International Journal of Statistics and Applied Mathematics 2016; 1(3): 21-25

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2016; 1(3): 21-25

www.mathsjournal.com
Received: 05-07-2016
Accepted: 06-08-2016

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Solution of fuzzy multi-objective nonlinear programming problem using interval arithmetic based alpha-cut

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Abstract
In this paper, a fuzzy multi-objective nonlinear programming problem is presented. All the coefficients of the nonlinear multi-objective functions and the constraints are fuzzy numbers. Here we find the solution of the nonlinear programming problem by using Interval arithmetic based on Alpha-cut. A numerical example is presented.

Keywords: Fuzzy multi-objective nonlinear programming (FMONLP), Triangular fuzzy number, Interval arithmetic, Optimal solution, α - cut.

1. Introduction
The concept of fuzzy non linear programming was proposed by zimmermann [12]. Fuzzy non linear programming problem is useful in solving problems which are difficult, impossible to solve to the imprecise, subjective nature of the problem formulation or have an accurate solution. Bellman and Zadeh [1] proposed the concept of decision making in fuzzy environment. A new method for solving linear programming problem with vagueness in constraints by using ranking function was introduced by Maleki [8]. Tanaka .et.al, [11] adopted this concept for solving mathematical programming problems. Pandian and Jayalakshmi [9] proposed a new method for solving fully fuzzy linear programming problem with fuzzy variables. Compos and Vardegay [2] considered linear programming problems with fuzzy constraints and fuzzy coefficients in both left and right hand of the constraints set. Kirtiwant and Tanaji [5] introduced a new approach for finding an optimal fuzzy solution for fuzzy non linear programming problem. In this paper, we propose the basic definitions of fuzzy set and Interval arithmetic operations on fuzzy numbers based on α - cut to solve a fuzzy multi objective non linear programming problem.

2. Preliminaries
2.1 Definition: [4]
Let A be a classical set. $\mu_a(x)$ be a real valued function defined from R → [0, 1]. A fuzzy set A^* with the function $\mu_a(x)$ is defined by $A^* = \{ (x , \mu_a(x)) ;\ x \in A$ and $\mu_a(x) \in [0, 1]\}$. The function $\mu_a(x)$ is known as the membership function of A^*.

2.2 Definition: [3]
A fuzzy number is a convex normalized fuzzy set of the real line R whose membership function is piecewise continuous.

2.3 Definition: [12]
A fuzzy number $\widetilde{A}$ in R is said to be a triangular fuzzy number if its membership function

$$\mu_{\tilde{A}}(X) = \begin{Bmatrix} \frac{x-p}{q-p}, & p \le x \le q \\ \frac{r-x}{r-q}, & q \le x \le r \\ 0 & otherwise \end{Bmatrix}$$

2.4 Definition: [4]

Given a fuzzy set A defined on X and any number α ε[0,1] , the α - cut α_A is the crisp set

$$\alpha_A = \{x / A(x) \ge \alpha\}.$$

2.5 Definition: [9]

Given a fuzzy set A defined on X and any number α ε[0,1] , the strong α - cut α_{+A} is the crisp set

$$\alpha_{+A} = \{x / A(x) \ge \alpha\}.$$

3. Interval arithmetic [10]

Interval arithmetic, interval mathematics, interval analysis, or interval was developed by various mathematicians in between 1950 and 1960. This is an approach to putting bounds on rounding errors and measurement of errors in mathematical computation and thus developing numerical methods that yield reliable results. This represents each value as a range of possibilities.

The following are the basic operations of interval arithmetic, for two variables [p, q] and [r, s] that are subsets of the real line (- ∞, ∞)

(i) Addition: [p, q] + [r, s] = [p + r, q + s]
(ii) Subtraction: [p, q] - [r, s] = [p - r, q - s]
(iii) Multiplication: [p, q]. [r ,s] = [min (pr, ps, qr, qs) , max(pr, ps, qr, qs)]
(iv) Division: $\left[\frac{p,q}{r,s}\right]$ = min $\left(\frac{p}{r}\frac{p}{s}\frac{q}{r}\frac{q}{s}\right)$, max $\left(\frac{p}{r}\frac{p}{s}\frac{q}{r}\frac{q}{s}\right)$, when 0 is not in[c, d]

4. Arithmetic operations of fuzzy numbers using α - cut method

In this section, we consider addition, subtraction, multiplication and division of fuzzy numbers using α - cut method [10].

4.1 Addition of Fuzzy Numbers

Let X = [p, q, r] and Y = [a, b, c] be two fuzzy numbers whose membership functions are defined by

$$\mu_x(X) = \begin{Bmatrix} \frac{x-p}{q-p}, & p \le x \le q \\ \frac{r-x}{r-q}, & q \le x \le r \\ 0 & otherwise \end{Bmatrix}$$

$$\mu_x(X) = \begin{Bmatrix} \frac{x-a}{b-a}, & a \le x \le b \\ \frac{c-x}{c-b}, & b \le x \le c \\ 0 & otherwise \end{Bmatrix}$$

Then, α_x = [(q - p)α + p , r - (r - q)α] and α_y = [(b - a)α + a , c - (c - b)α] are the α -cuts of the fuzzy numbers X and Y respectively. To calculate the addition of fuzzy numbers X and Y using interval arithmetic, that is

α_x + α_y = [(q - p)α + p , r - (r - q)α] + [(b - a)α + a , c - (c - b)α]
= [p + a + (q – p + b - a)α , r + c - (r – q + c - b)α .

4.2 Subtraction of Fuzzy Numbers
Let X = [p, q, r] and Y = [a, b, c] be two fuzzy numbers.

Then, α_x = [(q - p)α + p, r - (r - q)α] and α_y = [(b - a)α + a, c - (c - b)α] are the α - cuts of the fuzzy numbers X and Y respectively. To calculate the subtraction of fuzzy numbers X and Y using interval arithmetic, that is

α_x - α_y = [(q - p)α + p, r - (r - q)α] - [(b - a)α + a, c - (c - b)α]
= [(p - c) + (q – p + c – b)α , (r - a) - (q – r + b - a)α]

4.3 Multiplication of Fuzzy Numbers
Let X = [p, q, r] and Y = [a, b, c] be two fuzzy numbers.

Then, α_x = [(q - p)α + p, r - (r - q)α] and α_y = [(b - a)α + a, c - (c - b)α] are the α - cuts of the fuzzy numbers X and Y respectively. To calculate the multiplication of fuzzy numbers X and Y using interval arithmetic, that is

α_x * α_y = [(q - p)α + p, r - (r - q)α] * [(b - a)α + a, c - (c - b)α]
= [((q - p)α + p)* (b - a)α + a)), (r - (r - q)α)*(c - (c - b)α)]

4.4 Division of Fuzzy Numbers
Let X = [p, q, r] and Y = [a, b, c] be two fuzzy numbers.

Then, α_x = [(q - p)α + p, r - (r - q)α] and α_y = [(b - a)α + a, c - (c - b)α] are the α - cuts of the fuzzy numbers X and Y respectively. To calculate the division of fuzzy numbers X and Y using interval arithmetic, that is,

$$\frac{\alpha_x}{\alpha_y} = \frac{[(q-p)\alpha + p,\ r-(r-q)\alpha]}{[(b-a)\alpha + a,\ c-(c-b)\alpha]}$$

$$= \left(\frac{(q-p)\alpha + p}{c-(c-b)\alpha}, \frac{r-(r-q)\alpha}{(b-a)\alpha + a} \right)$$

5. Multiobjective nonlinear programming (monlp) formulation

Maximum Z = $\sum_j c_j x_j^{\alpha_j}$
Subject to the constraints,

$$\sum_{j=1}^{n} a_{ij} x_j \le b_i \qquad (i = 1, 2, …., m)$$

$$x_j \ge 0 \qquad (j = 1, 2, …., n)$$

The function to be maximized is called an objective function. This is denoted by Z. c = (c_1, c_2, ….,c_n) is called a cost vector. The matrix [a_{ij}] is called a constraint matrix and the vector b_i = (b_1, b_2,…., b_m) is called right hand side vector.

6. Procedure
The arithmetic operations of fuzzy numbers using α -cut operations discussed in the earlier sections are used below to solve the fuzzy nonlinear programming problem.

Step 1: Find the value of α_X and α_Y
Step 2: Add the α - cuts of X and Y using interval arithmetic
Step 3: The values obtained in step1 and step2 is converted into a crisp MONLPP.
Step 4: By solving the above nonlinear programming problem, we obtain the optimal solution.

7. Numerical problem [5]
Using the procedure an interval arithmetic nonlinear programming problem with triangular fuzzy numbers is considered.

Maximum Z = [(5, 6, 7) + (6, 7, 8)] x_1 + [(3, 4, 5) + (1, 2, 3)] x_2^2 (1)

Maximum Z = [(6, 7, 8) + (8, 9, 10)] x_1 + [(4, 5, 6) + (6, 7, 8)] x_2^2 (2)
Subject to the constraints

[(0, 1, 2) + (2, 3, 4)] x_1 + [(3, 4, 5) + (1, 2, 3)] x_2 $\le$ [(6, 8, 10) + (10, 12, 14)]

[(0, 1, 2) + (2, 3, 4)] x_1 + [(2, 3, 4) + (4, 5, 6)] x_2 $\le$ [(4, 6, 8) + (8, 10, 12)] (3)

Step 1: Determine α_X and α_Y for an objective function (1)
The α - cut of the fuzzy number (5, 6, 7) is

$\alpha = \frac{x-5}{1}, \frac{7-x}{1}$

$\alpha_X = (\alpha + 5, 7 - \alpha)$

The α - cut of the fuzzy number (6, 7, 8) is

$\alpha = \frac{x-6}{1}, \frac{8-x}{1}$

$\alpha_X = (\alpha + 6, 8 - \alpha)$

Step 2: Adding the α - cuts of X and Y using interval arithmetic we obtain

$\alpha_x + \alpha_y = (\alpha + 5, 7 - \alpha) + (\alpha + 6, 8 - \alpha) = 28$

Similarly the remaining fuzzy numbers of objective functions are

[(3, 4, 5) + (1, 2, 3)] = 12

[(6, 7, 8) + (8, 9, 10)] = 32

[(4, 5, 6) + (6, 7, 8)] = 24

Similarly the constraint matrix a_{ij} and the right hand side number b_i are

a_{11} = [(0, 1, 2) + (2, 3, 4)] = 8

a_{12} = [(3, 4, 5) + (1, 2, 3)] = 12

a_{21} = [(0, 1, 2) + (2, 3, 4)] = 8

a_{22} = [(2, 3, 4) + (4, 5, 6)] = 16

b_1 = [(6, 8, 10) + (10, 12, 14)] = 40

b_2 = [(4, 6, 8) + (8, 10, 12)] = 32

Step 3: Now the given problem becomes the crisp MONLPP as

Maximum Z = 28 x_1 + 12 x_2^2 (4)

Maximum Z = 32 x_1 + 24 x_2^2 (5)

Subject to the constraints

$8x_1 + 12x_2 \leq 40$

$8x_1 + 16x_2 \leq 32$ (6)

Step 4: Solving the nonlinear equation (4) with (6). We obtain the optimal solution.

Max Z = 48, $x_1 = 0$, $x_2 = 1.8$

Solving the non linear equation (5) with (6), we obtain the optimal solution.

Max Z = 31.68, $x_1 = 0$, $x_2 = 1.2$

8. Conclusion

Fuzzy multi-objective nonlinear programming problem has been solved by using interval arithmetic based on α - cut operation without converting them into a classical NLPP. This is an easy approach for solving FNLPP by using interval arithmetic when compared to the earlier approaches .This is also can be applied in trapezoidal, hexagonal, octagonal fuzzy numbers.

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