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On prime labeling of cubic graph with 12 vertices

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Abstract
In this paper, we show that the Cubic graph on 12 vertices admits prime labeling, we also proved that the graphs obtained by merging (or) fusion of two vertices, Duplication of an arbitrary vertex and Switching of an arbitrary vertex in the Cubic graph with 12 vertices are prime graphs.

Keywords: Prime graphs, Fusion, Duplication, Switching & Path union

1. Introduction

We begin with Simple, Finite, Connected and undirected graph $G = (V, E)$ with p vertices and q edges. For all other standard terminology and notations we refer to J.A. Bondy and U.S.R. Murthy [1]. We give a brief summary of definitions and other information which are useful for the present investication. A current survey of various graphs labeling problem can be found in [7] (Gallian J, 2009)

Following are the common features of any graph Labeling problem.

- A set of numbers from which vertex labels are assigned.
- A rule that assigns value to each edge.
- A condition that these values must satisfy.

The notion of Prime labeling was introduced by Roger Entringer and was discussed in a paper by A. Tout (1982 P 365-368) [2]. Many researches have studied prime graph for example in H.C. Fu (1994 P 181-186) [5] have proved that path P_n on n vertices is a prime graph.

T.O. Dertsky (1991 P 359-369) [4] have proved that the cycle C_n on n vertices is a prime graph. S.M. Lee (1998 P 59 -67) [3] have proved that wheel W_n is a prime graph iff n is even. Around 1980 Roger Entringer conjectured that all tress have prime labeling, which is not settled till today. The prime labeling for planner grid is investigated by M. Sundaram (2006 P205-209) [6]. In [8] S.K. Vaidhya and K.K. Kanmani) have proved that the prime labeling for some cycle related graphs. In [9] S. Meena and K. Vaithilingam, Prime Labeling for some Helm related graphs.

Definition 1: If the vertices of the graph are assigned values subject to certain conditions then it is known as (vertex) graph labeling.

Definition 2: Let $G = \big(V(G), E(G)\big)$ be a graph with p vertices. A bijection $f: V(G) \to \{1, 2, \dots p\}$ is called a prime labeling if for each edge $e = uv$, $gcd\big(f(u), f(v)\big) = 1$. A graph which admits prime labeling is called a prime graph.

Definition 3: An independent set of vertices in a graph G is a set of mutually non-adjacent vertices.

Definition 4: Let u and v be two distinct vertices of a graph G. A new graph G_1 is constructed by fusing (identifying) two vertices u and v by a single vertex x in G_1 such that every edge which was incident with either u (or) v in G now incident with x in G_1.

Definition 5: Duplication of a vertex v_k of a graph G produces a new graph G_1 by adding a vertex v'_k with $N(v'_k) = N(v_k)$. In other words, a vertex v'_k is said to be a duplication of v_k if all the vertices which are adjacent to v_k are now adjacent to v'_k.

Definition 6: A vertex switching G_v of a graph G is obtained by taking a vertex v of G, removing the entire edges incident with v and adding edges joining v to every vertex which are not adjacent to v in G.

Definition 7: A simple graph G is called a regular graph if each vertex of G has an equal degree.
Definition 8: A regular graph G is called a Cubic graph if all the vertices of G are of degree 3.
(i.e) A 3-regular graph is called a Cubic graph.

Proposition 1: A Cubic graph with 12 vertices is a prime graph.
Proof:
Let $G = (V, E)$ be a Cubic graph with 12 vertices and 18 edges
Let $V(G) = \{u_1, u_2, u_3, u_4, u_5, u_6, v_1, v_2, v_3, v_4, v_5, v_6\}$
$E(G) = \{u_i u_{i+1} / 1 \leq i \leq 5,\ u_1 u_6\} \cup \{v_i v_{i+1} / 1 \leq i \leq 5, v_1 v_6\} \cup \{u_i v_i / 1 \leq i \leq 6\}$
Define a labeling $f: V(G) \to \{1, 2, 3, \ldots..12\}$
such that $f(u_i) = i$ for $1 \leq i \leq 6$
(i.e.) $f(u_1) = 1$
$f(u_2) = 2$
$f(u_3) = 3$
$f(u_4) = 4$
$f(u_5) = 5$
$f(u_6) = 6$
and let $f(v_i) = 12 - (i - 1)$ for $1 \leq i \leq 6$
(i.e.) $f(v_1) = 12$
$f(v_2) = 11$
$f(v_3) = 10$
$f(v_4) = 9$
$f(v_5) = 8$
$f(v_6) = 7$
Clearly, for each edge $e = u_i u_j \in G$ for the edges $v_i v_j \in G$ and $u_i v_i \in G$,
$\gcd\big(f(u_i), f(u_j)\big) = 1$, $\gcd\big(f(v_i), f(v_j)\big) = 1$ and $\gcd(f(u_i), f(v_i)) = 1$
Therefore, G admits a prime labeling
Hence G is a prime graph.

1.1 Illustration

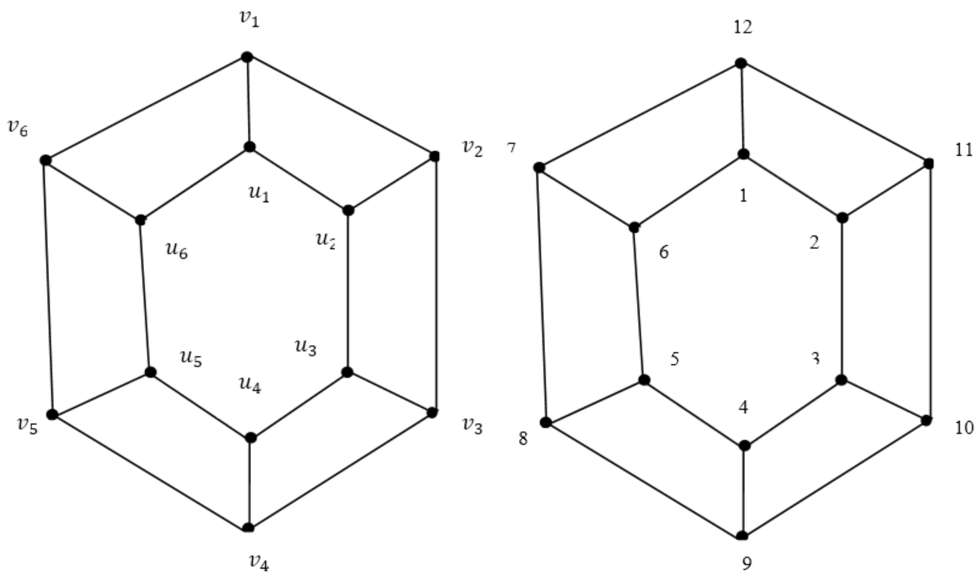

Fig 1: A Cubic graph with 12 vertices is a Prime graph.

Proposition 2: The Fusion(or merging) of two vertices in a Cubic graph with 12 vertices is a prime graph.
Proof:
Let $G = (V, E)$ be a cubic graph with 12 vertices and 18 edges
Let $V(G) = \{u_1, u_2, u_3, u_4, u_5, u_6;\ v_1, v_2, v_3, v_4, v_5, v_6\}$
and $E(G) = \{u_i u_{i+1} / 1 \leq i \leq 5, u_1 u_6\} \cup \{v_i v_{i+1} / 1 \leq i \leq 5, v_1 v_6\} \cup \{u_i v_i / 1 \leq i \leq 6\}$
Let G_f be a graph obtained from G by fusing two consecutive vertices (say v_1 and v_2)
Then $|V(G_f)| = 11$ and $|E(G_f)| = 17$
Define a label $f: V(G_f) \to \{1, 2,3, \ldots \ldots 11\}$
such that $f(u_i) = i$ for $1 \leq i \leq 6$
(i.e)$f(u_1) = 1$
$f(u_2) = 2$
$f(u_3) = 3$

$f(u_4) = 4$
$f(u_5) = 5$
and $f(u_6) = 6$
and let $f(v_1, v_2 = v) = 11$, where v is the new vertex after fusion of the vertices v_1 and v_2.
$f(v_{i+1}) = 11 - (i-1)$ for $2 \leq i \leq 5$
(i.e) $f(v_3) = 10$
$f(v_4) = 9$
$f(v_5) = 8$
and $f(v_6) = 7$

Clearly, for each edge $e = u_i u_j \in G_f$ for the edges $(u_1, v), (u_2, v), (v_3, v), v_i v_j, u_i v_j \in G_f$

We have $gcd\left(f(v), f(u_1)\right) = 1$
$gcd\left(f(v), f(u_2)\right) = 1$
$gcd\left(f(v), f(u_3)\right) = 1$
$gcd\left(f(v_i), f(v_j)\right) = 1,$
$gcd\left(f(u_i), f(v_i)\right) = 1$
and $gcd\left(f(u_i), f(u_j)\right) = 1$

Therefore, G_f admits prime labeling. Hence G_f is a prime graph.

1.2 Illustration

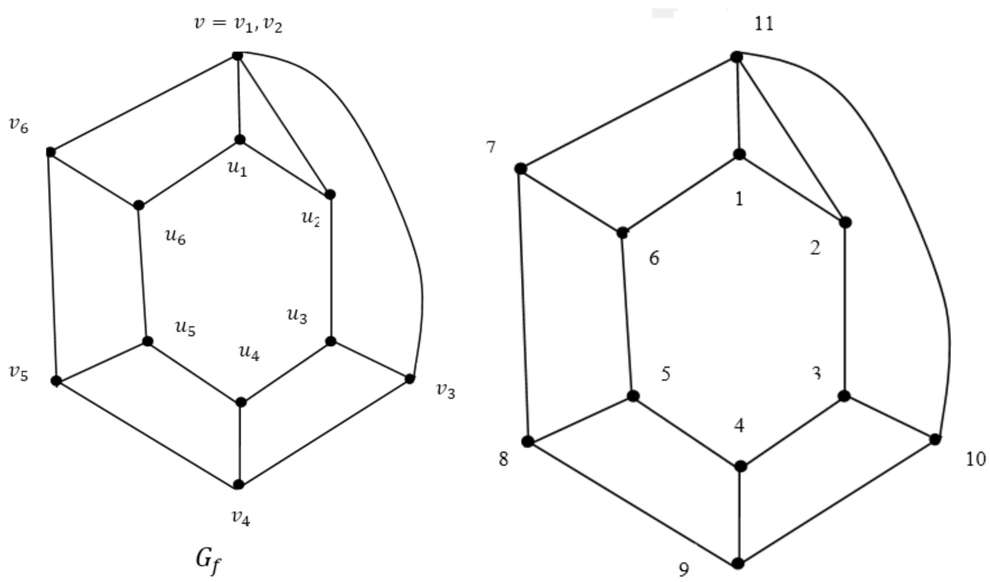

Fig 2: G_f is a prime graph

Proposition 3: The Duplication of any arbitrary vertex in a Cubic graph with 12 vertices is a prime graph.

Proof:

Let $G = (V, E)$ be a cubic graph with 12 vertices
Let $V(G) = \{u_1, u_2, u_3, u_4, u_5, u_6, v_1, v_2, v_3, v_4, v_5, v_6\}$
$E(G) = \{u_i u_{i+1}/1 \leq i \leq 5, u_1 u_6\} \cup \{v_i v_{i+1}/1 \leq i \leq 5, v_1 v_6\} \cup \{u_i v_i/\ 1 \leq i \leq 6\}$

Let G_k be a graph obtained from G by duplicating the vertex v_k in G and v_k^* duplication vertex of the vertex v_k. Without loss of generality, we may duplicate vertex v_1 and v_1^* be its duplication vertex.

Then $|V(G_k)| = 13$
Define a label $f: V(G_k) \to \{1, 2, \ldots \ldots .13\}$
such that $f(u_i) = i$ for $1 \leq i \leq 6$
$f(u_1) = 1$
$f(u_2) = 2$
$f(u_3) = 3$
$f(u_4) = 4$
$f(u_5) = 5$
$f(u_6) = 6$
And let $f(v_1^*) = 13$, where v_1^* is the duplication vertex of vertex v_1
$f(v_i) = 12 - (i-1)$ for $1 \leq i \leq 6$
(i.e) $f(v_1) = 12$
$f(v_2) = 11$
$f(v_3) = 10$

$f(v_4) = 9$
$f(v_5) = 8$
$f(v_6) = 7$

Clearly, for the edges $e = u_i u_j \in G_k$ and for the edges and $v_i v_j \in G_k, u_i v_i \in G_k, v_1^* v_1, v_1^* v_2, v_1^* v_6 \in G_k$

$gcd\big(f(u_i), f(u_j)\big) = 1$
$gcd\big(f(v_i), f(v_j)\big) = 1$
$gcd(f(u_i), f(v_i)) = 1$
$gcd(f(v_1^*), f(v_6)) = 1$
$gcd(f(v_1^*), f(v_1)) = 1$
$gcd(f(v_1^*), f(v_2)) = 1$

Therefore, G_k admits prime labeling
Hence G_k is a prime graph.

1.3 Illustration

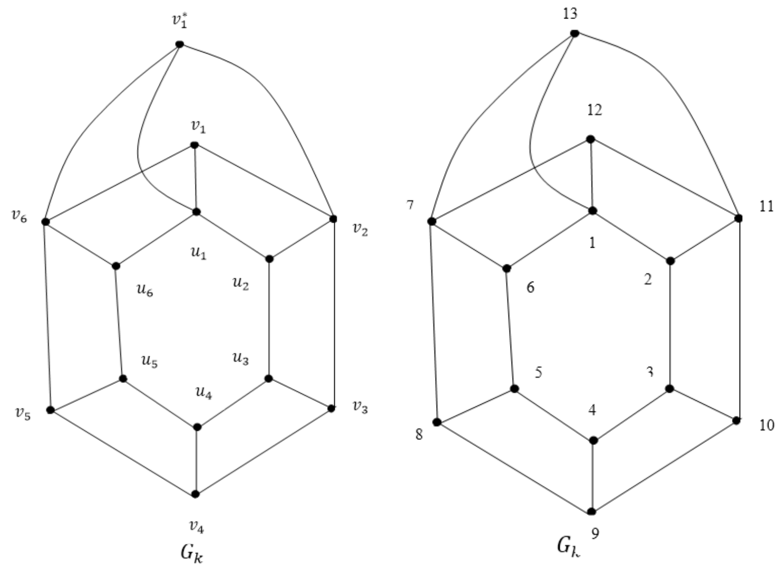

Fig 3: Duplication of a vertex v_1 in a Cubic graph G is a prime graph

Proposition 4: The switching of any vertex in a Cubic graph with 12 vertices is a prime graph.
Proof:
Let $G = (V, E)$ be a Cubic graph with 12 vertices and 18 edges
Let $V(G) = \{u_1, u_2, u_3, u_4, u_5, u_6;\ v_1, v_2, v_3, v_4, v_5, v_6\}$
$E(G) = \{u_i u_{i+1}/\ 1 \leq i \leq 5,\ u_1 v_6\} \cup \{v_i v_{i+1}/1 \leq i \leq 5, v_1 v_6\} \cup \{u_i v_i/1 \leq i \leq 6\}$
Let G_s be the graph obtained by switching an arbitrary vertex of G. Without loss of generality let this vertex be v_1 and $|\,V(G_s)| = 12$ and $|E(G_s)| = 23$
Define a label $f: V(G_s) \to \{1, 2, 3 \ldots \ldots 12\}$
Such that $f(u_i) = 12 - (i-1)$ for $1 \leq i \leq 6$
$f(u_1) = 12$
$f(u_2) = 11$
$f(u_3) = 10$
$f(u_4) = 9$
$f(u_5) = 8$
$f(u_6) = 7$
and $f(v_1) = 1$ where v_1 is a switching vertex
$f(v_i) = i$, for $2 \leq i \leq 6$
$f(v_2) = 2$
$f(v_3) = 3$
$f(v_4) = 4$
$f(v_5) = 5$
$f(v_6) = 6$

Clearly, for each edge $e = u_i u_j \in G_s$, for the edges $v_i v_j, u_i v_i, v_1 u_i, v_1 v_j \in G_s$
We have $gcd\big(f(u_i), f(u_j)\big) = 1$
$gcd\big(f(v_i), f(v_j)\big) = 1$
$gcd(f(u_i), f(v_i)) = 1$
$gcd(f(v_1), f(u_i)) = 1$ for $i = 2, 3, 4, 5, 6$

$gcd(f(v_1), f(v_j)) = 1$ for $j = 3, 4, 5$

Clearly, G_s admits a prime labeling

Hence G_s is a prime graph.

1.4 Illustration

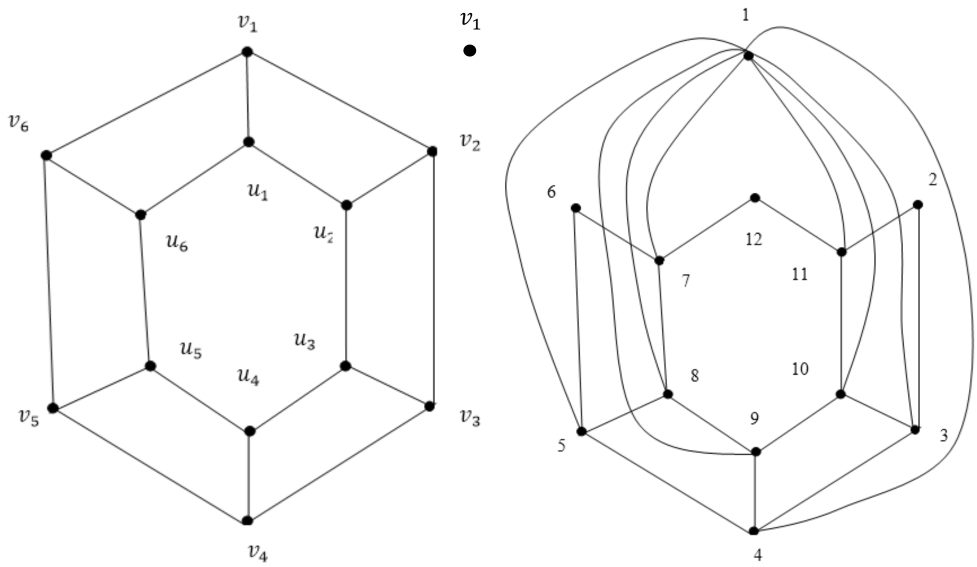

Fig 4: switching a vertex v_1 in a cubic graph with 12 vertices is a prime graph

Proposition 5: The Path union of two pieces of Cubic graph with 12 vertices is a prime graph.

Proof:

Let $G = (V, E)$ be a Cubic graph with 12 vertices and 18 edges

Let $V(G) = \{u_1, u_2, u_3, u_4, u_5, u_6, v_1, v_2, v_3, v_4, v_5, v_6\}$

$E(G) = \{u_i u_{i+1}/\ 1 \le i \le 5,\ u_1 v_6\} \cup \{v_i v_{i+1}/1 \le i \le 5, v_1 v_6\} \cup \{u_i v_i/1 \le i \le 6\}$ Let G_p be the graph obtained by taking the path union of two pieces of Cubic graphs with 12 vertices G and G' respectively. Then$\left| V(G_p) \right| = 24$. Define a label $f: V(G) \to \{1, 2, 3 \ldots \ldots .24\}$

1.5 For labeling of G

$f(u_i) = i$, for $1 \le i \le 6$

$f(u_1) = 1$

$f(u_2) = 2$

$f(u_3) = 3$

$f(u_4) = 4$

$f(u_5) = 5$

$f(u_6) = 6$

$f(v_i) = 12 - (i - 1)$ for $1 \le i \le 6$

$f(v_1) = 12$

$f(v_2) = 11$

$f(v_3) = 10$

$f(v_4) = 9$

$f(v_5) = 8$

$f(v_6) = 7$

1.6 For labeling G'

let $f(u_i') = 13 + (i - 1)$ for $1 \le i \le 6$

f(u'_1) = 13

$f(u_2') = 14$

$f(u_3') = 15$

$f(u_4') = 16$

$f(u_5') = 17$

$f(u_6') = 18$

And $f(v_i') = 24 - (i - 1)$ for $2 \le i \le 6$

$f(v_1') = 24$

$f(v_2') = 23$

$f(v_3') = 22$

$f(v_4') = 21$

$f(v_5') = 20$
$f(v_6') = 19$

Clearly, $gcd(f(u_i), f(u_j)) = 1$
$gcd(f(v_i), f(v_j)) = 1$
$gcd(f(u_i), f(v_i)) = 1$
$gcd(f(u_i'), f(u_j')) = 1$
$gcd(f(v_i'), f(v_j')) = 1$
and $gcd(f(u_i'), f(v_i')) = 1$
and $gcd(f(v_2), f(v_6')) = 1$

Clearly, G_p admits a prime labeling and G_p is a prime graph.

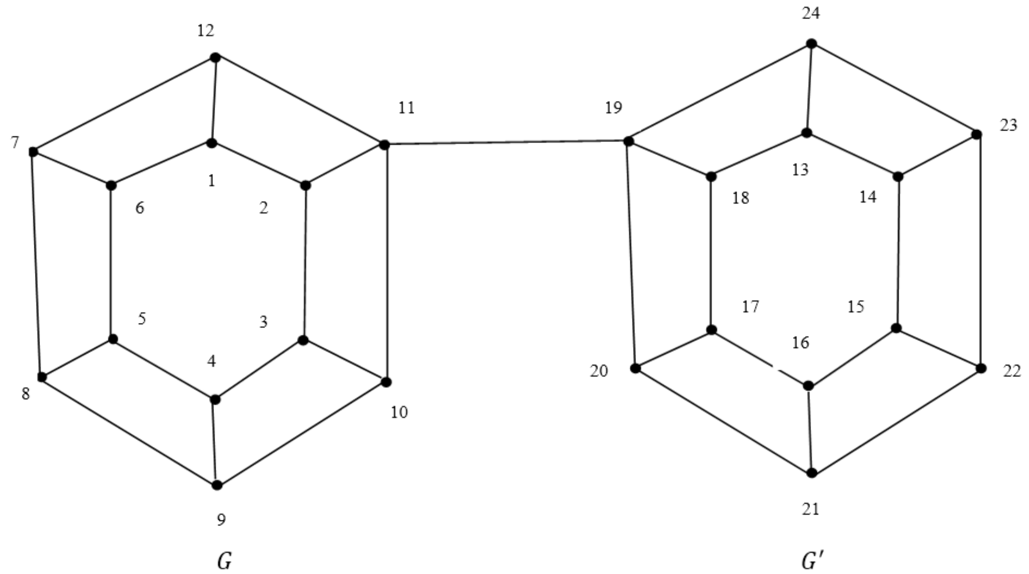

Fig 5: Path union of two pieces of Cubic graphs G and G' is a prime graph

1.7 Concluding Remarks and further scope: As all graphs are not prime graphs it is very interesting to investigate graphs which admits prime labeling it is possible to investigate similar results for other graph families and in the context of different labeling techniques.

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