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Study of two theorem on fuzzy fields

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Abstract
Fuzzy fields, which was proposed by S. Nanda in 1990. Here we have studied the properties of fuzzy algebra. Finally, we have proposed a redefined version of fuzzy algebras over fuzzy fields and observe that our definition is more general than the definition of Nanda, because if A is a fuzzy algebra under Nanda's definition, it will also be a fuzzy algebra under our definition but not conversely.

Keywords: Fuzzy set, fuzzy algebraic structure, fuzzy field etc.

1. Introduction
Algebraic properties of fuzzy sets, fuzzy measures, probability measures of fuzzy events, fuzzy mathematical programming, fuzzy dynamic programming and decision making on fuzzy environment. Fuzzy groups were defined by Rosenfeld and subsequently the theory was enhanced by Anthony and Sherwood, Osmand and W Liu. Fuzzy rings and fuzzy ideals have been studied by Liu. The notion of fuzzy field and fuzzy algebra were introduced by Nanda. In this paper our aim is to discuss the concept of fuzzy field.

1.1 Fuzzy Field: Let X be a field and F be a fuzzy set in X with membership function μ_F. then F will be called a fuzzy field, if the following properties are satisfied.

(i) $\mu_F(x+y) \geq \min\{\mu_F(x), \mu_F(y)\}$ for all x, y ϵX

(ii) $\mu_F(-x) \geq \mu_F(x)$, for all x ϵ X

(iii) $\mu_F(xy) \geq \min\{\mu_F(x), \mu_F(y)\}$ for all x, y ϵX

(iv) $\mu_F(x^{-1}) \geq \mu_F(x)$ for all $x \neq 0 \in X$

(v) $\mu_F(0) = 1$ and $\mu_F(1) = 1$

Let x, y be any two elements of X and F be a fuzzy set in X then we have

$$\mu_F(x-y) = \mu_F\{x+(-y) \geq \min\{\mu_F(x), \mu_F(-y)\} \geq \min\{\mu_F(x), \mu_F(y)\}$$

$$\therefore \mu_F(x-y) \geq \min\{\mu_F(x), \mu_F(y)\} \text{..............}(1)$$

And $\mu_F(xy^{-1}) \geq \min\{\mu_F(x), \mu_F(y^{-1})\} \geq \min\{\mu_F(x), \mu_F(y)\}$, using (iv)

$$\mu_F(xy^{-1}) \geq \min\{\mu_F(x), \mu_F(y)\} \text{.............}(2)$$

Now we assume that F is an ordinary subset of x.
Then $\mu_F(x) = 1 \quad \mu_F(y) = 1 \ \forall x, y \in F$

Hence, equation (1) implies that

$$\Rightarrow \mu_F(x-y) \geq \min\{1,1\} = 1$$

i.e $\mu_F(x-y) \geq 1$

But $\mu_F(x-y) \not> 1$

$$\therefore \mu_F(x-y) = 1 \Rightarrow x-y \in F$$

Thus, we observe that for all x, y$\in$ F $\Rightarrow$ x-y $\in$F
Similarly, from equation (2), we will get

$$\mu_F(xy^{-1}) \geq \min\{1,1\} =1$$

i.e. $\mu_F(xy^{-1}) \geq 1$

But $\mu_F(xy^{-1})1 \not> 1$

$$\therefore \mu_F(xy^{-1}) = 1 \Rightarrow xy^{-1} \in F$$

That is for all x, y $\in$F $\Rightarrow$ $xy^{-1}\in$F

In this way we conclude that the notion of fuzzy field has been taken in such a way that when F is taken to be an ordinary subset of X under the definition of Nanda F turns out to be a sub field of X. form the definition of fuzzy field, we have

$$\mu_F(x) = \mu_F\{-(-x)\} \geq \mu_F(-x)$$

$$\forall x \in X$$

i.e $\mu_F(x) \geq \mu_F(-x)$.................(i)

But from condition (2) of the fuzzy field we have

$$\mu_F(-x) \geq \mu_F(x) \text{.................}(ii)$$

(i)& (ii) $\Rightarrow$ $\mu_F(-x) = \mu_F(x)$.................(iii)

Further, we have $\mu_F(x) = \mu_F\{(x^{-1})^{-1}\} \geq \mu_F(x^{-1})$

i.e $\mu_F(x) \geq \mu_F(x^{-1})$.................$(iv)$

And from the (iv[th]) condition of fuzzy field, we have

$$\mu_F(x^{-1}) \geq \mu_F(x) \text{.................}(v)$$

From (iv) and (v), we get

$$\mu_F(x^{-1}) = \mu_F(x), \forall x \in X \text{.............}(vi)$$

Theorem 1.1:- If F is a fuzzy field on a field X, then

1. $\mu_F(x-y) \geq \min\{\mu_F(x), \mu_F(y)\}\ forall x, y \in X$
2. $\mu_F(xy^{-1}) \geq \min\{\mu_F(x), \mu_F(y)\}\ forall, x \in X, Y \neq 0 \in X$

Proof: - Suppose that X is a field and F is a fuzzy field on X. Hence, we have

$$\mu_F(x-y) = \mu_F\{x+(-y)\} \geq \min\{\mu_F(x), \mu_F(y)\}$$

$$= \min\{\mu_F(x), \mu_F(y)\}$$

$\therefore \mu_F(x-y) \geq \min\{\mu_F(x), \mu_F(y)\}$ Using condition (ii)

And, $\mu_F(xy^{-1}) \geq \min\{\mu_F(x), \mu_F(y^{-1})\}$

$$= \min\{\mu_F(x), \mu_F(y)\}$$

$\therefore \mu_F(xy^{-1}) \geq \min\{\mu_F(x), \mu_F(y)\}$ Using (iv)

Conversely, we assume that F is a fuzzy set in X and is such that

$\mu_F(x-y) \geq \min\{\mu_F(x), \mu_F(y)\}$, For all x, y$\in$ X………..….(i)

And, $\mu_F(xy^{-1}) \geq \min\{\mu_F(x), \mu_F(y)\}$, for all x $\in$X, Y$\neq$0$\in$X…….(ii)

Then, we claim that F is a fuzzy field in X.
On putting x=0 in (i), we get

$$\mu_F(0-y) \geq \min\{\mu_F(0), \mu_F(y)\}$$

$$\Rightarrow \mu_F(-y) \geq \min\{1, \mu_F(y)\},$$

$\Rightarrow \mu_F(-y) \geq \mu_F(y)$ for all y$\in$X.

Again on taking x=1 in (ii), we get

$$\mu_F(1y^{-1}) \geq \min\{\mu_F(1), \mu_F(y)\}$$

$$= \min\{1, \mu_F(y)\}$$

$$= \mu_F(y)$$

$\therefore \mu_F(y^{-1}) \geq \mu_F(y)$, For all y$\neq$0$\in$X

On taking y =-y in (1), we get

$$\mu_F\{x-(-y)\} \geq \min\{\mu_F(x), \mu_F(y)\}$$

$= \min\{\mu_F(x), \mu_F(y)\}$, Using (iii)

$$\therefore \mu_F\{x+y)\} \geq \min\{\mu_F(x), \mu_F(y)\}$$

And on taking y^{-1} for Y in (ii), we get

$$\mu_F\{x(y^{-1})^{-1}\} \geq \min\{\mu_F(x), \mu_F(y^{-1})\}$$

$= \min\{\mu_F(x), \mu_F(y)\}$, Using (iv)

$$\therefore \mu_F(xy) \geq \min\{\mu_F(x), \mu_F(y)\}$$

Thus, we observe that the above theorem cannot give the result,

$\mu_F(0) = 1$ And $\mu_F(1) = 1$

Which are also the basic requirements for F to be a fuzzy field in X. So the definition of fuzzy field needs modification. Keeping these things in his mind, R. Biswas redefined fuzzy fields the definition of fuzzy field proposed by Biswas is as follows:

1.2 Redefinition of Fuzzy Field

Let X be a field and F be a fuzzy set in X with membership function μ_F. Then F be called a fuzzy field in X if and only if the following condition are satisfied:

1. $\mu_F\{x+y)\} \geq \min\{\mu_F(x), \mu_F(y)\}$ for all x, y $\in$X
2. $\mu_F(-x) \geq \mu_F(X)$ for all x $\in$X
3. $\mu_F(xy) \geq \min\{\mu_F(x), \mu_F(y)\}$ for all x, y$\in$ X

4. $\mu_F(x^{-1}) \geq \mu_F(x)$ x≠0ϵX

Now we can see that the above theorem 1.1 holds good.

Theorem 1.2.: If F is a fuzzy field on a field X and if $\mu_F(x) = 1$ for at least one x in X, then

a. $\mu_F(0)=1$

b. $\mu_F(1)=1$

Proof: - Let us suppose that X is field and F is a fuzzy field in X and is such that $\mu_F(x)=1$, For at least one x ϵX……………. (i)

Hence, from the (i) condition for F to be a fuzzy field, we have

$$\mu_F\{x-x\} \geq \min\{\mu_F(x), \mu_F(-x)\}$$

$$= \min\{\mu_F(x), \mu_F(x)\}$$

$$= \mu_F(x), \forall x \in X$$

i.e. $\mu_F(0) \geq \mu_F(x) \quad \forall x \in X$

$$\Rightarrow \mu_F(0) \geq 1$$ using (i)

But we know that $\mu_F(0) \leq 1$

Hence $\mu_F(0) = 1$

Further, we have on putting x^{-1} for Y in (ii) condition of the definition of the fuzzy field

$$\mu_F(xx^{-1}) \geq \min\{\mu_F(x), \mu_F(x^{-1})\}, \forall x \neq 0 \in X$$

$$= \min\{\mu_F(x), \mu_F(x)\}$$

$$= \mu_F(x)$$

Thus $\mu_F(1) \geq \mu_F(X)$

i.e. $\mu_F(1) \geq 1$ using (i)

But, we have

$$\mu_F(1) \leq 1$$

$$\therefore \mu_F(1) = 1$$

2. Reference

1. Katsars AK, Liu DB. fuzzy vector spaces and fuzzy topological vector spaces. J Math. Anal. Appl. 1977; 58:135-146.
2. Nanda S. fuzzy fields and fuzzy linear spaces, fuzzy sets system. 1986; 19(1):89-94.
3. Wing-Jin-Liu, fuzzy invariant subgroups and fuzzy ideals, fuzzy sets and systems 1982; 8:133-139.
4. Zadeh LA. fuzzy sets, informs. And control. 1965; 8:338-353.