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## Study of some important theorem on a fuzzy prime radical and fuzzy prime ideal of Gamma- Ring

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### Abstract

Fuzzy Prime radical and fuzzy primary ideal of a  $\Gamma$ -ring and have derived several interesting and appealing result on it.

**Keywords:** Fuzzy ideal,  $\Gamma$ -ring, fuzzy prime radical, fuzzy primary ideal

### 1. Introduction

The notion of fuzzy ideal in  $\Gamma$ -ring was introduced by Y.B. Jun and C.Y. Lee in 1992. Some earlier work on fuzzy ideals of  $\Gamma$ -ring may be found in the work of Dutta & Chanda, Jun & Lee, Hang & Jun, Jun and Jun & Lee. In this paper our aim is to introduce the notions of fuzzy prime radical and fuzzy primary ideal of a  $\Gamma$ -ring and study them. We will obtain a characterization of fuzzy primary ideal of a  $\Gamma$ -ring M.

**1.1  $\Gamma$ -RING:** Let M and  $\Gamma$  be two additive abelian groups. M is called a  $\Gamma$ -ring if there exists a mapping  $F: M \times \Gamma \times M \rightarrow M$ ,  $f(a, \alpha, b)$  is denoted by  $a\alpha b$ ,  $a, b \in M, \alpha \in \Gamma$ , satisfying the following conditions for all  $a, b, c \in M$  and for all  $\alpha, \beta, \gamma \in \Gamma$ ;

- (i)  $(a + b)\alpha c = a\alpha c + b\alpha c$ ,
- (ii)  $a(\alpha + \beta)b = a\alpha b + a\beta b$ ,
- (iii)  $a\alpha(b + c) = a\alpha b + a\alpha c$ ,
- (iv) and  $a\alpha(b\beta c) = (a\alpha b)\beta c$ ,

**1.2 Left Ideal of  $\Gamma$ -Ring:** A subset A of a  $\Gamma$ -ring M is called a left ideal of M if A is an additive subgroups of M and  $m\alpha a \in A$  for all  $m \in M, \alpha \in \Gamma, a \in A$ .

**1.3 Right Ideal of  $\Gamma$ -Ring:** The subset A of the  $\Gamma$ -ring M, be called a right ideal of M if A is an additive sub group of M and  $a\alpha m \in A$  for all  $m \in M, \alpha \in \Gamma, a \in A$ . if A is a left and right of M, then A is called a two sided ideal of M or simply an ideal of M

### 1.4 Left Primary Ideal & Right Primary Ideal

$\Gamma$ -ring M is said to be left primary if for any two ideal A and B,  $A\Gamma B \subseteq Q$  implies  $A \subseteq Q$  or  $B \subseteq PR(Q)$ , Where  $PR(Q)$  is the prime radical of Q. Similarly we can define right primary ideal. If in a commutative  $\Gamma$ -ring each left primary ideal is also a right primary ideal and conversely; we call it a primary ideal.

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**1.5 Fuzzy Left Ideal:** A non empty fuzzy subset  $\mu$  (i.e.  $\mu(x) \neq 0$  for some  $x \in M$ ) of a  $\Gamma$ -ring  $M$  is called a fuzzy left ideal of  $M$  if,

1.  $\mu(x-y) \geq \min\{\mu(x), \mu(y)\}$ ,
2.  $\mu(x\alpha y) \geq \mu(y)$ , for all  $x, y \in M$  and for all  $\alpha \in \Gamma$ .

**1.6 Fuzzy Right Ideal:** A non empty fuzzy subset  $\mu$  ( $\mu(x) \neq 0$  for some  $x \in M$ ) of a  $\Gamma$ -ring  $M$  is called a fuzzy right ideal of  $M$  if

1.  $\mu(x-y) \geq \min\{\mu(x), \mu(y)\}$ ,
2.  $\mu(x\alpha y) \geq \mu(x)$ , for all  $x, y \in M$  and for all  $\alpha \in \Gamma$ .

A non- empty fuzzy subset  $\mu$  of a  $\Gamma$ -ring  $M$  is called a fuzzy ideal of  $M$  if it is a fuzzy left ideal and a fuzzy right ideal of  $M$ .

**1.7 Fuzzy Prime Ideal:-** A non constant fuzzy ideal  $\mu$  of a  $\Gamma$  ring  $M$  is called a fuzzy prime ideal of  $M$ , if for any two fuzzy ideals  $\sigma$  and  $\theta$  of  $M$ ,  $\sigma \Gamma \theta \subseteq \mu$  implies that either  $\sigma \subseteq \mu$  or  $\theta \subseteq \mu$ .

We denote the set of all fuzzy prime ideals of  $M$  and the set of all fuzzy prime ideals of  $R$  by  $FpI(M)$  and  $FpI(R)$  respectively.

**Theorem 1.1:-** A fuzzy ideal  $\mu$  of a  $\Gamma$ -ring  $M$  is fuzzy prime if and only if  $\mu(o_m) = 1$ ,  $IM_\mu = \{1, \alpha\}$ ,  $\alpha \in [0, 1]$  and  $\mu_o = \{X \in M : \mu(X) = \mu(o_m)\}$  is a prime ideal of  $M$ .

**Proof:** - It is obvious from the definition of fuzzy ideal and fuzzy prime ideal.

Let  $G$  and  $H$  be two groups,  $A$  and  $B$  a  $t$ -fuzzy sub groups of  $G$  and  $H$ , respectively. Then the  $t$ -level subset  $(A \times B)_r^\Gamma$  for  $r \in [0, 1]$  is a sub group of  $G \times H$ , where  $e_G$  and  $e_H$  are identities of  $G$  and  $H$  respectively.

**Theorem 1.2:-** Let  $\mu$  be a non-constant fuzzy ideal of a  $\Gamma$ -ring  $M$ , then the following conditions are equivalent:

1.  $\mu$  is a fuzzy semi prime ideal of  $M$ ;
2. For any  $a \in M$ ,  $\inf\{\mu(a\gamma_1 m \gamma_2 a), m \in M, \gamma_1 \gamma_2 \in \Gamma\} = \mu(a)$

**Proof:** - It follows from that  $A, B$  be fuzzy subsets of the sets  $G$  and  $H$  respectively.  $\Gamma$  be a  $t$ -norm and  $r \in [0, 1]$ . Then  $A_r^\Gamma \times B_r^\Gamma = (A \times B)_r^\Gamma$ .

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