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A cable connected satellites system non-linear oscillation and the main resonance

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Abstract
We study about the non-linear Oscillation of a system of two-Cable connected artificial satellites under several influences of general nature like magnetic force of the earth solar radiation pressure and shadow of the earth and the case of main resonance the cable is light flexible inelastic and non conducting in nature. We applied B.K.M. method for analysis of the motion of the system concerned we investigate the system moves like a dumbbell Satellite with stationary amplitude.

Keywords: Two cable connected satellites, main resonance, non-linear planner oscillation, B.K.M. method

Introduction

Beletsky (1965) and Beletsky and Novikova(1969) studied the motion of a system of two cable-connected satellites in the central gravitational field of force relative to its center of mass. This study assumed that the two satellites are moving in the plane of the center of mass. Singh and Demin (1972) and Singh (1973) investigated the problem in two and three dimensional cases. Das *et al*. (1976) studied the effect of magnetic force on the motion of a system of two cable-connected satellites in orbit.Sinha and Singh (1987) studied effect of solar radiation pressure on the motion and stability of the system of two inter connected satellites when their center of mass moves in circular orbit.Again, Sinha and Singh (1988) could generalize the above problem by considering the center of mass of the system moving in elliptical orbit. Beletsky and Levin (1993) studied dynamics of space tether‘s systems. Kurpa *et al*. (2000) explained about a new concept of space flight for tethered satellite systems. Again Kurpa *et al*. (2006) studied about modelling, dynamics and control of tethered satellite systems. Sharma and Narayan (2001) studied non – linear oscillation of inter connected satellite system under the combined influence of the solar radiation pressure and dissipative forces of general nature. Again Sharma and Narayan (2002) investigated effect of solar radiation pressure on the motion and stability of inter connected satellites system in orbit. Singh *et al*. (2001) studied non- linear effects on the motion and stability of an inter connected satellites system orbiting around an oblate Earth. Celled and Sidorenko (2008) studied some properties of dumb-bell satellite altitude dynamics. Umar (2013) explained development of satellite technology and its impact on social life. Kumar and Srivastava (2006) studied evolutional and non-evolutional motion of a system of two cable-connected artificial satellites under some perturbative forces. Kumar and Prasad (2015) studied aboutnon - linear planer oscillation of a cable – connected satellites system and non – resonance. Kumar and Kumar (2016) studied equilibrium positions of a cable- connected satellites system under several influences. Kumar (2018) studied liberation points of a cable – connected satellites system under the influence of solar radiation pressure, earth’s magnetic field, shadow of the earth and air resistance: circular orbit. Kumar and Bhattacharya [4] studied the stability of equilibrium positions of two cable-connected satellites under the influence of solar radiation pressure, earth’s oblateness and earth’s magnetic field. Kumar and Srivastava [5] obtained the equations of motion of a system of two cable-connected artificial satellites under the influence of some perturbative forces. Kumar *et al*. [6] studied the boundary of motion of a system of two cable-connected satellites under some.

Treatment of the problem

We write equation of small oscillation of the system about the equilibrium position as:-

$$\left(1 + e\cos v\right)n'' - 2en'\sin v + 3\left[A_0\sqrt{1-A_0^2} + (1-2A_0^2)n - 2A_0\sqrt{1-A_0^2}\,.\,n^2 - \frac{2}{3}(1-2A_0^2)n^3\right]$$

$$= 2e\sin v + B(1+e\cos v)[A_0 + \sqrt{1-A_0^2}\,n - \frac{A_0 n^2}{2} - \sqrt{1-A_0^2}.\frac{n^3}{6}$$

$$-Be\sin v\,[\sqrt{1-A_0^2} - A_0 n - \sqrt{1-A_0^2}.\frac{n^2}{2} + A_0\frac{n^3}{6}] + k_1(1+e\cos v)^{-3}[A_0 + \sqrt{1-A_0^2}n - A_0\frac{n^2}{2} - \sqrt{1-A_0^2}.\frac{n^3}{6}]$$

$$-k_2(1+e\cos v)^{-3}[\sqrt{1-A_0^2} - A_0 n - \sqrt{1-A_0^2}\frac{n^2}{2} + A_0\frac{n^3}{6}] \qquad \text{... (1)}$$

Condition of Constant for equatorial orbit is given by

$$(1+e\cos v)(\lambda'+1)^2 + 3\cos^2\Psi - 1 - \{\cos\Psi\cos\alpha + \sin\Psi\sin\alpha\}\frac{B\cos i}{\rho}$$

$$-\frac{A\cos\epsilon}{\pi}\frac{[\cos\Psi\cos\alpha + \sin\Psi\sin\alpha]}{(1+e\cos v)^3} \geq 0 \qquad \text{…(2)}$$

Assuming the eccentricity to be a small quantity of the first order infinitesimal the equation of the motion (1) can be written as:

$$n'' + n''e\cos v\left(1 + e\cos v\right)n'' - 2en'\sin v + 3\left[A_0\sqrt{1-A_0^2} + (1-2A_0^2)n - 2A_0\sqrt{1-A_0^2}\,.\,n^2 - \frac{2}{3}(1-2A_0^2)n^3\right]$$

$$= 2e\sin v + B(1+e\cos v)[A_0 + \sqrt{1-A_0^2}\,n - \frac{A_0 n^2}{2} - \sqrt{1-A_0^2}.\frac{n^3}{6}$$

$$-Be\sin v\,[\sqrt{1-A_0^2} - A_0 n - \sqrt{1-A_0^2}.\frac{n^2}{2} + A_0\frac{n^3}{6}] + k_1(1 - 3e\cos v + 6e^2\cos^2 v)[A_0 + \sqrt{1-A_0^2}n - A_0\frac{n^2}{2} - \sqrt{1-A_0^2}.\frac{n^3}{6}]$$

$$-k_2(1 - 3e\cos v + 6e^2\cos^2 v)\left[\sqrt{1-A_0^2} - A_0 n - \sqrt{1-A_0^2}\frac{n^2}{2} + A_0\frac{n^3}{6}\right] \qquad \text{….(3)}$$

Since the non-linearity is sufficiently weak so we get:

$$\left.\begin{aligned}
&\left[-3A_0\sqrt{1-A_0^2} + BA_0 + k_1A_0 - k_2\sqrt{1-A_0^2}\right] = eD_1 \\
&\left[-B\frac{A_0}{2} - \frac{k_1A_0}{2} + 6A_0\sqrt{1-A_0^2} + \frac{k_2\sqrt{1-A_0^2}}{2}\right]n^2 = eD_2n^2 \\
&\left[+2(1-2A_0^2) - \frac{B\sqrt{1-A_0^2}}{6} - \frac{k_1\sqrt{1-A_0^2}}{6} - \frac{k_2A_0}{6}\right]n^3 = eD_3n^3
\end{aligned}\right\} \qquad \text{….(4)}$$

Hence we get from (3) on using (4) and neglecting terms containing e^2 and the highest power of a

$$\eta'' + \eta^2\eta = e[\left(2 - B\sqrt{1-A_0^2}\right)\sin v + 2\eta'\sin v - \eta''\cos v$$

$$+A_0B\cos v + B\sqrt{1-A_0^2}\eta\cos v - B\frac{A_0}{2}\cos v.\eta^2 - \frac{B\sqrt{1-A_0^2}}{6}\cos v.\eta^3$$

$$+A_0B\eta\sin v + \frac{B\sqrt{1-A_0^2}}{2}\eta^2\sin v - \frac{BA_0\eta^3}{6}\sin v - 3k_1A_0\cos v - 3k_1\sqrt{1-A_0^2}\eta\cos v$$

$$+\frac{3k_1A_0}{2}\eta^2\cos v+\frac{k_1\sqrt{1-A_0^2}}{2}\eta^3\cos v+3k_2A_0\eta\cos v-\frac{3k_2\sqrt{1-A_0^2}}{2}\eta^2\cos v$$

$$+\frac{k_2A_0}{2}n^3\cos v+D_1+D_2\eta^2+D_3\eta^3] \quad \text{……(5)}$$

Where $\eta^2=[3(1-2A_0^2)-(B+k_1)\sqrt{1-A_0^2}-k_2A_0$

To study the phenomenon of main resonance and to solve the equation of motion we applying B.K.M. Method. Assuming e to be a small parameter, the solution in the first approximation of the equation of motion (3.2.5) can be sought in the form:-

$$\eta=a\cos k,\ where\ k=v+\theta \quad \text{…(6)}$$

Where amplitude a and phase θ must satisfy the system ay ordinary differential equations

$$\frac{da}{dv}=eA_1(a,\theta)$$

$$\frac{d\theta}{dv}=(n-1)+eB_1(a,\theta)$$

Where A_1 (a,θ) and B_1(a,θ) are periodic solutions periodic with respect to θ of the system of partial differential equations given below:-

$$(n-1)\frac{\partial A_1}{\partial\theta}-2anB_1=\frac{1}{\pi}\int_0^{2\pi}f_0(v,\eta,\eta',\eta'')\cos k\,dk$$

And $a(n-1)\frac{\partial B_1}{\partial\theta}+2nA_1=-\frac{1}{\pi}\int_0^{2\pi}f_0(v,\eta,\eta',\eta'')\sin k\,dk$ ….(7)

Where, $f_0(v,\eta,\eta',\eta'')$ is obtained by using (6) on the R.H. side of (5) in the form:-

$$f_0(v,\eta,\eta',\eta'')=\left(2-B\sqrt{1-A_0^2}\right)\sin(k-\theta)+(A_0B-3k_1A_0+3k_2\sqrt{1-A_0^2})\cos(k-\theta)$$

$$-2\,a\sin(k-\theta)\sin k+A_0B\,a\sin(k-\theta)\cos k+B\frac{\sqrt{1-A_0^2}}{2}a^2\sin(k-\theta)cos^2k$$

$$B\frac{A_0}{6}a^3\sin(k-\theta)cos^3k+B\sqrt{1-A_0^2}\,a\cos(k-\theta)\cos k+a\cos k\cos(k-\theta)$$

$$-3k_1\sqrt{1-A_0^2}a\cos(k-\theta)\cos k-3k_2A_0\,a\cos(k-\theta)\cos k-B\frac{A_0}{2}a^2\cos(k-\theta)cos^2k$$

$$+\frac{3k_1A_0}{2}a^2\cos(k-\theta)\,cos^2k-\frac{B\sqrt{1-A_0^2}}{6}a^3\cos(k-\theta)\,cos^3k+\frac{k_1\sqrt{1-A_0^2}}{2}a^3\cos(k-\theta)\,cos^3k$$

$$+\frac{k_2A_0}{2}a^3\cos(k-\theta)\,cos^3k+D_1+D_2a^2cos^2k+D_3cos^3k.a^3 \quad \text{…(8)}$$

Putting the value of $f_0(v,\eta,\eta',\eta'')$ from (8) on the R.H. side of (7) under the integral sign and integrating, we get:-

$$(n-1)\frac{\partial A_1}{\partial\theta}-2anB_1=\left[2-B\sqrt{1-A_0^2}+\frac{3}{4}\frac{B}{2}\sqrt{1-A_0^2}a^2\right]\sin\theta$$

$$+\left[\left(A_0B-3k_1A_0+3k_2\sqrt{1-A_0^2}\right)+\left(\frac{9}{8}k_1A_0-\frac{9}{8}k_2\sqrt{1-A_0^2}-\frac{3}{8}A_0B\right)a^2\right]\cos\theta+\frac{3}{4}D_3a^3$$

i.e $(n-1)\frac{\partial A_1}{\partial\theta}-2anB_1=-\mu\sin\theta+\lambda\cos\theta+\frac{3}{4}D_3a^3$

and $a(n-1)\frac{\partial B_1}{\partial\theta}+2nA_1=-\mu\cos\theta-\lambda\sin\theta$ …..(10)

where $\mu=\left(2-B\sqrt{1-A_0^2}+\frac{3}{8}B\sqrt{1-A_0^2}.a^2\right)$

and $\lambda = [A_0B - 3k_1A_0 + 3k_2\sqrt{1-A_0^2} + (\frac{9}{8}k_1A_0 - \frac{9}{8}k_2\sqrt{1-A_0^2} - \frac{3}{8}A_0B)a^2$ …(11)

The particular solution periodic with respect to θ of the system of equations (10) can be easily obtained as:-

$$A_1 = \frac{1}{(n+1)}[-\mu\cos\theta - \lambda\sin\theta]$$

And $B_1 = [\frac{1}{a(n+1)}[\mu\sin\theta - \lambda\cos\theta - \frac{3D_3a^2}{8n}]$ ….(12)

Now substituting the values of A_1 and B_1 from (12) in (10), we get

$$\frac{da}{dv} = \frac{e}{(n+1)}[-\mu\cos\theta - \lambda\sin\theta]$$

And $\frac{d\theta}{dv} = (n-1)\frac{e}{a(n+1)}\{[\mu\sin\theta - \lambda\cos\theta]\} - \frac{3eD_3a^2}{8n}$ ….(13)

The system of equations (13) may be written as

$$\frac{da}{dv} = \frac{1}{a}\frac{\partial H}{\partial\theta}$$

And $\frac{d\theta}{dv} = -\frac{1}{a}\frac{\partial H}{\partial\theta}$

Where H = $-\frac{ea}{(n+1)}(\mu\sin\theta - \lambda\cos\theta) + \frac{3eD_3a^2}{32n} - \frac{(n-1)a^2}{2}$ …(14)

Clearly, the system of equations (13) is in conical form and hence it has a first integral of the form:-

H = C_0’ …(15)

Which reduces the problem to quadrative and here C_o’ is the constant of integration.
The stationary regions of the amplitude a and phase θ are given by equating the R.H. side of equations (13) to zero.

Thus, $\mu\cos\theta + \lambda\sin\theta = 0 \Longrightarrow \sin\theta = -\frac{\mu}{\lambda}\cos\theta \;\; i.e \;\; \tan\theta = -\frac{\mu}{\lambda}$

And $(n-1) + \frac{e}{a(n+1)}(\mu\sin\theta - \lambda\cos\theta) - \frac{3eD_3a^2}{8n} = 0$

i.e $(n-1) - \frac{e}{a(n+1)}(\frac{\mu^2}{\lambda}\cos\theta + \lambda\cos\theta) - \frac{3eD_3a^2}{8n} = 0$

Thus, we get $\tan\theta = -\frac{\mu}{\lambda}$

And $(n-1) - \frac{e}{a(n+1)}\frac{\mu^2\lambda^2}{\lambda}.\frac{\lambda}{\sqrt{\mu^2+\lambda^2}} - \frac{3eD_3a^2}{8n} = 0$

i.e $(n-1) - \frac{e}{a(n+1)}\sqrt{\mu^2\lambda^2} - \frac{3eD_3a^2}{8n} = 0$ …(16)

Actually, it is very difficult to get the algebraic solutions of (16).
Hence integral curves are not drawn.

Conclusion

We have studied the non-linear osccilation of system of two satellites connected by cable under several influence of general nature. differential equation of motion of the system is obtained we apply B.K.M. Metod to solve the equation. the algebraic solution opf the equation is not possible hence we do not get the integral curves at the main resonance we assume that eccentricity of the orbit is a very small parameter but eccentricity which not equal to zero. In fact motion of the system in the whole planes will be non-evoluational. As a result of which we conclude that the system will always moves like a dumbbell satellites with stationary amplitude.

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