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Three body problem in which the bigger primary is a source of radiation and the smaller one is an oblate spheroid

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Abstract
The present Paper deals with the linear stability of the equilibrium points of the photogravitational restricted three body problem in which the bigger primary is a source of radiation and the smaller primary is an oblate spheroid. The problem possesses five equilibrium points, three collinear and two triangular. The triangular equilibrium points make nearly equilateral triangles with primaries. The results differ slightly with those of the classical problem due to radiation effect of the bigger primary and the oblateness of the smaller one.

Keywords: Restricted, oblate spheroid, smaller primary, triangular equilibrium points

1. Introduction

The elliptic restricted three-body problem (ER3BP) determines the motion of infinestimal body under the gravitational attraction of two finite bodies known as primaries, which revolve in elliptic orbits around their center of mass. The term "restricted" means that the third body gets influenced by the primaries whereas it does not affect their motion. It describes the dynamical system more accurately as the motion of the primaries are mostly elliptical. The photogravitational circular restricted three-body problem (PCR3BP) studied by. It is understood that the rotation of planets produces an equatorial bulge due to centrifugal force; resulting in an oblate spheroid. Some of the planets in the solar system such as Jupiter, Saturn, Uranus and Neptune are sufficiently oblate. Various authors such as Sahoo and Ishwar (2000), have studied the effect of oblateness in ER3BP by taking one or both primaries as a source of radiation or oblate spheroid.Some of the authors have calculated the mean motion n and others have used the previous values of this important parameter. Essentially the true anomaly was used for averaging the radial distance r over a revolution. Some of the authors did not explain the method of evaluation. However, some of the authors have ignored the effect of eccentricity on the mean motion. Following Danby (1988, page 346), in this paper we have used the mean anomaly M to average the distance r between the primaries to get its average value. Then we have used this average value in computation of the mean motion n of the primaries. This value of n is different from the mean motion which is derived from the true anomaly in the literature. The characteristic exponents can be analyzed by various perturbation techniques. The present paper uses the analytical technique developed by. The stability of the third body around the triangular equilibrium points results in the form of transition curves in the μ-e plane, accurate to O(e2).These curves separate the stable region from the unstable region.

The collinear points are generally unstable; whereas the triangular points are conditionally stable in the linear analysis. This results in long- and short- orbits around the triangular Libration points. The present study involves in examining the motion of the third body in the ER3BP when the larger primary is a source of radiation and the smaller primary is an oblate spheroid. Analysis of the stability of the triangular points using the analytical technique with the effect of radiation pressure through generation of transition curves is carried out. Tadpole orbits for Sun-Jupiter system are generated. The motion around L4 is studied for the Sun-Saturn system with and without radiation and conclusion.

2. Periodic Orbit
If the period of the generating solution is same as the period of the analytic solution, then such a continuation is called a periodic orbit in the sense of Poincare. According to him there are three kinds of periodic orbits.

a. Periodic Orbits of First Kind
The orbits generated from the circular planar orbits are called periodic orbits of first kind. In this kind eccentricity **e = 0**, inclination of the orbital plane with a fixed plane θ = 0.

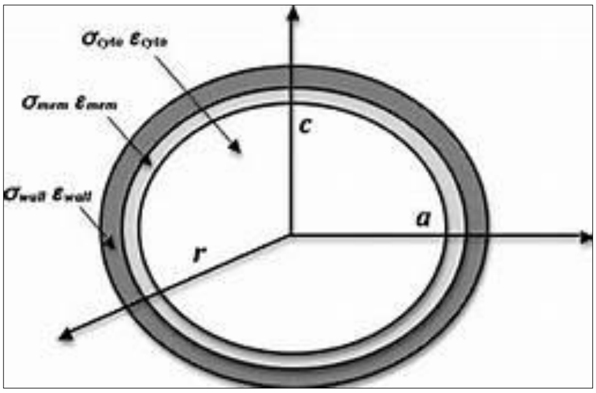

Fig 1: Circular planar orbits

b. Periodic Orbits of Second Kind
The orbits generated from elliptic two – body orbits in the plane of the primaries $(e \neq 0, \theta = 0)$ are known as periodic orbits of second kind

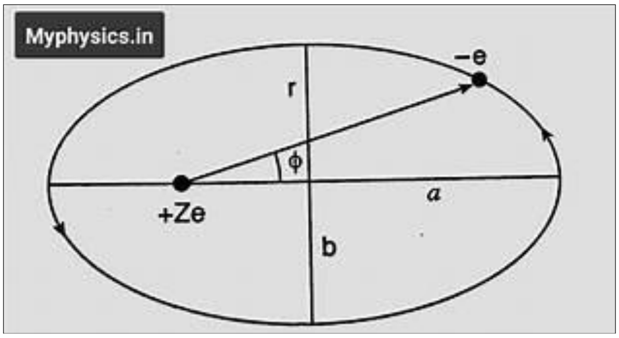

Fig 2: Elliptic periodic orbits

c. Periodic Orbits of Third Kind
The orbits generated from two – body orbits of $e \neq 0$ and $i \neq 0$ are named as periodic orbits of third kind.

3.Equations of Motion of the System
Let us consider a restricted three body problem in which the bigger primary is a source of radiation and the smaller one is an oblate spheroid. Let m_1 and m_2 be the masses of the bigger and smaller primaries and m be the mass of the third body (infinitesimal). Let us assume that the equatorial plane of the smaller primary coincides with the plane of motion of the third body. Let us take the line joining the primaries as x-axis and a line lying in the plane of motion of the primaries and perpendicular to x-axis, as y – axis. Z – Axis is in a direction perpendicular to the plane of motion. Let the masses m_1 and m_2 move in the circular orbits around their centre of mass with mean motion n about z-axis. Let R_0 be the distance between the primaries m_1 and m_2. Let $\vec{r}_1$ and $\vec{r}_2$ be the distances of m_1 and m_2 from the third body m respectively. We suppose that the third body moves in the xy – plane. Let (x, y) be the coordinate of the third body. As the third body due to its infinitesimal mass does not influence the motion of the primaries. Therefore the motion of the primaries will be governed by the problem of two bodies. Here, we assume that the radiation effect of the bigger primary influences the motion of the infinitesimal mass but does not influence the motion of the smaller primary. Thus the motion of the primaries takes place according to the law of two body problem in which law of gravitation and the oblateness effect of the smaller primary hold. Consequently the relative motion of the primaries is described by the differential equation:

$$\ddot{\vec{R}} = \frac{\partial U}{\partial R} \frac{\vec{R}}{|\vec{R}|}$$

Where the force function U is given by:

$$U = \frac{\bar{\mu}}{R} + \frac{\bar{\mu}\bar{A}}{2R^3}$$

Where

$$\bar{A} = \in R_E^{\ 2}$$

R_E being the equatorial radius of the oblate primary, $\in$ is the non – dimensional oblateness parameter and $\bar{\mu} = k^2(m_1 + m_2) = K^2 M$, where M $= m_1 + m_2$.
As the force function is the function of radius vector only hence the area integral exists and thus the motion takes place in an invariable plane assumed to coincide with the equatorial plane of the oblate primary. Let the motion of the primaries be a circle of radius R_0. Then the mean motion of the primaries is given by:

$$n^2 R_0 = -\left(\frac{\partial U}{\partial R}\right)_0 \text{s}$$

$$= -\bar{\mu}\left(-\frac{1}{R^2} - \frac{3\bar{A}}{2R^4}\right)_{R=R_0} \quad \text{Or,}$$

$$= -\frac{\bar{\mu}}{R_0^{\ 2}}\left(1 + \frac{3\bar{A}}{2R_0^{\ 2}}\right) \qquad (3.1)$$

The force of radiation is taken as.

$$F = F_g - F_p = F_g\left(1 - \frac{F_g}{F_p}\right) = (1-q)F_g$$

Where
F_g is the gravitational attraction force
F_p is the radiation pressure forceandq is the mass reduction factor. Here it is assumed that gravitation prevails i.e., (1-q)> 0 and 0<q<<1. Here the Poynting – Robertson drag effect is ign

4. Barycentric coordinate system

In the rotating barycentric coordinate system (x, y) the Lagrangian for the motion of the infinitesimal mass in the circular restricted three body problem is given by

$$I=\frac{m}{2}\left[\left(\frac{dX}{d\Gamma}-nY\right)^2+\left(\frac{dY}{d\Gamma}-nX\right)^2\right]+\bar{U} \quad \ldots \qquad (4.1)$$

Where

$$\bar{U}=K^2 m\left[\frac{m_1(1-q)}{R_1}+\frac{m_2}{R_2}+\frac{m_2\bar{A}}{2R_2^3}\right]$$

Using the non – dimensional variables

$$\frac{m_2}{M}=\mu;\frac{m_1}{M}=1-\mu, M=m_1+m_2$$

$$x=\frac{X}{R_0},\ y=\frac{Y}{R_0}\ R_1=R_0 r_1, R_2=R_0 r_2 \ \ldots. \qquad (4.2)$$

and

T=nt … (4.3)

The Langrangian (4.1) takes the form

$$\bar{L}=\frac{mn^2R_0^2}{2}\left[\left(\dot{x}-y\right)^2+\left(\dot{y}+x\right)^2\right]+k^2mM\left[\frac{(1-\mu)(1-q)}{r_1R_0}+\frac{\mu}{r_2R_0}+\frac{\mu\bar{A}}{2R_0^3r_2^3}\right]$$

or,

$$\frac{\bar{L}}{mn^2R_0^2}=\frac{1}{2}\left[\left(\dot{x}-y\right)^2+\left(\dot{y}+x\right)^2\right]+$$

$$+\frac{k^2M}{n^2R_0^3}\left[\frac{(1-\mu)(1-q)}{r_1}+\frac{\mu}{r_2}+\frac{\mu\bar{A}}{2r_2^3R_0^2}\right]$$

Here dot denotes differentiation with respect to non – dimensional time T.

i.e. $$\frac{\bar{L}}{mn^2R_0^2}=\frac{1}{2}\left(\dot{x}^2-\dot{y}^2\right)+\frac{1}{2}\left(x^2+y^2\right)+\left(x\dot{y}-y\dot{x}\right)$$

$$+\frac{k^2M}{n^2R_0^3}\left\{\frac{(1-\mu)(1-q)}{r_1}+\frac{\mu}{r_2}+\frac{\mu A}{2r_2^3}\right\}$$

Where

$$A=\frac{\bar{A}}{R_0^2}=\frac{\in R_E^2}{R_0^2}$$

is also a non – dimensional number.

Putting $L=\frac{\bar{L}}{mn^2R_0^2}$, we get

$$L=\frac{1}{2}\left(\dot{x}^2+\dot{y}^2\right)+\frac{1}{2}\left(x^2+y^2\right)+\left(x\dot{y}-y\dot{x}\right)$$

$$+\frac{k^2M}{n^2R_0^3}\left\{\frac{(1-\mu)(1-q)}{r_1}+\frac{\mu}{r_2}+\frac{\mu A}{2r_2^3}\right\}$$

Where,

$$r_1^2=(x+\mu)^2+y^2$$

$$r_2^2=(x-1+\mu)^2+y^2$$

Also, we obtain

$$\frac{n^2R_0^3}{k^2M}=1+\frac{3A}{2}$$

As the unit of distance is taken as the distance between the primaries. The unit of time is so chosen that the Gaussian constant of gravitation is unity. Hence the Lagrangian is obtained as

$$L=\frac{1}{2}\left(\dot{x}^2+\dot{y}^2\right)+\left(x\dot{y}-y\dot{x}\right)+\frac{1}{2}\left(x^2+y^2\right)+$$

$$+\frac{1}{1+\frac{3A}{2}}\left\{\frac{(1-\mu)(1-q)}{r_1}+\frac{\mu}{r_2}+\frac{\mu A}{2r_2^3}\right\} \quad \ldots \qquad (4.4)$$

Now,

$$\frac{\partial L}{\partial \dot{x}}=\dot{x}-y$$

$$\frac{\partial L}{\partial \dot{y}}=\dot{y}+x$$

$$\frac{\partial L}{\partial x}=\dot{y}+\frac{\partial\Omega}{\partial x}$$

$$\frac{\partial L}{\partial y}=\dot{x}+\frac{\partial\Omega}{\partial y}$$

Where

$$\Omega=\frac{1}{2}\left(x^2+y^2\right)+\frac{1}{\left(1+\frac{3A}{2}\right)}\left\{\frac{(1-\mu)(1-q)}{r_1}+\frac{\mu}{r_2}+\frac{\mu A}{2r_2^3}\right\} .. \qquad (4.5)$$

As the Lagrangian does not involve time T explicitly, the dynamical system admits Jacobis integral given by:

$$\sum \frac{\partial L}{\partial \dot{q}_i} q_i = h$$

i.e., $-\frac{1}{2}\left(\dot{x}^2+\dot{y}^2\right)-\left(x\dot{y}-y\dot{x}\right)-\Omega+\dot{x}\left(\dot{x}-y\right)+\dot{y}\left(\dot{y}+x\right)=h$

or

$$\frac{1}{2}\left(\dot{x}^2+\dot{y}^2\right)-\Omega=h$$

Now the equations of motion of the third body can be written as

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right)-\frac{\partial L}{\partial x}=0$$

And

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right)-\frac{\partial L}{\partial y}=0$$

or, $\ddot{x}-\dot{y}-\left(\dot{y}+\frac{\partial \Omega}{\partial x}\right)=0$

$$\ddot{y}-\dot{x}-\left(-\dot{x}+\frac{\partial \Omega}{\partial y}\right)=0$$

Or

$$\left.\begin{aligned}\ddot{x}-2\dot{y}&=\frac{\partial \Omega}{\partial x}\\ \ddot{y}-2\dot{x}&=\frac{\partial \Omega}{\partial y}\end{aligned}\right\} \quad \ldots (4.6)$$

Where

$$\Omega=\frac{x^2+y^2}{2}+\frac{1}{\left(1+\frac{3A}{2}\right)}\left\{\frac{(1-\mu)(1-q)}{r_1}+\frac{\mu}{r_2}+\frac{\mu A}{2r_2^3}\right\} \quad \ldots (4.7)$$

and the non – dimensional variables (x, y) are coordinates of the third particle with reference to a cartesian rectangular rotating system of axes around its origin located at the centre of mass of the primaries. Dots denote differentiation with respect to the non – dimensional time T.

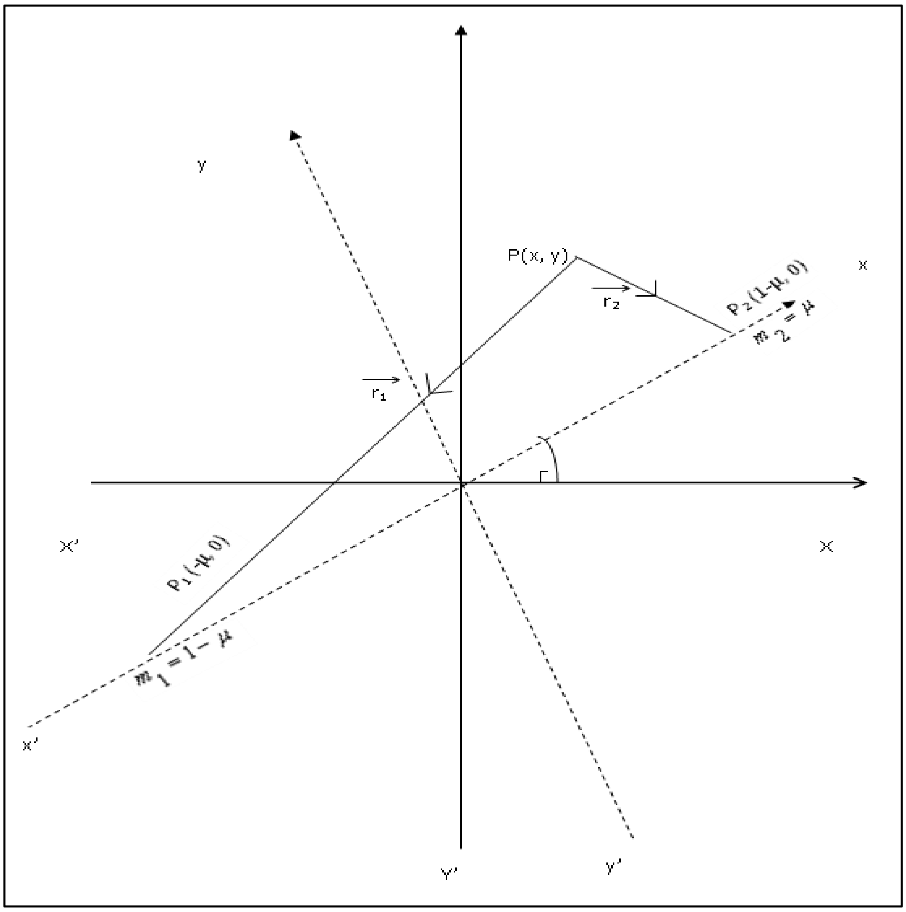

Fig 3: The rotating coordinate system (O*xy*) with angular velocity μ relative to the inertial frame (O*XY*). The two primaries and the massless body are denoted by $P1$, and $P2$, respectively.

Conclusions

In this paper, we have studied the linear stability of triangular equilibrium points in the photogravitational restricted three body problem with triaxial rigid bodies, the bigger one an oblate spheroid and source of radiation. The co-ordinates of the triangular equilibrium points are given in equations (4.2) and (4.3). We see that the non – dimensional variables (x, y) are coordinates of the third particle with reference to a

Cartesian rectangular rotating system of axes around its origin located at the centre of mass of the primaries displacements of the new triangular equilibrium points from the classical triangular equilibrium points are small and depend upon the quantities.

References

1. Douskos CN, Perdios EA. On the Stability of Equilibrium Points in the Relativistic Restricted Three-body Problem, CeMDA 2002;52:312.
2. Ishwar B. Non-linear Stability in the Generalized Re-stricted Three Body Problem, CeMDA 1997;65:253.
3. Ishwar B, Elipe A. Secular Solutions at Triangu-lar Equilibrium Point in the Generalise Photogravitational Restricted Three{Body Problem, Ap&SS 2001;277:437.
4. Sanjay J, Ajay K,Bhatnagar KB. Periodic Orbits Around the Collinear Libration Point in the Re-stricted Three-body Problem When the Smaller Primaryis a Triaxial Rigid Body and Bigger Primary is a Source of Radiation Pressure, Indian J phys 2009;83(2):171.
5. Khanna M, Bhatnagar KB. Existence and Stability of Libration Points in the Restricted Three-body Prob-lem when Thesmaller Primary is a Triaxial Rigid Body, Indian J Pure. Appl. Math 1998;29(10):1011.
6. Kalantonis VS, Perdios EA, Perdiou AE. The Sitnikov Family and the Associated Families of 3D Periodic Orbits in the Photogravitational RTBP with Oblate-ness, ApSS 2008;315:323.
7. Avdhesh K, Ishwar B. Linear Stability of Triangular Equilibrium Points in the Photogravitational Restricted Three Body Problem with Triaxial Rigid Bodiesand Oblate Spheroid, Nova Science Publishers, 6, Number1, (JNSST 2012) 2012.
8. Lara M, Elipe A. Periodic Orbits in the Restricted Three Body Problem with Radiation Pressure, CeMDA 1997;68:1.
9. Mittal A, Ahmad I, Bhatnagar KB. Periodic Orbits Generated by Lagrangian Solutions of the Restricted Three Body Problem When One of the Primaries is an Oblate Body, A p& SS 2009;319:63.
10. EIA Bouelmagd, Existence and stability of triangular points in the restricted three–body problem with numerical applications, Astrophys Space Sci 2012;342:45-53.
11. EI Abouelmagd. The effect of photogravitational force and oblateness in the perturbed restricted three–body problem. Astrophys Space Sci 2013;346:51-69.
12. Barrar RB. Existence of periodic orbits of second kind in the restricted problem of three bodies, Astronaumical Journal 1965;70:3-4.