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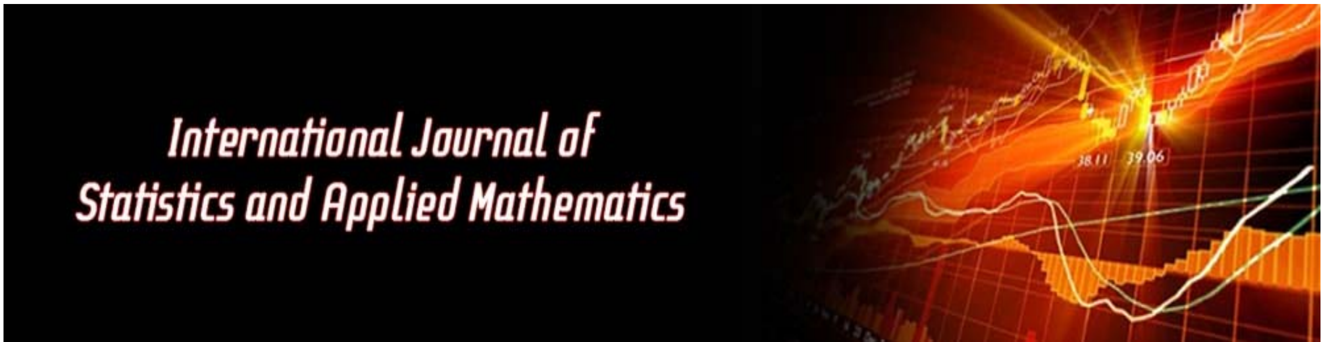

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A Relation between Laplace and Hankel transform of three variables

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Abstract
In this paper we have been evaluated a relation between Laplace and Hankel transform of three variables and Hankel transform of three variables of Meijer's G-function based theorems and application.

Keywords: Laplace transform, Hankel transform, three variables

1. Introduction

In the present note a relation between Laplace and Hankel of three variables has been established. The result may be found useful in evaluating Hankel transform of three variables by means of tables of Laplace transform of three variables. By Humbert (1934) results of two variables. let us consider the well-known Laplace transform of a function f(x, y, z) of three variables

F (p, q, r) = $\iiint_0^\infty e^{-px-qy-rz}$ f(x, y, z) $dx\,dy\,dz$,
Re (p, q, r) > 0…………….. (1.1)

and its Hankel transform of orders n_1, n_2 and n_3 is given by in the form

F (p, q, r) = $\iiint_0^\infty \sqrt{pqr\,xyz}$ $Jn_1(p\,x)\,Jn_2(qy)\,Jn_3(rz)\,dx\,dy\,dz$
p>0, q>0, r>0 …………………. (1.2)

We shell denote (1.1) and (1.2) symbolically as

L $[f(x,y,z); p,q,r]$ = F (p, q, r)

and F (p, q, r) $\overset{J}{\overline{n1\ n2\ n3}}$ f (x, y, z) respectively.

2. Formulae used

We shall use the following formula in the sequel, which can be proved easily form the definition:

L $[x^{n1/2}\, y^{n2/2}\, z^{n3/2}\, Jn_1(2\sqrt{sx})\, Jn_2(2\sqrt{ty})\, Jn_3(2\sqrt{uz})]$

$= \dfrac{s^{n1/2}\, t^{n2/2}\, u^{n3/2}\, e^{-\frac{s}{p}-\frac{t}{q}-\frac{u}{r}}}{p^{n1+1}\, q^{n2+1}\, r^{n3+1}}$ ……………… (2.1), Provided Re (p, q, r) > -1

L $\left[x^{c1} y^{c2} z^{c3} G^{h,l,m}_{q,r,s}\left(\begin{array}{c} a1 \ldots aq \\ b1 \ldots br \end{array}\right); p, q, r\right]$ [

$= \dfrac{1}{p^{c1+1} q^{c2+1} r^{c3+1}} G^{h,l,m}_{q,r,s}\left(\dfrac{z}{pqr}\middle| \begin{array}{c} -c1, -c2, -c3, a1 \ldots aq \\ b1 \ldots br \end{array}\right)$ ……..(2.2)

Provided
$q+r+s < 2(h+l+m)$, Re $(c_k+b_j+a_i)>0$, k= 1, 2, j= 1…..h;

$|zp^{-1}q^{-1}r^{-1}| < (h+l+m-\frac{1}{2}q-\frac{1}{2r}-\frac{1}{2s})\pi.$

3. Theorem

If h (p, q, r) $\frac{J}{n1\ n2\ n3}$f(x, y, z) …………………………………..(3.1)
And

F (p, q, r) = L [$x^{\frac{n1}{2}-\frac{1}{4}}y^{\frac{n2}{2}-\frac{1}{4}}z^{\frac{n3}{2}-\frac{1}{4}}h(\sqrt{2x},\sqrt{2y},\sqrt{2z}); p, q, r$]………… (3.2)
Then

$p^{n1+1}q^{n2+2}r^{n3+1}$F (p, q, r) = L [$x^{\frac{n1}{2}-\frac{1}{4}}y^{\frac{n2}{2}-\frac{1}{4}}z^{\frac{n3}{2}-\frac{1}{4}}h(\sqrt{2x},\sqrt{2y},\sqrt{2z})$; $p^{-1}q^{-1}r^{-1}$]. (3.3)

Provided Hankel transform of $|f(x,y,z)|$ and Laplace transform of $\left|x^{\frac{n1}{2}-\frac{1}{4}}y^{\frac{n2}{2}-\frac{1}{4}}z^{\frac{n3}{2}-\frac{1}{4}}h(\sqrt{2x},\sqrt{2y},\sqrt{2z})\right|$, $\left|x^{\frac{n1}{2}-\frac{1}{4}}y^{\frac{n2}{2}-\frac{1}{4}}z^{\frac{n3}{2}-\frac{1}{4}}f(\sqrt{2x},\sqrt{2y},\sqrt{2z})\right|$ in (3.2) and (3.3) exist and

Re (n_1, n_2, n_3) > -1 , Re (p, q, r)>0.

Proof: Substituting the value of $h(\sqrt{2x},\sqrt{2y},\sqrt{2z})$ with the help of (3.1) in (3.2), we get

F (p, q, r) = $\iiint_0^\infty e^{-px-qy-rz}x^{\frac{n1}{2}-1/4}y^{\frac{n2}{2}-1/4}z^{\frac{n3}{2}-1/4}$ [$\iiint_0^\infty \sqrt{2stu}\sqrt{xyz}$ Jn $_1$($\sqrt{2xs}$) Jn$_2$ ($\sqrt{2yt}$) Jn$_3$ ($\sqrt{2zu}$) $\times$ f(s, t, u) ds dt du] $dx\,dy dz$

= $\iiint_0^\infty \sqrt{2stu}$ f(s, t, u) ds dt du [$\iiint_0^\infty e^{-px-qy-rz}x^{\frac{n1}{2}}y^{\frac{n2}{2}}z^{\frac{n3}{2}}\times$ Jn$_1$ ($\sqrt{2xs}$) Jn$_2$ ($\sqrt{2yt}$) Jn$_3$ ($\sqrt{2zu}$) $dx\,dy dz$]
………. (3.4)

The change in the order of integration is justified under the conditions stated in the theorem. Evaluating the inner triple integral by (2.1), and simplifying, we obtain

$p^{n1+1}q^{n2+1}r^{n3+1}$F (p, q, r) = $\iiint_0^\infty \sqrt{2stu}$ f(s, t, u) $(\frac{s^2}{2})^{n1/2}(\frac{t^2}{2})^{n2/2}(\frac{u^2}{2})^{n3/2}e^{-\frac{s^2}{2p}-\frac{t^2}{2q}-\frac{u^2}{2r}}$ ds dt du,

Re (n_1, n_2, n_3)> -1, Re (p, q, r)>0 ………… (3.5)
Substituting $s^2 = 2S$, $t^2 = 2T$ and $u^2 = 2U$ in (3.5), the result (3.3) follows on simplification.

4. Applications

Let f(x, y, z) =x,

Where q+r+s < 2(h+l+m), $|\arg z|$< (h+l $-\frac{1}{2q}-\frac{1}{2}r-\frac{1}{2s})\pi$.

Then, form (2.2), we have L [$x^{\frac{n1}{2}-\frac{1}{4}}y^{\frac{n2}{2}-\frac{1}{4}}z^{\frac{n3}{2}-\frac{1}{4}}h(\sqrt{2x},\sqrt{2y},\sqrt{2z})$; $p^{-1}q^{-1}r^{-1}$] = $p^{n1+1}q^{n2+1}r^{n3+1}$ F (p, q, r)

$= 2^{\rho1+\rho2+\rho3-\frac{1}{2}}p^{\rho1+\frac{n1}{2}+\frac{1}{2}}q^{\rho2+\frac{n2}{2}+\frac{1}{2}}r^{\rho3+\frac{n3}{2}+\frac{1}{2}}\ G^{h,l,m}_{q,r,s}\left(6zpqr\middle|^{k1,k2,k3\ a1\ldots\ldots aq}_{b1\ldots\ldots br}\right)$

Where

$K_t = -p_t - \frac{nt}{2}+\frac{1}{2}$, Re ($p_t + \frac{nt}{2}$ +bj+$\frac{1}{2}+$ a_i+$\frac{1}{2}$)>0, t= 1, 2, j= 1, 2…h

$|\arg 6z\,pqr|$ < (1+h+l+m-$\frac{1}{2}q-\frac{1}{2}r-\frac{1}{2}s)\pi$.

And

F (p, q, r) = $2^{\rho1+\rho2+\rho3-\frac{1}{2}}p^{\rho1-\frac{n1}{2}-\frac{1}{2}}q^{\rho2-\frac{n2}{2}-\frac{1}{2}}r^{\rho3-\frac{n3}{2}-\frac{1}{2}}\ G^{h,l+2,m}_{q+2,r,s}\left(6zpqr\middle|^{k1,k2,k3\ a1\ldots\ldots aq}_{b1\ldots\ldots br}\right)$

$= L[2^{\rho1+\rho2+\rho3-\frac{1}{2}}x^{\frac{n1}{2}-\rho1-\frac{1}{2}}q^{\frac{n2}{2}-\rho2-\frac{1}{2}}r^{\frac{n3}{2}-\rho3-\frac{1}{2}}$

$G^{h,l+2,m}_{q+2,r,s}\left(\frac{6z}{xyz}\middle|^{k1,k2,k3\ a1\ldots\ldots aq,h1,h2,h3}_{b1\ldots\ldots br}\right)$; p, q, r]

= L [$x^{\frac{n1}{2}-\frac{1}{4}}y^{\frac{n2}{2}-\frac{1}{4}}z^{\frac{n3}{2}-\frac{1}{4}}h(\sqrt{2x},\sqrt{2y},\sqrt{2z}); p, q, r$]

Where $h_t = -p_t + \frac{nt}{2} + \frac{1}{2}$

Re ($\frac{nt}{2}-pt+at+bj+\frac{1}{2}$)>0, j= 1… h, t= 1, 2.

Hence h ($\sqrt{2x}, \sqrt{2y}, \sqrt{2z}$) =

$$= [2^{p1+p2+p3-\frac{1}{2}} x^{\frac{m1}{2}-p1-\frac{1}{2}} q^{\frac{m2}{2}-p2-\frac{1}{2}} r^{\frac{m3}{2}-p3-\frac{1}{2}}$$

$$G^{h,l+2,m}_{q+6,r,s}\left(\frac{6z}{xyz}\middle|\begin{matrix}k1,\ k2, k3\ a1 \ldots\ldots\ldots aq, h1, h2, h3\\ b1 \ldots\ldots\ldots\ldots br\end{matrix}\right)$$

On applying (3.1) of the theorem, we get

$$\frac{2^{2p1+2p2+2p3}}{p^{2p1+\frac{1}{2}} q^{2p2+\frac{1}{2}} r^{2p3+\frac{1}{2}}}$$

$$G^{h,l+2,m}_{q+2,r,s}\left(\frac{36z}{x^2y^2z^2}\middle|\begin{matrix}k1,\ k2, k3\ a1 \ldots\ldots\ldots aq, h1, h2, h3\\ b1 \ldots\ldots\ldots\ldots br\end{matrix}\right)\frac{J}{n1, n2}$$

$$x^{2p1-\frac{1}{2}} y^{2p2-\frac{1}{2}} z^{2p3-\frac{1}{2}} G^{h,l,m}_{q,r,s}\left(zx^2y^2\middle|\begin{matrix}a1 \ldots\ldots\ldots aq\\ b1 \ldots\ldots\ldots\ldots br\end{matrix}\right)$$

Provided q+r+s<2(h+l+m), $|\arg z|$< (h+l+m $-\frac{1}{2}q - \frac{1}{2}r\ \frac{1}{2}$s)$\pi$.

Re ($b_j+p_t+a_i+\frac{1}{2}n_t$)> $-\frac{1}{2}$, t= 1, 2, j= 1, 2 ….h;

Re ($a_j+p_t+b_i$) < $\frac{3}{4}$, t=1, 2, j =1, 2, ……., l.

5. References

1. Humbert P. Le caloul symbolique a deux variables, C. r. hebd. Seanc. Acad. sci, paris, 1934; 199:657-60.
2. Jain NC. Laplace transform of two variables and self – reciprocal functions. J.math. pures appl., 1968; 47:47-55.
3. verma RS. A relation between Laplace and Hankel transform of two variables, 1969.
4. Widder DV. The Laplace transform, Princeton Univ. Press, 1946,
5. Mainara VP. A new generalization of the Laplace transforms Bull. of the cal. Math. Soc.