International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2016; 1(3): 36-42

www.mathsjournal.com
Received: 09-07-2016
Accepted: 10-08-2016

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A single server compulsory vacation queue with three type of services and with restricted admissibility

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Abstract
A single server queue with compulsory server vacation has been considered. In addition the admission to queue is based on a Bernoulli process and the server gives two type of services and an optional service. For this model the probability generating function for the number of customers in the queue at different server's state are obtained using supplementary variable technique. Some performance measures are calculated.

Keywords: Compulsory vacation - bernoulli process - supplementary variable technique performance measures

AMS Subject classification number: 90B22, 60K25 and 60K30.

1. Introduction

In some queueing situations, it can be seen that, after serving a certain number of customers, the server becomes unavailable for a random period of time. Such queueing systems are called vacation queueing systems and the random period is called vacation period. Various modifications have been made on the vacation period by researchers and different systems have been defined. One such system is queueing system with compulsory server vacation. The eariler works on vacation systems are by Miller [19], Cooper [3-4], Levy and Yechiali [13], Keilson and Servi [10], Levy *et al*. [12] and Doshi [5]. Madan [14] analysed single server queue with compulsory server vacation. Neuts [20] considered an *M/G/*1 queue with restriction on the number of customers to be admitted during a service period or with restriction on the time period at which the customers are admitted. Madan and Dayyeh [17] investigated a bulk queue with restricted admissibility of batches and with Bernoulli scheduled server vacation. Anabosi and Madan[2] have analyzed a single server queue with two types of service under Bernoulli schedule server vacation. The server provides two types of heterogeneous exponential service with single vacation policy and a customer may choose either type of service.

In day to day life, one encounters numerous queueing situations in which all the arriving customers are given the essential service and only some of them may require additional optional service. Such a model was studied by Madan [15]. The other works to be noted here are Madan [16], Medhi [18], Al-Jararah and Madan[1], Jinting Wang[11], Kalyanaraman *et al*. [9] and Jau-Chuan [6]. Kalyanaraman and Suvitha [7] have analysed a single server Bernoulli vacation queue with two type of services and with restricted admissibility in steady state. The same authors [8] have considered an M/G/1 queue with compulsory server vacation and with two type services, restricted admissibility. They obtained the probability generating function of number of customers the queue and some performance measures in time independent domain.

In this article, a single server infinite capacity Poisson arrival queue with three types of services, with compulsory server vacation and with restriction on arrivals has been studied. The corresponding mathematical model is defined in section 2 and the governing differential difference equations, the boundary conditions and the normalizing conditions are given in section 3. For this model the probability generating function of the number of customers in the queue when the server is on vacation, the probability generating function of the number of customers in the queue when the server provides i^{th} type of service $(i = 1,2,3)$ and the probability generating function of the number of customers in the queue irrespective of the server states are derived in section 4. Also, some performance measures related to this queueing model are obtained from these probability generating functions and are given in section 5. In sections 6 and 7, particular model is derived by assigning particular values to the parameters and a numerical study is carried out respectively.

2. Model Description

Consider a single server queue with the customers arrival follows, according to a Poisson process of intensity λ and the server provides two types of services, respectively called type 1 service and type 2 service. At the beginning of a service, it is assumed that the customer has the choice of selecting type 1 service with probability p_1 or type 2 service with probability $p_2 (p_1 + p_2 = 1).$ The type 1 service is a phase type service (two phases). After completion of type 1 service, the customer leaves the system, whereas after completion of type 2 service, the customer leaves the system with probability $1 - p_3$ or choose an optional service with probability p_3. After completion of optional service, the customer leaves the system. Here the optional service is called type 3 service. The service discipline is assumed to be first come, first served (FCFS). The service time distributions are general, the distribution functions are $B_{1,j}(x)$ for type 1 and $j^{th} (j = 1,2)$ phase of service, $B_2(x)$ for type 2 service, $B_3(x)$ for type 3 service. The Laplace- Stieltjes transform (LST) for $B_{1,j}(x), B_2(x), B_3(x)$ are $B_{1,j}^*(\theta), B_2^*(\theta), B_3^*(\theta)$ and the respective finite moments are $E(B_{1,j}^k), E(B_2^k), E(B_3^k), k \geq 1.$

After completion of each service the server takes vacation for a random period of time, called vacation period. This vacation period V is independently and identically distributed with distribution function $V(x)$, Laplace- Stieltjes transform (LST) $V^*(\theta)$ and finite moments $E(V^k), k \geq 1.$ Further, it is assumed that not all the arriving customers are allowed to join the system at all time. Let r $(0 < r < 1)$ be the probability that an arriving customer will be allowed to join the system while the server is busy or idle and let p $(0 < p < 1)$ be the probability that an arriving customer will be allowed to join the system while the server is on vacation.

It may be noted that $B_{1,j}(x), B_2(x), B_3(x)$ and $V(x)$ $(B_{1,j}(\infty) = 1, B_{1,j}(0) = 0, B_2(\infty) = 1, B_2(0) = 0, B_3(\infty) = 1, B_3(0) = 0, V(\infty) = 1,$ $V(0) = 0)$ are continuous, so that $\mu_{1,j}(x)dx = \dfrac{b_{1,j}(x)}{1 - B_{1,j}(x)}, \mu_2(x)dx = \dfrac{b_2(x)}{1 - B_2(x)}, \mu_3(x)dx = \dfrac{b_3(x)}{1 - B_3(x)}, \gamma(x)dx = \dfrac{v(x)}{1 - V(x)}$ and are the first order differential functions (hazard rates) of $B_{1,j}(j = 1,2), B_2, B_3$ and V respectively. For the analysis the supplementary variable (the variable is elapsed time) technique has been used.

Let $N(t)$ be the queue size at time t and $X(t)$ is the state of the server at time $t.$ Then $\{(E(t), N(t)) : t \geq 0\}$ be a bivariate Markov process, where $E(t)$ be the elapsed (service or vacation) time at time $t.$ For the Mathematical definition of the models we introduce the following notations:

$P_n^{(i)}(x,t) =$ Pr{ at time $t,$ there are n customers in the queue excluding one receiving the type 1 service and is in the j^{th} phase of service and the elapsed service time is $x\}; j = 1,2; n \geq 0,$

$P_n^{(2)}(x,t) =$ Pr{ at time $t,$ there are n customers in the queue excluding one receiving the type 2 service and the elapsed service time is $x\}; n \geq 0,$

$P_n^{(3)}(x,t) =$ Pr{ at time $t,$ there are n customers in the queue excluding one receiving the type 3 service and the elapsed service time is $x\}; n \geq 0,$

$V_n(x,t) =$ Pr{ at time $t,$ the server is on vacation with elapsed vacation time is x and the number of customers in the queue is $n\}; n \geq 0$ and

$Q(t) =$ Pr{at time $t,$ there are no customers in the system and the server is idle}.

Let $P_n^{(1,j)}(x), P_n^{(2)}(x), P_n^{(3)}(x), V_n(x)$ and Q denote the corresponding steady state probabilities.

The probability generating functions for the probabilities $\{P_n^{(1,j)}(x)\}, \{P_n^{(2)}(x)\}, \{P_n^{(3)}(x)\}, \{V_n(x)\}$ are respectively defined as $P^{(1,j)}(x,z) = \sum_{n=0}^{\infty} z^n P_n^{(1,j)}(x); j = 1,2, P^{(2)}(x,z) = \sum_{n=0}^{\infty} z^n P_n^{(2)}(x), P^{(3)}(x,z) = \sum_{n=0}^{\infty} z^n P_n^{(3)}(x)$ and $V(x,z) = \sum_{n=0}^{\infty} z^n V_n(x).$

Further, it may be noted that $\mu_{1,j}(x)dx$ be the conditional probability of completion of the j^{th} $(j = 1,2)$ phase of type 1 service during the interval $(x, x + dx]$ given that the elapsed service time is $x,$ $\mu_2(x)dx$ be the conditional probability of completion of the type 2 of service during the interval $(x, x + dx]$ given that the elapsed service time is $x,$ $\mu_3(x)dx$ be the conditional probability of completion of the type 3 service during the interval $(x, x + dx]$ given that the elapsed service time is x and $\gamma(x)dx$ be the conditional probability of completion of the vacation during the interval $(x, x + dx]$ given that the elapsed vacation time is $x.$

3. The Governing Equations

We utilize the argument of Cox [1] and obtain the following Kolmogorov forward equations related to the above defined model under the steady state conditions:

$$\frac{d}{dx}P_0^{(1,1)}(x)+(\lambda+\mu_{1,1}(x))P_0^{(1,1)}(x)=\lambda(1-r)P_0^{(1,1)}(x) \tag{1}$$

$$\frac{d}{dx}P_n^{(1,1)}(x)+(\lambda+\mu_{1,1}(x))P_n^{(1,1)}(x)=\lambda(1-r)P_n^{(1,1)}(x)+\lambda rP_{n-1}^{(1,1)}(x);n\ge 1 \tag{2}$$

$$\frac{d}{dx}P_0^{(1,2)}(x)+(\lambda+\mu_{1,2}(x))P_0^{(1,2)}(x)=\lambda(1-r)P_0^{(1,2)}(x) \tag{3}$$

$$\frac{d}{dx}P_n^{(1,2)}(x)+(\lambda+\mu_{1,2}(x))P_n^{(1,2)}(x)=\lambda(1-r)P_n^{(1,2)}(x)+\lambda rP_{n-1}^{(1,2)}(x);n\ge 1 \tag{4}$$

$$\frac{d}{dx}P_0^{(2)}(x)+(\lambda+\mu_2(x))P_0^{(2)}(x)=\lambda(1-r)P_0^{(2)}(x) \tag{5}$$

$$\frac{d}{dx}P_n^{(2)}(x)+(\lambda+\mu_2(x))P_n^{(2)}(x)=\lambda(1-r)P_n^{(2)}(x)+\lambda rP_{n-1}^{(2)}(x);n\ge 1 \tag{6}$$

$$\frac{d}{dx}P_0^{(3)}(x)+(\lambda+\mu_3(x))P_0^{(3)}(x)=\lambda(1-r)P_0^{(3)}(x) \tag{7}$$

$$\frac{d}{dx}P_n^{(3)}(x)+(\lambda+\mu_3(x))P_n^{(3)}(x)=\lambda(1-r)P_n^{(3)}(x)+\lambda rP_{n-1}^{(3)}(x);n\ge 1 \tag{8}$$

$$\frac{d}{dx}V_0(x)+(\lambda+\gamma(x))V_0(x)=\lambda(1-p)V_0(x) \tag{9}$$

$$\frac{d}{dx}V_n(x)+(\lambda+\gamma(x))V_n(x)=\lambda(1-p)V_n(x)+\lambda pV_{n-1}(x);n\ge 1 \tag{10}$$

$$\lambda rQ=\int_0^\infty V_0(x)\gamma(x)dx \tag{11}$$

The above set of equations is to be solved under the following boundary conditions at $x=0$:

$$P_0^{(1,1)}(0)=\lambda rp_1Q+p_1\int_0^\infty V_1(x)\gamma(x)dx \tag{12}$$

$$P_n^{(1,1)}(0)=p_1\int_0^\infty V_{n+1}(x)\gamma(x)dx;n\ge 1 \tag{13}$$

$$P_n^{(1,2)}(0)=\int_0^\infty P_n^{(1,1)}(x)\mu_{1,1}(x)dx;n\ge 0 \tag{14}$$

$$P_0^{(2)}(0)=\lambda rp_2Q+p_2\int_0^\infty V_2(x)\gamma(x)dx \tag{15}$$

$$P_n^{(2)}(0)=p_2\int_0^\infty V_{n+1}(x)\gamma(x)dx;n\ge 1 \tag{16}$$

$$P_n^{(3)}(0)=p_3\int_0^\infty P_n^{(2)}(x)\mu_2(x)dx;n\ge 0 \tag{17}$$

$$V_n(0)=\int_0^\infty P_n^{(1,2)}(x)\mu_{1,2}(x)dx+(1-p_3)\int_0^\infty P_n^{(2)}(x)\mu_2(x)dx+\int_0^\infty P_n^{(3)}(x)\mu_3(x)dx;n\ge 0 \tag{18}$$

and the normalization condition is

$$Q+\sum_{n=0}^\infty\int_0^\infty\left[P_n^{(1,1)}(x)+P_n^{(1,2)}(x)+P_n^{(2)}(x)+P_n^{(3)}(x)+V_n(x)\right]dx=1 \tag{19}$$

4. The Analysis

Multiplying equation (2) by z^n, summing from $n=1$ to ∞ and then adding (1), we get

$$\frac{\frac{d}{dx}P^{(1,1)}(x,z)}{P^{(1,1)}(x,z)}=-T-\mu_{1,1}(x) \tag{20}$$

where $T=\lambda r(1-z)$.

Integration of the equation (20) leads to

$$P^{(1,1)}(x,z)=C(1-B_{1,1}(x))e^{-Tx} \tag{21}$$

Taking $x=0$ in equation (21), the constant C is obtained as

$$C=P^{(1,1)}(0,z) \tag{22}$$

Using equation (22) in equation (21), we get

$$P^{(1,1)}(x,z) = P^{(1,1)}(0,z)(1-B_{1,1}(x))e^{-Tx} \quad (23)$$

Proceeding the same manner with the equations (3)-(10), we obtain

$$P^{(1,2)}(x,z) = P^{(1,2)}(0,z)(1-B_{1,2}(x))e^{-Tx} \quad (24)$$

$$P^{(2)}(x,z) = P^{(2)}(0,z)(1-B_{2}(x))e^{-Tx} \quad (25)$$

$$P^{(3)}(x,z) = P^{(3)}(0,z)(1-B_{3}(x))e^{-Tx} \quad (26)$$

$$V(x,z) = V(0,z)(1-V(x))e^{-Rx} \quad (27)$$

where $R = \lambda p(1-z)$.

Next, we multiply equations (12)-(18) by appropriate powers of z and then taking the summation over all possible values of n and useing (11), (23)-(27) respectively. We get

$$zP^{(1,1)}(0,z) = \lambda r p_1 Q + p_1 V^*(R)V(0,z) \quad (28)$$

$$P^{(1,2)}(0,z) = B_{1,1}^*(T)P^{(1,1)}(0,z) \quad (29)$$

$$zP^{(2)}(0,z) = \lambda r p_2 Q + p_2 V^*(R)V(0,z) \quad (30)$$

$$P^{(3)}(0,z) = p_3 B_2^*(T)P^{(2)}(0,z) \quad (31)$$

$$V(0,z) = B_{1,2}^*(T)P^{(1,2)}(0,z) + (1-p_3)B_2^*(T)P^{(2)}(0,z) + B_3^*(T)P^{(3)}(0,z) \quad (32)$$

Using equations (28)-(31) in (32), we get

$$V(0,z) = \frac{\lambda r(z-1)C_1 Q}{z - C_1 V^*(R)} \quad (33)$$

where $C_1 = p_1 B_{1,1}^*(T)B_{1,2}^*(T) + p_2 B_2^*(T)(1 - p_3 + p_3 B_3^*(T))$.

Using equation (33) in (28) and (30), we get

$$P^{(1,1)}(0,z) = \frac{\lambda r p_1(z-1)Q}{z - C_1 V^*(R)} \quad (34)$$

$$P^{(2)}(0,z) = \frac{\lambda r p_2(z-1)Q}{z - C_1 V^*(R)} \quad (35)$$

Using equations (34), (35) in (29) and (31), we get

$$P^{(1,2)}(0,z) = \frac{\lambda r p_1(z-1)B_{1,1}^*(T)Q}{z - C_1 V^*(R)} \quad (36)$$

$$P^{(3)}(0,z) = \frac{\lambda r p_2 p_3(z-1)B_2^*(T)Q}{z - C_1 V^*(R)} \quad (37)$$

Integration of equations (23)-(27) with respect to x and using (33)-(37), we get

$$P^{(1,1)}(z) = \frac{p_1(B_{1,1}^*(T)-1)Q}{z - C_1 V^*(R)} \quad (38)$$

$$P^{(1,2)}(z) = \frac{p_1 B_{1,1}^*(T)(B_{1,2}^*(T)-1)Q}{z - C_1 V^*(R)} \quad (39)$$

$$P^{(2)}(z) = \frac{p_2(B_2^*(T)-1)Q}{z - C_1 V^*(R)} \quad (40)$$

$$P^{(3)}(z) = \frac{p_2 p_3 B_2^*(T)(B_3^*(T)-1)Q}{z - C_1 V^*(R)} \quad (41)$$

$$V(z) = \frac{rC_1(V^*(R)-1)Q}{p[z - C_1 V^*(R)]} \quad (42)$$

Putting $z = 1$ in equations (38)-(42) and using equation (19) leads to

$$Q = \frac{(1-\rho)}{1 - \lambda E(V)(p-r)} \quad (43)$$

Equation (43) is called the probability that the server is idle and no one in the system.

Equations (38)-(42) together with equation (43) are respectively, the probability generating functions of the number of customers in the queue when the server is serving type 1 service and is in the j^{th} $(j = 1,2)$ phase of service, serving type 2 service and type 3 service respectively, the server is on vacation.

Here $Q > 0$ guarantees the existence of the probability generating functions in equations (38)-(42) and therefore the stability condition for the system is $\rho < 1$ where $\rho = \lambda r[p_1(E(B_{1,1}) + E(B_{1,2})) + p_2 E(B_2) + p_2 p_3 E(B_3)] + \lambda p E(V)$.

The probability generating function that the number of customers in the queue irrespective of the server state is

$$U(z)=Q+P^{(1,1)}(z)+P^{(1,2)}(z)+P^{(2)}(z)+P^{(3)}(z)+V(z)$$
$$=\frac{(1-\rho)[p(z-1)+C_1(p-r)(1-V^*(R))]}{p[1-\lambda(p-r)E(V)][z-C_1V^*(R)]} \qquad (44)$$

5. The Performance Measures

Using straightforward calculations the following performance measures have been obtained:

(i) The mean number of customers in the queue is

$$L_q=\lim_{z\to 1}\frac{dU(z)}{dz}$$
$$=\frac{\lambda^2 r}{2(1-\rho)[1-\lambda(p-r)E(V)]}\{r[1-\lambda(p-r)E(V)][p_1(E(B_{1,1}^2)+2E(B_{1,1})E(B_{1,2})+E(B_{1,2}^2))+p_2(E(B_2^2)+2p_3E(B_2)E(B_3)$$
$$+p_3E(B_2^2))]+[1+\lambda(p-r)(p_1(E(B_{1,1})+E(B_{1,2}))+p_2E(B_2)+p_2p_3E(B_3))][pE(V^2)+2rE(V)(p_1(E(B_{1,1})+E(B_{1,2}))$$
$$+p_2E(B_2)+p_2p_3E(B_3))]\}$$

where $U(z)$ is given in equation (44).

(ii) The mean waiting time in the queue is

$$W_q=\frac{L_q}{\lambda'}$$
$$=\frac{\lambda}{2(1-\rho)}\{r[1-\lambda(p-r)E(V)][p_1(E(B_{1,1}^2)+2E(B_{1,1})E(B_{1,2})+E(B_{1,2}^2))]+p_2(E(B_2^2)+2p_3E(B_2)E(B_3)+p_3E(B_2^2))]$$
$$+[1+\lambda(p-r)(p_1(E(B_{1,1})+E(B_{1,2}))+p_2E(B_2)+p_2p_3E(B_3))][pE(V^2)+2rE(V)(p_1(E(B_{1,1})+E(B_{1,2}))+p_2E(B_2)$$
$$+p_2p_3E(B_3))]\}$$

where

λ' = actual arrival rate

$$=\lambda r[Q+P^{(1,1)}(1)+P^{(1,2)}(1)+P^{(2)}(1)+P^{(3)}(1)]+\lambda pV(1)$$
$$=\frac{\lambda r}{1-\lambda(p-r)E(V)}$$

Special Case

If $p_1=p_3=0; p_2=p=r=1$ and $E(B_2)=E(B)$ then $Q=1-\lambda(E(B)+E(V))$ and $L_q=\frac{\lambda^2[E(B^2)+E(V^2)+2E(B)E(V)]}{2[1-\lambda(E(B)+E(V))]}$.

The results coincides with the result of Madan[14].

6. Particular Model

In this section, a particular model has been calculated by taking known distribution to service time and vacation time. The service times distribution are negative exponential with parameters $\mu_{1,1}$ for phase 1 service, $\mu_{1,2}$ for phase 2 service, μ_2 for type 2 service, μ_3 for type 3 service and vacation time distribution is hyper exponential with parameters θ_1, θ_2, q_1, q_2; (q_1+q_2). The following performance measures have been calculated.

$$Q=\frac{C_2}{\mu_{1,1}\mu_{1,2}\mu_2\mu_3[\theta_1\theta_2-\lambda(p-r)(q_1\theta_2+q_2\theta_1)]}$$
$$L_q=\frac{\lambda^2 rC_3}{\mu_{1,1}\mu_{1,2}\mu_2\mu_3C_2[\theta_1\theta_2-\lambda(p-r)(q_1\theta_2+q_2\theta_1)]}$$
$$W_q=\frac{\lambda C_3}{\mu_{1,1}\mu_{1,2}\mu_2\mu_3C_2}$$

where

$$C_2=\mu_{1,1}\mu_{1,2}\mu_2\mu_3[\theta_1\theta_2-\lambda p(q_1\theta_2+q_2\theta_1)]-\lambda r[p_1\mu_2\mu_3(\mu_{1,1}+\mu_{1,2})+p_2\mu_{1,1}\mu_{1,2}(\mu_3+p_3\mu_2)]$$
$$C_3=[\mu_{1,1}\mu_{1,2}\mu_2\mu_3+\lambda(p-r)(p_1\mu_2\mu_3(\mu_{1,1}+\mu_{1,2})+p_2\mu_{1,1}\mu_{1,2}(\mu_3+p_3\mu_2))][r\theta_1\theta_2(q_1\theta_2+q_2\theta_1)(p_1\mu_2\mu_3(\mu_{1,1}+\mu_{1,2})$$
$$+p_2\mu_{1,1}\mu_{1,2}(\mu_3+p_3\mu_2))+\mu_{1,1}\mu_{1,2}\mu_2\mu_3(q_1\theta_2^2+q_2\theta_1^2)]+r\theta_1\theta_2[\theta_1\theta_2-\lambda(p-r)(q_1\theta_2+q_2\theta_1)][p_1\mu_2^2\mu_3^2(\mu_{1,1}^2+\mu_{1,2}^2$$
$$+\mu_{1,1}\mu_{1,2})+p_2(\mu_3^2+p_3\mu_2^2+p_3\mu_2\mu_3)]$$

7. The Numerical Study

In this section, numerical results are pressented related to the model discussed in section 6. We fix some parameter values $(\mu_{1,1}=5.0,\ \mu_{1,2}=8.0, \mu_2=6.0, \mu_3=7.0, q_1=0.6, q_2=0.4, p_1=0.2, p_2=0.8, p_3=0.5, p=0.6 r=0.9)$ and vary other parameters $(0.0\le\lambda\le 1.0$, by fixing $\theta_1=2.0$ $(\theta_2=2.0)$, θ_2 has been varied from 1.1, 1.5, 1.9, (θ_1 has been varied from 1.1, 1.5, 1.9)). Figures 1, 2 represents the mean number of customers in the queue whereas 3, 4 indicates that the mean waiting time in the queue with respect to arrival rate. All the functions are increasing function. Table 1 represents the probability that the server is idle. From the table it is clear that the arrival rate increases, the corresponding probability is also increases.

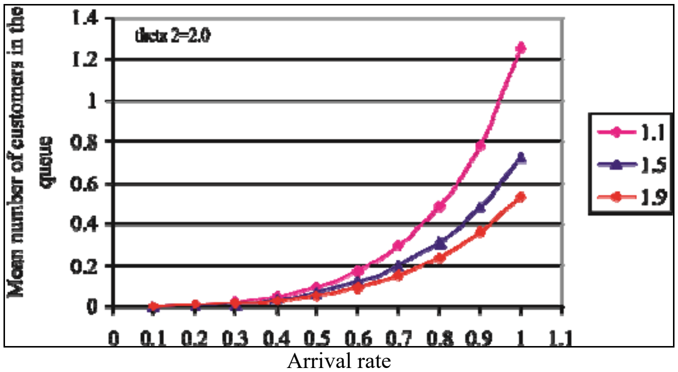

Fig 1: Arrival rate versus mean number of customers in the queue

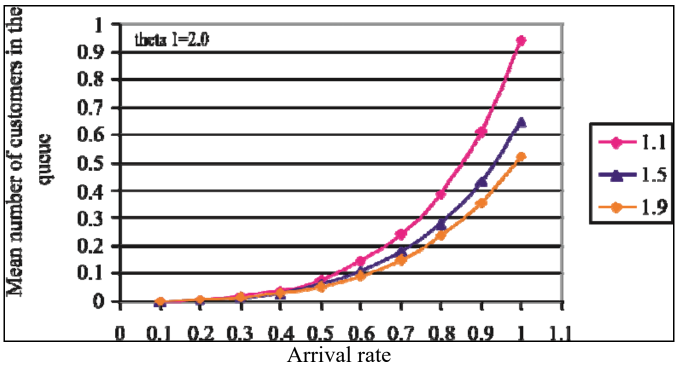

Fig 2: Arrival rate versus mean number of customers in the queue

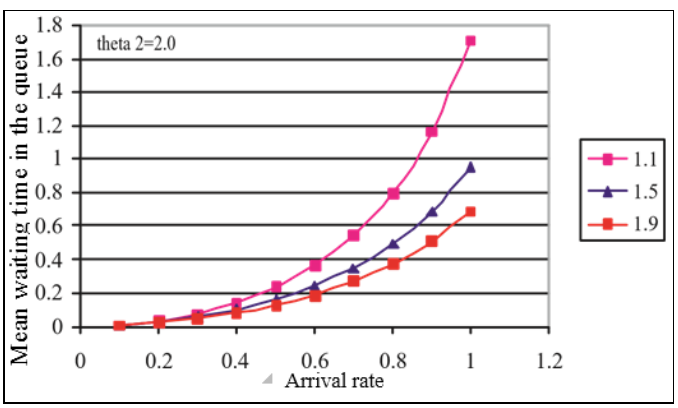

Fig 3: Arrival rate versus mean waiting time in the queue

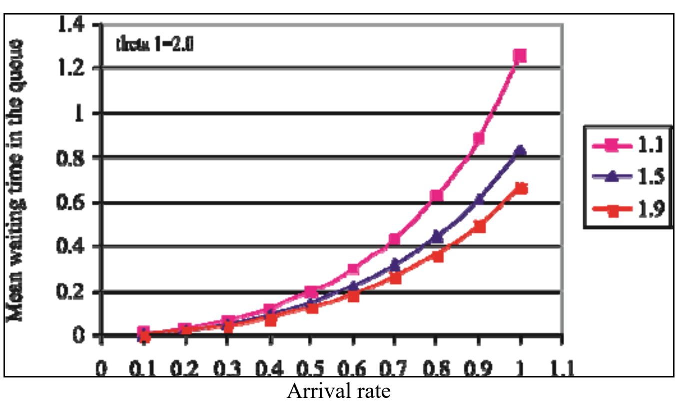

Fig 4: Arrival rate versus mean waiting time in the queue

Table 1: The idle probability

λ	$\theta_1 = 2.0$ $\theta_2 = 1.1$	$\theta_1 = 1.1$ $\theta_2 = 2.0$	$\theta_1 = 2.0$ $\theta_2 = 1.5$	$\theta_1 = 1.5$ $\theta_2 = 2.0$	$\theta_1 = 2.0$ $\theta_2 = 1.9$	$\theta_1 = 1.9$ $\theta_2 = 2.0$
0.1	0.9189	0.9119	0.9272	0.9244	0.9321	0.9316
0.2	0.8409	0.8275	0.8569	0.8514	0.8662	0.8653
0.3	0.7658	0.7467	0.7888	0.7809	0.8023	0.8010
0.4	0.6935	0.6693	0.7229	0.7127	0.7402	0.7385
0.5	0.6238	0.5949	0.6590	0.6468	0.6798	0.6779
0.6	0.5566	0.5234	0.5971	0.5831	0.6212	0.6189
0.7	0.4918	0.4548	0.5371	0.5214	0.5641	0.5616
0.8	0.4292	0.3887	0.4789	0.4616	0.5087	0.5059
0.9	0.3686	0.3251	0.4224	0.4037	0.4547	0.4516
1.0	0.3101	0.2638	0.3676	0.3475	0.4022	0.3989

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