left(G(PMp, Qq, l, t)\right) = f\left(G(MMp, Nq, l, t)\right)$$
$$\Rightarrow MMp = Mp \text{ and } Mp = MMp = PMp.$$

Hence, Mp is a common fixed point of M and P. Similarly from (ii), we can say that $Nq\ (= Mp)$ is a common fixed point of N and Q. Thus Mp is a common fixed point of M, N, P and Q.
For uniqueness of fixed point:
Consider Mq is another common fixed point of M, N, P and Q. Then,
by (ii), we have

$$R\left(f\left(G(MMp, NMq, l, kt)\right)\right) \leq$$
$$R\left(f\left(G(PMp, QMq, l, t)\right)\right)$$
$$\Rightarrow f\left(G(MMp, NMq, l, kt)\right) \leq$$
$$f\left(G(PMp, QMq, l, t)\right) = f\left(G(Mp, Mq, l, t)\right)$$
$$\Rightarrow Mp = Mq.$$

Hence, Mp is a unique common fixed point of M, N, P and Q.

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