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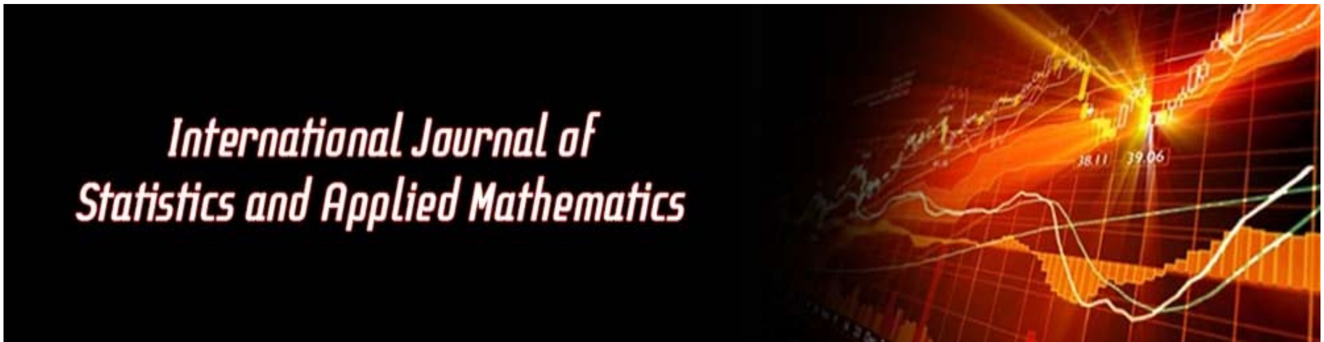

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Extension of G-Symbol of three variables

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Abstract
The aim of this paper is to evaluate and extend the G-Symbol into three variables which is a special cases of Appell's Function

Keywords: R-norm Entropy, 'Useful' R-norm Information, Utilities, Kraft inequality, Holder's inequality

Introduction

Meijer in 1941 defined his G-Function by means of a Mellin-Barnes type of integral in the form

$$G_{p,q}^{m,n}\, x\begin{matrix} a1, a2 \ldots\ldots ap \\ b1, b2 \ldots\ldots pq \end{matrix} = \frac{1}{2\pi i}\int \frac{\prod \Gamma(bj-s) \prod_{J=1}^{N} \Gamma(1-aj+s)}{\prod_{J=m+1}^{q} \Gamma(1-bj+s) \prod_{J=n+1} \Gamma(aj+s)} x_s ds,$$

where an empty product is interpreted as i,o $\leq$ m $\leq$ q, o $\leq$ n $\leq$ p, and the parameters are such that no pole of $\Gamma(bj\text{-}s)$, j =1, 2, …. m coincides with any pole of $\Gamma(1\text{-}a_k\text{+}s)$, k = 1, 2, ……n. The path of integration L runs from -i∞ to i∞ so that all poles of $\Gamma(bj\text{-}s)$, j = 1, 2, …. m are to the right, and all the poles of $\Gamma(1\text{-}a_k\text{+}s)$, k = 1, 2, ……. n to the left of L. The integral converges for $p+q<2(m+n)$ and larg x| < $\pi(m+n-½p-½q)$. The importance of the G-Function lies in the great many special functions that can be represented as its particular cases.

The object of this paper is define a G-Function of three variables which not only includes the Meijer's G-Function as a particular cases but also most of the known function of three variables, e.g. the Appell's function F1, F2, F3, F4, the Whittaker function of three variables, etc. besides including the known function of three variables as particular cases, it leaves the possibility of defining, through this new G-Symbol of three variables, a great many special function of three variables not hither to mentioned.

$$\text{Let } (a_m) = a(a+1)\,(a+2) \ldots\ldots\ldots\ldots (a+m-1);\ (a)_0 = 1$$

Also, the symbol (α_p) denotes the sequence of elements $\alpha_1, \alpha_2, \ldots\ldots \alpha_p$ and $(\alpha_{m,p})$ denotes the sequence $\alpha_m, \alpha_{m+1}, \ldots\ldots\ldots\ldots \alpha_p$. Thus the triple hyper geometric function of higher order in three variables.

p	ε_1	$,\varepsilon_2$	$,\varepsilon_3$	,……………………………………	ε_p			
t	Υ_1	$,\Upsilon_1'$	$,\Upsilon_1''$	,……………………………………	Υ_t	$,\Upsilon_t'$	$,\Upsilon_t''$	x
ϑ	δ_1	$,\delta_2$	$,\delta_3$	,……………………………………	δ_ϑ			
q	β_1	$,\beta_2$	$,\beta_3$	,……………………………………	β_q	$,\beta_q'$	$,\beta_q''$	y
l	ρ_1	$,\rho_2$	$,\rho_3$		ρ_l			
ω	υ_1	$,\upsilon_2$	$,\upsilon_3$	,……………………………………	υ_ω	$,\upsilon_\omega'$	$,\upsilon_\omega''$	z

$$= \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} \begin{array}{|c|} \hline (\varepsilon_1)_{m+n+k} \ldots\ldots\ldots (\varepsilon_p)_{m+n+k} (\Upsilon_1)_m (\Upsilon_1')_n (\Upsilon_1'')_k \ldots\ldots\ldots\ldots (\Upsilon_t)_m (\Upsilon_t')_n (\Upsilon_t'')_k \\ \hline [(\delta 1)_{m+n+k} \ldots\ldots\ldots\ldots (\delta_s)_{m+n+k} (\beta_1)_{m+n+k} (\beta_1')_{m+n+k} (\beta_1'')_{m+n+k} \ldots\ldots\ldots (\beta_q)_m (\beta_q')_n (\beta_q'')_k]\ [(\rho_1)_{m+n+k} \\ \hline \ldots\ldots\ldots (\rho_l)_{m+n+k} \ldots\ldots\ldots (\upsilon_1)_{m+n+k} (\upsilon_1')_{m+n+k} (V_1'')_{m+n+k} \ldots\ldots (\upsilon_\omega)_m (\upsilon_\omega')_n (\upsilon_\omega'')_k (1)_m (1_n) (1)_k \\ \hline \end{array}$$

Where p+t < s+q+l+ω+1 shall be a abbreviated to

p	$(\mathcal{E}_p)$			
t	(Υ_t)	$;(\Upsilon_t')$	$;(\Upsilon_t'')$	x
ȣ	$(\delta_ȣ)$			
q	(β_q)	$;(\beta_q')$	$;(\beta_q'')$	y
l	(ρ_l)			
ω	(υ_ω)	$;(\upsilon_\omega')$	$;(\upsilon_\omega'')$	z

The series for the F-Function converges absolutely for all complex values of x, y and z if p + t < s+q+l+ω+1. In case p+t = s+q+l+ω+1, it converges absolutely for all complex values of x, y and z and **|x|+|y|+|z| <** min $(1, 2^{l+s-p+1})$.

2. The $G^{m1,m2,m3,n,v1,v2,v3}_{p,q,s,t,l,w}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$

Consider the triple contour integral.

(1) $I = \frac{-1}{4\pi 2}\int_{-i\infty}^{i\infty}\int_{-i\infty}^{i\infty}\int_{-i\infty}^{i\infty} \emptyset(\xi+\eta)\psi(\eta+\tau)\chi(\eta,\tau)x^{\xi}y^{\eta}z^{\tau}d\xi d\eta d\tau$

Where

$$\chi(\eta,\tau) = \frac{\prod_{j=1}^{m2}\Gamma(v_{j-\eta})\prod_{j=1}^{v2}\Gamma(\beta_{j+\eta})\prod_{j=1}^{m3}\Gamma(v''_{j-\tau})\prod_{j=1}^{v3}\Gamma(\beta''_{j+\tau})}{\prod_{j=1}^{w}\Gamma(1-v_{j+\eta})\prod_{j=v_2+1}^{q}\Gamma(1-\beta_{j-\eta})\prod_{j=1+m3}^{w}\Gamma(1-v''_{j+\tau})\prod_{j=1+v_3}^{q}\Gamma(1-\beta''_{j-\eta})}$$

$$\psi(\eta+\tau) = \frac{\prod_{j=1}^{n}\Gamma(1-\delta_{j+\eta+\tau})}{\prod_{j=n+1}^{s}\Gamma(\delta j-\eta-\tau)\prod_{j=1}^{l}\Gamma(\delta j+\eta+\tau)}$$

and $\emptyset(\xi+\eta) = \frac{\prod_{j=1}^{n}\Gamma(1-\varepsilon j+\xi+\eta)}{\prod_{j=n+1}^{\alpha}\Gamma(\varepsilon j-\xi-\eta)\prod_{j=1}^{\gamma}\Gamma(\delta j+\xi+\eta)}$

and $0 \leqq m_1 \leqq q$, $0 \leqq m_2 \leqq w$, $0 \leqq m_3 \leqq w$, $0 \leqq v_1 \leqq t$, $0 \leqq v_2 \leqq t$, $0 \leqq v_3 \leqq t$, $0 \leqq n \leqq p$,

The sequence of parameter (υ_{m1}), $(\upsilon_m{'}_2)$, $(\upsilon_m{''}_3)$, $(\beta_{\upsilon 1})$, $(\beta_\upsilon{'}_2)$, $(\beta_\upsilon{''}_3)$ and (En) are s.t. none of the poles of the integrand coincide. The Paths of integration are indented, if necessary, in such a manner that all the poles of $\Gamma(\upsilon_{j-n})$, j = 1, 2, ………… m_2 and $\Gamma(\upsilon_{k-\tau})$, k = 1, 2, ………… υ_3 lie to the right, and those of $\Gamma(\beta_{k+\eta})$, k = 1, 2, ………… υ_2 and $\Gamma(\beta_k{''}_{+\tau})$, k = 1, 2, ………… υ_3 and $\Gamma(1 - \varepsilon_k + \xi + \eta + \tau)$, k = 1, 2, ………… n lie to the left of the imaginary axis.

The integral (2.1) converges if

(2) p + q + l + w + s + t < 3 $(m_1 + \upsilon_1 + n)$, p + q + l + w + s + t < 3 $(m_2 + \upsilon_2 + n)$

p + q + l + w + s + t < 3 $(m_3 + \upsilon_3 + n)$

and

$|\arg x| < \Pi\,(m_1 + \upsilon_1 + n - \frac{1}{2}(p + q + l + w + s + t))$, $|\arg y| < \Pi\,(m_2 + \upsilon_2 + n - \frac{1}{2}(p + q + l + w + s + t))$, $|\arg z| < \Pi\,(m_3 + \upsilon_3 + n - \frac{1}{2}(p + q + l + w + s + t))$

Evaluating (2.1) by considering the residues at the poles of integrand that lie to the right of imaginary axis, we have

$I = \sum_{n=1}^{m1}\sum_{k=1}^{m2}\sum_{d=1}^{m3} x^{\upsilon_n} y^{\upsilon'_k} z^{\upsilon''_d}$

$$\frac{\prod_{j=1}^{\upsilon_1}\Gamma(\gamma_j+\beta_j+\upsilon_n)\prod_{j=1}^{\upsilon_2}\Gamma(\gamma'_j+\beta'_j+\upsilon'_k)\prod_{j=1}^{\upsilon_3}\Gamma(\gamma''_j+\beta''_j+\upsilon''_d)\prod_{j=1}^{m3}\Gamma(\upsilon''_j-\upsilon''_d)}{\prod_{j=\upsilon_1+1}^{t}\Gamma(1-\gamma_j-\beta_j-\upsilon_n)\prod_{j=1+\upsilon_2}^{t}\Gamma(1-\gamma'_j-\beta'_j-\upsilon'_k)\prod_{j=1+\upsilon_3}^{t}\Gamma(1-\gamma''_j-\beta''_j-\upsilon''_d)\prod_{j=1+m_1}^{w}\Gamma(1-\upsilon_j-\upsilon_n)}$$

(3) $$\frac{x\ \prod_{j=1}^{n}\Gamma(1-\varepsilon_j+\upsilon_n+\upsilon'_k+\upsilon'_d)\ \prod_{j=1}^{m1}\Gamma(\upsilon_j-\upsilon_n)}{\prod_{j=1+m_2}^{w}\Gamma(1+\upsilon'_j+\upsilon'_k)\prod_{j=1+m_3}^{w}\Gamma(1-\upsilon''_j-\upsilon''_k)\prod_{j=1+n}^{q}\Gamma(\varepsilon_j-\upsilon_n-\upsilon'_k-\upsilon''_d)\prod_{j=1}^{l}\Gamma(\delta_j+\upsilon_n+\upsilon'_k+\upsilon''_d)}$$

	p	$(1-\mathcal{E}_p)\ \upsilon_n+\upsilon'_k+\upsilon_d''$			
XF	t	$(\Upsilon_t+\upsilon_n)$;	$(\Upsilon'_t+\upsilon'_k)$;	$(\Upsilon''_t+\upsilon''_d)$	$(-)^{m1+p-n+t+q-\upsilon 1}x$
	ȣ	$(\delta_ȣ+\upsilon_n+\upsilon'_k+\upsilon''_d)$			
	q	$(\beta_q+\upsilon_n)$;	$(\beta'_q+\upsilon'_k)$;	$(\beta''_q+\upsilon''_d)$	$(-)^{m2+p-n+t+q-\upsilon 2}y$
	l	$(\rho_l+\upsilon_n+\upsilon'_k+\upsilon''_d)$			
	ω-1	$(1-\upsilon_\omega+\upsilon_n)$*	$;(1-\upsilon'_\omega+\upsilon'_k)$*	$;(1-\upsilon''_\omega+\upsilon''_d)$	$(-)^{m3+p-n+t+q-\upsilon 3}z$

Where the prime in Π' indicates the omission of the factor of the type $\Gamma(\upsilon_j - \upsilon_n)$; the asterisk in the F denotes the omission of the parameter of the type $(1-\upsilon_n+\upsilon'_k+\upsilon''_d)$, (2.3) converges absolutely for $p + t < s + q + l + w$ or $p + t = s + q + l + w$, and $|x| + |y| + |z| < \min(1, 2^{(l+s-p+1})$)

The right hand side of (2.3) shall, henceforth, be symbolically denoted by

		x	$(\mathcal{E}_p)$		
			(Υ_t)	;(Υ_t')	;(Υ_t'')
G	n,υ1,υ2,υ3,m1,m2,m3	y	(δ_{ε})		
	p, t, s, q, l, w		(β_q)	;(β_q')	;(β_q'')
		z	(ρ_l)		
			(υ_ω)	;(υ_ω')	;(υ_ω'')

Or

		x			x
G	n,υ1,υ2,υ3,m1,m2,m3	y	or simply	G	y
	p, t, s, q, l, w				
		z			z

$$= G^{m1,\,\upsilon 1}_{t,q,w}\left(\begin{array}{l}(1-\Upsilon_t)\\(\beta_q)\\(1-\beta_q)\\(\upsilon_w)\end{array}\right)\begin{array}{l}(q \geq t)\\(w \geq t)\end{array}$$

Whenever there is no chance of misunderstanding is the required extension of Meijer's G-Function to three variables.

3. Creation Particular Cases of $G\left[\begin{smallmatrix}x\\y\\z\end{smallmatrix}\right]$

(i)

G	o,υ1,υ2,υ3,m1,m2,m3	x	= G	m1, υ1	(	x	$(1-\Upsilon_t)$	)	= G	m2, υ2	(	y	$(1-\Upsilon'_t)$	)
		y					(β_q)						(β'_q)	
	o, t, s, q, o, w	z		t,s,q,w			(υ_w)			t,s,q,w			(υ'_w)	
									= G	m3, υ3	(	z	$(1-\Upsilon''_t)$	)
													(β''_q)	
										t,s,q,w			(υ''_w)	

(ii)

				……………	
G	o,υ1,t,t,m1,1,1	x	(Υ_t) ;	(Υ'_t) ;	(Υ''_t)
				……………	
	o, t, o, q, o, w	y	(β_q) ;	(β'_q) ;	(β''_q)
				……………	
		o	(υ_ω);	o ; o;	$(\upsilon''_{3,\omega})$

$$= G^{m1,\,\upsilon 1}_{t,q,w}\left(\begin{array}{l}(1-\Upsilon_t)\\(\beta_q)\\(1-\beta_q)\\(\upsilon_w)\end{array}\right)\begin{array}{l}(q \geq t)\\(w \geq t)\end{array} = \frac{\prod_{j=1}^{t}\Gamma(\Upsilon''_j)\quad \Gamma(\beta''_j)}{\prod_{j=2}^{q}\Gamma(1-\beta''_q)\prod_{j=3}^{w}\Gamma(1-\upsilon''_\omega)}$$

(iii)

$$G^{m,t,t,t,1,1,1}_{n,t,q,l,w}\begin{pmatrix}x\\y\\z\end{pmatrix} = x^{\upsilon_1}\, y^{\upsilon'_1} z^{\upsilon''_1}\,\frac{\prod_{j=1}^{t}\{\Gamma(\Upsilon_j+\beta_1+\upsilon_1)\ \prod_{j=1}^{t}\{\Gamma(\Upsilon'_j+\beta'_1+\upsilon'_1)\ \prod_{j=1}^{t}\{\Gamma(\Upsilon''_j+\beta''_1+\upsilon''_1)\}}{\prod_{j=3}^{w}\{\Gamma(1-\upsilon_j+\upsilon_1)\ \Gamma(1-\upsilon'_j+\upsilon'_1)\,\Gamma(1-\upsilon''_j+\upsilon''_1)\}}$$

$$\text{x}\ \frac{\prod_{j=1}^{n}\Gamma(1-\varepsilon_j+\upsilon_1+\upsilon'_1+\upsilon''_1)}{\prod_{j=1}^{l}\Gamma(\delta_j+\upsilon_1+\upsilon'_1+\upsilon''_1)}\ F\left[\begin{array}{c|lll|c} n & (1-\mathcal{E}_n)\ \upsilon_1+\upsilon'_1+\upsilon_1'' & & & \\ t & (\Upsilon_t+\upsilon_1)\ ; & (\Upsilon'_t+\upsilon'_1)\ ; & (\Upsilon''_t+\upsilon''_1) & -x \\ \varepsilon & (\delta_\varepsilon+\upsilon_1+\upsilon'_1+\upsilon''_1) & & & \\ q & (\beta_q+\upsilon_1)\ ; & (\beta'_q+\upsilon'_1)\ ; & (\beta''_q+\upsilon''_1) & -y \\ l & (\rho_l+\upsilon_1+\upsilon'_1+\upsilon''_1) & & & \\ \omega-1 & (1-\upsilon_\omega+\upsilon_1)^*\upsilon 1 & ;(1-\upsilon'_\omega+\upsilon'_1)^* & ;(1-\upsilon''_\omega+\upsilon''_1) & -z \end{array}\right](n+t \leq s+q+l+w)$$

(iv)

$$\lim_{\substack{Y\to 0\\ Z\to 0}} G^{\,n,o,o,o,m,1,1,1}_{\,p,o,o,q,o,w}\left[\begin{array}{c|l} & (\varepsilon p)\\ x & \ldots\ldots\ldots\ldots\\ & \ldots\ldots\ldots\ldots\\ y & (\beta_q),(o)\\ & \ldots\ldots\ldots\ldots\\ o & (\upsilon_\omega),(o),(o)\end{array}\right] \quad G^{\,m,n}_{\,p,q}\left(x\left|\begin{array}{l}(\varepsilon p)\\(\beta_q)\\(\upsilon_w)\end{array}\right.\right),\ \left(\begin{array}{l}p\le q\\ p\le w\end{array}\right)$$

(v)

$$G^{\,1,1,1,1,1,1,1}_{\,1,1,1,1,1,1}\begin{pmatrix}x\\y\\z\end{pmatrix} = x^{\upsilon_1}\, y^{\upsilon'_1} z^{\upsilon''_1}\ \frac{\Gamma(\square_1+\beta_1+\upsilon_1)\,\Gamma(\square'_1+\beta'_1+\upsilon'_1)\,\Gamma(\square''_1+\beta''_1+\upsilon''_1)}{\Gamma(\delta_1+\beta'_1+\beta'_1+\beta''_1)\,\Gamma(\rho_1+\upsilon_1+\upsilon'_1+\upsilon''_1)}$$

$$\mathrm{X}F_1\,[(1-\varepsilon_1+\beta_1+\beta'_1+\beta''_1);(1-\varepsilon_1+\upsilon_1+\upsilon'_1+\upsilon''_1);(\square_1+\beta_1+\upsilon_1),(\square'_1+\beta'_1+\upsilon'_1),(\square''_1+\beta''_1+\upsilon''_1);$$
$$(\delta_1+\beta_1+\beta'_1+\beta''_1);(\rho_1+\upsilon_1+\upsilon'_1+\upsilon''_1);\ -x,-y,-z]$$

(vi)

$$G^{\,1,1,1,1,1,1,1}_{\,1,1,0,2,0,2}\begin{pmatrix}x\\y\\z\end{pmatrix}$$

$$= x^{\upsilon_1}\, y^{\upsilon'_1} z^{\upsilon''_1}\ \frac{\Gamma(\square_1+\beta_1+\upsilon_1)\,\Gamma(\square'_1+\beta'_1+\upsilon'_1)\,\Gamma(\square''_1+\beta''_1+\upsilon''_1)\Gamma(1-\varepsilon_1+\beta_1+\beta'_1+\beta''_1)\Gamma(1-\varepsilon_1+\upsilon_1+\upsilon'_1+\upsilon''_1)}{[\Gamma(1-\beta_2+\beta_1)\,\Gamma(1-\beta'_2+\beta'_1)\,\Gamma(1-\beta''_2+\beta''_1)][\Gamma(1-\upsilon_2+\upsilon_1)\Gamma(1-\upsilon'_2+\upsilon'_1)\Gamma(1-\upsilon''_2+\upsilon''_1)]}$$

$$\mathrm{X}\,F_2\quad[(1+\beta_1+\beta''_1-\xi_1);(1+\upsilon_1+\upsilon''_1-\xi_1)(\square_1+\beta_1+\upsilon_1),(\square'_1+\beta'_1+\upsilon'_1),(\square''_1+\beta''_1+\upsilon''_1);\ (1-\beta_2+\beta_1)\,(1-\beta'_2+\beta'_1)\,(1-\beta''_2+\beta''_1);$$
$$(1-\upsilon_2+\upsilon_1),(1-\upsilon'_2+\upsilon'_1),(1-\upsilon''_2+\upsilon''_1);\ -x,-y,-z]$$

(vii)

$$G^{\,0,2,2,2,1,1,1}_{\,0,2,2,1,1,1}\begin{bmatrix}x\\y\\z\end{bmatrix} = x^{\upsilon_1}y^{\upsilon'_1}z^{\upsilon''_1}\ \frac{\Gamma(\square_1+\beta_1+\upsilon_1)\,\Gamma(\square_2+\beta_1+\upsilon_1)\,\Gamma(\square'_1+\beta'_1+\upsilon'_1)\,\Gamma(\square''_1+\beta''_1+\upsilon''_1)}{\Gamma(\delta_1+\beta_1+\beta'_1)\,\Gamma(\delta_1+\beta_1+\beta''_1)\,\Gamma(\rho_1+\upsilon_1+\upsilon'_1)\,\Gamma(\rho_1+\upsilon_1+\upsilon''_1)}$$

$$\mathrm{X}\ \ \Gamma(\square'_2+\beta'_1)\,\Gamma(\square''_2+\beta''_1)\,\Gamma(\square'_2+\upsilon'_1)\,\Gamma(\square''_2+\upsilon''_1)\ \ F_3\,[(\square_1+\beta_1+\upsilon_1),(\square'_1+\beta'_1+\upsilon'_1),(\square''_1+\beta''_1+\upsilon''_1),(\square_2+\beta_1)(\square'_2+\beta'_1)$$
$$(\square'_2+\upsilon'_1);(\delta_1+\beta_1+\beta'_1);\ (\delta_1+\beta_1+\beta''_1);\ (\rho_1+\upsilon_1+\upsilon'_1);(\rho_1+\upsilon_1+\upsilon''_1);\ -x,-y,-z\,]$$

(viii)

$$G^{\,1,0,0,0,1,1,1}_{\,1,0,0,0,2,2}\begin{bmatrix}x\\y\\z\end{bmatrix} = x^{\upsilon_1}y^{\upsilon'_1}z^{\upsilon''_1}$$

$$\frac{\Gamma(1-\varepsilon_1+\beta_1+\beta'_1+\beta''_1)\,\Gamma(1-\varepsilon_2+\beta_1+\beta'_1+\beta''_1)\,\Gamma(1-\varepsilon_1+\upsilon_1+\upsilon'_1+\upsilon''_1)\,\Gamma(1-\varepsilon_2+\upsilon_1+\upsilon'_1+\upsilon''_1)}{\Gamma(1-\beta_2+\beta_1)\,\Gamma(1-\beta'_2+\beta'_1)\,\Gamma(1-\beta''_2+\beta''_1)\,\Gamma(1-\upsilon_2+\upsilon_1)\,\Gamma(1-\upsilon'_2+\upsilon'_1)\,\Gamma(1-\upsilon''_2+\upsilon''_1)}$$

$$\mathrm{X}F_4\Big[\begin{array}{l}\Gamma(1-\varepsilon_1+\beta_1+\beta'_1+\beta''_1),(1-\varepsilon_2+\beta'_1+\beta''_1);\ (1+\beta_1-\beta_2),(1+\beta'_1-\beta''_2);\ (1+\beta''_1-\beta''_2):\\ (1-\varepsilon_1+\upsilon_1+\upsilon'_1+\upsilon''_1)\qquad :\\ (1-\varepsilon_2+\upsilon_1+\upsilon'_1+\upsilon''_1);\ (1+\upsilon_1-\upsilon_2),(1+\upsilon'_1-\upsilon'_2),(1+\upsilon''_1-\upsilon''_2);\ -x,-y,-z\end{array}\Big]$$

(ix)

$$G^{\,n,0,0,0,1,1,1}_{\,p,q,q,1,1,1}\begin{bmatrix}x\\y\\z\end{bmatrix} = x^{\upsilon_1}y^{\upsilon'_1}z^{\upsilon''_1}$$

$$\frac{\prod_{j=1}^{n}\Gamma(1-\varepsilon_j+\beta_1+\beta'_1+\beta''_1)\ \prod_{j=1}^{n}\Gamma(1-\varepsilon_j+\upsilon_1+\upsilon'_1+\upsilon''_1)}{\prod_{j=1}^{s}\Gamma(\delta_j+\beta_1+\beta'_1+\beta''_1)\ \prod_{j=n+1}^{p}\Gamma(\varepsilon_j-\beta_1-\beta'_1-\beta''_1)\ \prod_{j=1}^{l}\Gamma(\rho_j+\upsilon_1+\upsilon'_1+\upsilon''_1)\ \prod_{j=n+1}^{p}\Gamma(\varepsilon_j-\upsilon_1-\upsilon'_1-\upsilon''_1)}$$

$$\mathrm{X}\ {}_pF_s\left[\begin{array}{c}(1-\varepsilon_p+\beta_1+\beta'_1+\beta''_1);(-)^{n-p}\,(x+y)\\(1-\delta_s+\beta_1+\beta'_1+\beta''_1)\end{array}\right]\mathrm{X}\ {}_pF_l\left[\begin{array}{c}(1-\varepsilon_p+\upsilon_1+\upsilon'_1+\upsilon''_1);(-)^{n-p}\,(x+z)\\(1-\delta_l+\upsilon_1+\upsilon'_1+\upsilon''_1)\end{array}\right],\ \begin{array}{l}p\le s+1\\ p\le l+1\end{array}$$

(x)

$$G^{0,1,v_2,v_3,l,m_2,m_3}_{0,t,s,q,l,w}\left[\begin{matrix}x\\y\\z\end{matrix}\right]\sum_{n=1}^{t}\sum_{k=1}^{m2}\sum_{d=1}x^{\upsilon_n}y^{\upsilon'_k}z^{\upsilon''_d}\left[\Upsilon_1+\beta_1+\upsilon_n\right]_X$$

$$\frac{\prod_{j=1}^{v_2}\Gamma(\Upsilon'_j+\beta'_j+\upsilon'_k)\prod_{j=1}^{v_3}\Gamma(\Upsilon''_j+\beta''_j+\upsilon''_d)\prod_{j=1}^{l}\Gamma(\upsilon_j-\upsilon_n)\prod_{j=1}^{m_2}\Gamma(\upsilon'_j-\upsilon'_k)\prod_{j=1}^{m_3}\Gamma(\upsilon''_j-\upsilon''_k)}{\prod_{j=1}^{t}\Gamma(\rho_j+\upsilon_n+\upsilon'_k+\upsilon''_d)\prod_{j=1}^{s}\Gamma(1-\rho_j-\upsilon_n)\prod_{j=1+v_2}^{s}\Gamma(1-\rho'_j-\upsilon'_k)\prod_{j=1+v_3}^{s}\Gamma(1-\rho''_j-\upsilon''_d)\prod_{j=1+m_2}^{w}\Gamma(1+\upsilon'_k-\upsilon'_j)\prod_{j=1+m_3}^{w}\Gamma(1+\upsilon''_k-\upsilon''_j)}$$

	………………….				
X	$(\Upsilon_t+\upsilon_n)$;	$(\Upsilon'_t+\upsilon'_k)$;	$(\Upsilon''_t+\upsilon''_d)$	$(-)^{w+q+t+1}{}_x$	
	$(\delta_{\mathcal{Z}}+\upsilon_n+\upsilon'_k+\upsilon''_d)$				
	$(\beta_q+\upsilon_n)$;	$(\beta_q+\upsilon'_k)$;	$(\beta_q+\upsilon''_d)$	$(-)^{m2+t-\upsilon 2}{}_y$	, $(t \leq s+q,\ t \leq l+w)$
	$(\rho_l+\upsilon_n+\upsilon'_k+\upsilon''_d)$				
	$(1-\upsilon_\omega+\upsilon_n)$	;$(1-\upsilon'_\omega+\upsilon'_k)$	;$(1-\upsilon''_\omega+\upsilon''_d)$	$(-)^{m3+t+-\upsilon 3}{}_z$	

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