International Journal of Statistics and Applied Mathematics 2016; 1(5): 06-13

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2016; 1(5): 06-13

www.mathsjournal.com
Received: 03-09-2016
Accepted: 06-10-2016

Izevbizua O
Department of Mathematics, University of Benin. Nigeria/ Department of Mathematics, Delta State University, Abraka, Nigeria

Aanaudor JS
Department of Mathematics, University of Benin. Nigeria/ Department of Mathematics, Delta State University, Abraka, Nigeria

Corresponding Author:
Izevbizua O
Department of Mathematics, University of Benin. Nigeria/ Department of Mathematics, Delta State University, Abraka, Nigeria

Implementing Chiu's fixed lifetime inventory model

Izevbizua O and Aanaudor JS

Abstract
The fixed lifetime inventory system with zero or positive leadtime has being considered by various authors, with the aim of reducing the quantity outdating in the inventory system. Chiu (1995), considered the positive leadtime case by approximating the expected shortage quantity, the expected outdate quantity and the expected on hand inventory. His result was a Q, R ordering policy where Q and r are the ordered quantity and the reorder point respectively. In this work, we examine Chiu's implementation of the model, use simple minimization method to obtain the order quantity and modify the total cost function obtained by Chiu. Also, we consider the case where r the reorder point is not zero, which Chiu did not considered.

Keywords: Outdate, shortage, leadtime, reordered point, cost function, ordering policy, fixed, lifetime

Introduction

Many researchers have worked on the fixed lifetime inventory system. Schmidt and Nahmias (1985) [3], Liu and Lian (1999) [5], Omosigho (2002) [4], Olsson and Tydesjo (2010) [6], Shen *et al.* (2012) [7] to mention a few. All of these authors worked on reducing the amount of items outdating in a fixed lifetime inventory system. Fixed lifetime products are products with a pre-defined shelf life and normally carry expiration dates on them. While in inventory, these products are depleted by either demand or outdating. Outdating occur when a product is not used to meet demand by the end of its useful lifetime in inventory. Examples of such products include foodstuffs, drugs, blood, batteries etc. This work examine Chiu's model on fixed lifetime, redefine the outdate quantity and consider the case where the reorder point is not zero.

Chiu (1995), considered the case of an approximation to the continuous review inventory model with perishable items and positive lead time and came up with the total cost function

$$EAC(Q,r) = \{(k + cQ + pES + wER)/ET\} + hOH\,.$$

where

$$ES = \int_r^{\infty} (d_L - r) f_L(d_L) d_L$$

$$ER = \int_0^{r+Q} (r + Q - d_{m+L}) f_{m+L}(d_{m+L}) d_{m+L} - \int_0^{r} (r - d_{m+L}) f_{m+L}(d_{m+L}) d_{m+L}$$

$$ET = \frac{Q - ER}{D}$$

$$OH = r - (DL) + \frac{Q}{2}$$

So that the total cost function becomes

$$EAC(Q,r) = \{k + cQ + p\int_r^{\infty}(d_L - r) f_L(d_L) d_L + w\{\int_0^{r+Q}(r + Q - d_{m+L}) f_{m+L}(d_{m+L}) -$$
$$\int_0^r (r - d_{m+L}) f_{m+L}(d_{m+L}) d_{m+L}\}\} / ET + h(r - DL + \frac{Q}{2}) \quad (1)$$

where

$ES = shortage\ quantity$

$ER = outdate\ quantity$

$ET = cycle\ time$

$OH = on\ hand\ inventory$

$Q = ordered\ quantity$

$r = reorder\ point$

$k = fixed\ ordering\ \text{cost}\ per\ order$

$c = replenishment\ \text{cost}\ per\ unit$

$p = shortage\ \text{cost}$

$h = holding\ \text{cost}\ per\ unit\ per\ unit\ time$

$w = outdate\ \text{cost}\ per\ unit$

In order to search for a minimum Chiu took the partial derivatives of (1) with respect to Q and r, setting each to zero, and obtained

$$\frac{\partial EAC}{\partial Q} = \frac{-D}{(Q - ER)^2}[1 - F_{m+L}(r + Q)](k + cQ + pES + wER) + \frac{D}{(Q - ER)}[c + wF_{m+L}(r + Q)] + \frac{h}{2} = 0 \quad (2)$$

$$\frac{\partial EAC}{\partial r} = \frac{D}{(Q - ER)^2}[F_{m+L}(r + Q) - F_{m+L}(r)](k + cQ + pES + wER) + \frac{D}{(Q - ER)}\{p[-1 + F_L(r)]$$
$$+ w[F_{m+L}(r + Q) - F_{m+L}(r)]\} + h = 0 \quad (3)$$

Equations (2) and (3) could not be solved explicitly, so he developed a solution algorithm to solve the equations for Q and r. However, Chiu reported that, “ it requires an extremely large number of iterations to complete the steps of the solution algorithm and the intolerably computational efforts makes it impossible to obtain a convergent solution”. The result for this model is of the form (Q, r) where Q is ordered whenever on hand inventory drops to r, which imply that the decision to reorder depends on the quantity of products on hand. Furthermore, Chiu obtained numerical results for the case where the reorder point is zero.

To implement the model, we obtain the partial derivatives of equation (1) with respect to the ordered quantity using Mathematical 8, equate to zero and solve the resulting equation.

$$\frac{\partial EAC}{\partial Q} = \frac{h}{2} + \frac{D(c + \frac{1}{2}(Q + r)^2 wf_{m+l})}{Q + (\inf inity)^3 \frac{f_l}{3} - \frac{1}{2}(\inf inity)^2 rf_l + r^3 \frac{f_l}{6} + \frac{r^3 f_{m+l}}{6} - \frac{(Q + r)^3}{6} f_{m+l}} -$$
$$\frac{D(1 - \frac{1}{2}(Q + r)^2 f_{m+l})(k + cQ + p(\frac{(\inf inity)^3 f_l}{3} - \frac{(\inf inity)^2 rf_l}{2} + \frac{r^3 f_l}{6}) + w(-\frac{r^3 f_{m+l}}{6} + \frac{(Q + r)^3 f_{m+l}}{6})}{(Q + \frac{(\inf inity)^3 f_l}{3} - \frac{(\inf inity)^2 rf_l}{2} + \frac{r^3 f_l}{6} + \frac{r^3 f_{m+l}}{6} - \frac{(Q + r)^3 f_{m+l}}{6})^2}$$

Going forward, we take $f(\inf inity)$ to be zero and consider the cases where $r = 0\ \ and\ \ r \neq 0$ which was not considered by Chiu.

Case 1. assume r=0 and simplifying, we have

$$\Rightarrow \frac{h}{2}+\frac{D(c+\frac{Q^2}{2}wf_{m+l})}{Q-\frac{Q^3}{6}f_{m+l}}-\frac{D(1-\frac{Q^2}{2}f_{m+l})(k+cQ+w\frac{Q^3}{6}f_{m+l})}{(Q-\frac{Q^3}{6}f_{m+l})^2}=0$$

$$\Rightarrow h(Q-\frac{Q^3f_{m+l}}{6})^2+2D(c+\frac{Q^2wf_{m+l}}{2})(Q-\frac{Q^3f_{m+l}}{6})-2D(1-\frac{Q^2f_{m+l}}{2})(k+cQ+\frac{wQ^3f_{m+l}}{6})=0$$

$$\Rightarrow hQ^2-\frac{hQ^4f_{m+l}}{3}+\frac{hQ^6f_{m+l}}{36}-\frac{DcQ^3f_{m+l}}{3}+\frac{DwQ^3f_{m+l}}{3}-\frac{DQ^5wf_{m+l}}{6}-2Dk-\frac{DwQ^3f_{m+l}}{3}+DQ^2kf_{m+l}$$
$$+DcQ^3f_{m+l}+\frac{DQ^5wf_{m+l}}{3}=0$$

so for the case $r=0$, *our polynomial will be*

$$\Rightarrow \frac{hf_{m+l}Q^6}{36}+(\frac{Dwf_{m+l}}{3}-\frac{Dwf_{m+l}}{6})Q^5-\frac{hf_{m+l}Q^4}{3}+(-\frac{Dcf_{m+l}}{3}+Dwf_{m+l}-\frac{Dwf_{m+l}}{3}+Dcf_{m+l})Q^3$$
$$+(h+Dkf_{m+l})Q^2-2Dk=0 \qquad (4)$$

We solved equation (4) using mathematical 8 and the results obtained for the ordered quantity is compared with the results obtained by Chiu and shown in table1

Table 1: comparing values of Q from chiu's algorithm (Qc) and minimization method Q_m. Constant values, c=h=d=1

s/n	w	m	k	Q_c	Q_m
1	1	1	1	0.6635	1.0453
2			5	1.5545	1.6503
3			10	1.9307	1.8724
4	1	5	1	1.3663	1.4081
5			5	2.8115	3.0313
6			10	3.6735	4.0869
7	1	10	1	1.4139	1.4142
8			5	3.1452	3.1623
9			10	4.3962	4.4721
10	5	1	1	0.4936	0.8549
11			5	0.8068	1.3580
12			10	1.4524	1.5820
13	5	5	1	1.3036	1.3965
14			5	2.5544	2.9904
15			10	3.2741	4.0010
16	5	10	1	1.4115	1.4142
17			5	3.1258	3.1623
18			10	4.3259	4.4721
19	10	1	1	0.4057	0.7474
20			5	0.6928	1.1971
21			10	0.8322	1.4121
22	10	5	1	1.2397	1.3827
23			5	2.3538	2.8858
24			10	2.9866	3.7846
25	10	10	1	1.4115	1.4142
26			5	3.1045	3.1623
27			10	4.2472	4.4721

For the total cost function, we observe that the outdate quantity obtained by Chiu, shown below

$$ER=\int_0^{r+Q}(r+Q-d_{m+L})f_{m+L}(d_{m+L})d_{m+L}-\int_0^{r}(r-d_{m+L})f_{m+L}(d_{m+L})d_{m+L}$$

Is misleading for the following reasons

(1) r is designed to satisfy the leadtime demand and not demand during M+L units of time as giving in the second term of the outdate quantity

(2) The first term calculates the outdate quantities for both the previous and current orders. We believe that outdate quantity should be a function of previous order only, since the issuing policy is oldest unit first.

We this, the outdate quantity should be $\int_0^r (r-d_m) f_m(d_m) d_m$ so that the total cost function becomes

$$EAC(Q,r) = \{k + cQ + p\int_r^\infty (d_L - r) f(d_L) d_L + w\int_0^r (r - d_m) f\ (d_m) d_m\} / ET + h(r - DL + \frac{Q}{2}) \quad (5)$$

We compare the total cost using equations (1) and (5) for the values of Q obtained in table 1.

Table 2: Comparing total cost between Chiu and the modify model. Constant values c=h=d=1.

s/n	w	m	k	Q_c	EAC_c	Q_0	EAC_0
1	1	1	1	0.6635	3.8483	1.0453	2.6920
2			5	1.5545	9.7612	1.6503	5.8656
3			10	1.9307	15.6106	1.8724	9.2921
4	1	5	1	1.3663	2.4462	1.4081	2.4169
5			5	2.8115	4.3809	3.0313	4.1824
6			10	3.6735	6.0282	4.0869	5.5287
7	1	10	1	1.4139	2.4145	1.4142	2.4142
8			5	3.1452	4.1671	3.1623	4.1623
9			10	4.3962	5.4944	4.4721	5.4721
10	5	1	1	0.4936	4.7428	0.8549	2.9342
11			5	0.8068	12.4472	1.3580	6.5980
12			10	1.4524	19.5293	1.5820	10.3508
13	5	5	1	1.3036	2.4898	1.3965	2.4210
14			5	2.5544	4.5658	2.9904	4.2025
15			10	3.2741	6.3704	4.0010	5.5701
16	5	10	1	1.4115	2.4149	1.4142	2.4142
17			5	3.1258	4.1723	3.1623	4.1622
18			10	4.3259	5.5139	4.4721	5.4721
19	10	1	1	0.4057	5.5311	0.7474	3.1502
20			5	0.6928	14.1839	1.1971	7.2405
21			10	0.8322	23.0770	1.4121	11.3121
22	10	5	1	1.2397	2.5398	1.3827	2.4261
23			5	2.3538	4.7544	2.8858	4.2300
24			10	2.9866	6.7040	3.7846	5.6354
25	10	10	1	1.4115	2.4154	1.4142	2.4142
26			5	3.1045	4.1787	3.1623	4.1622
27			10	4.2472	5.5366	4.4721	5.4721

We observed that the cost for the modify model are lower when compared with Chiu's model. The reason for this is because the outdate quantities are lower with the redefined form as shown in the table3. Now the outdate quantity can be obtained from

$$EAC(Q) = \frac{(k + cQ + wER)}{ET} + HC$$

where

$$ET = \frac{Q - ER}{D}$$

$$HC = \frac{[h(\frac{Q}{2})ET]}{ET}$$

simplifying we have

$$ER = \frac{2EAC - 2Dk - 2DcQ - hQ^2}{2Dw + 2ECA - hQ}$$

$$ER = shortage \quad quantity$$

Table 3: Comparing outdate quantities between Chiu and modify Chiu's. Constant values, c=h=d=1.

s/n	w	k	Q_c	ER_c	Q_0	ER_{mc}	
1	1	1	0.6635	0.1483	1.0453	-0.0290	
2		5	1.5545	0.7423	1.6503	0.2226	
3		10	1.9307	1.0447	1.8724	0.4445	
4	1	1	1.3663	0.0154	1.4081	0.00092	
5		5	2.8115	0.1391	3.0313	0.0013	
6		10	3.6735	0.3320	4.0869	-0.0237	
7	1	1	1.4139	0.00015	1.4142	0.000007	
8		5	3.1452	0.0042	3.1623	0.000002	
9		10	4.3962	0.0221	4.4721	-0.00007	
10	5	1	0.4936	0.0764	0.8549	-0.0217	
11		5	0.8068	0.2294	1.3580	0.0723	
12		10	1.4524	0.6662	1.5820	0.2378	
13	5	1	1.3036	0.0135	1.3965	0.0004	
14		5	2.5544	0.1021	2.9904	-0.01035	
15		10	3.2741	0.2284	4.0010	-0.0444	
16	5	1	1.4115	0.00014	1.4142	0.000003	
17		5	3.1258	0.00403	3.1623	0.00012	
18		10	4.3259	0.0204	4.4721	-0.00131	
19	10	1	0.4057	0.0732	0.7474	0.01622	
20		5	0.6928	0.1633	1.1971	0.05425	
21		10	0.8322	0.2457	1.4121	0.0844	
22	10	1	1.2397	0.0118	1.3827	-0.00004	
23		5	2.3538	0.0786	2.8858	0.0128	
24		10	2.9866	0.1693	3.7846	0.04354	
25	10	1	1.4115	0.00014	1.4142	0.000002	
26		5	3.1045	0.00391	3.1623	-0.00020	
27		10	4.2472	0.0185	4.4721	-0.0019	

When the outdate quantity is negative, it imply that no outdating occur from that order, but there will be shortages.

Case 2: $when \quad r \neq 0$

Next we assume $r \neq 0$ and simplify.

$$\frac{h}{2}+\frac{D(c+\frac{(Q+r)^2}{2}wf_{m+l}}{Q+r^3\frac{f_l}{6}+r^3\frac{f_{m+l}}{6}-(Q+r)^3\frac{f_{m+l}}{6}}-\frac{D(1-\frac{(Q+r)^2}{2}f_{m+l})(k+cQ+p\frac{r^3 f_l}{6})+w(\frac{-r^3 f_{m+l}}{6}+\frac{(Q+r)^3 f_{m+l}}{6})}{(Q+\frac{r^3 f_l}{6}+\frac{r^3 f_{m+l}}{6}-\frac{(Q+r)^3 f_{m+l}}{6})^2}=0 \tag{6}$$

Simplifying equation (6) we have

$$h\{Q+\frac{r^3 f_l}{6}+r^3\frac{f_{m+l}}{6}-\frac{(Q+r)^3 f_{m+l}}{6}\}^2+2\{D(c+\frac{(Q+r)^2 wf_{m+l}}{2})(Q+\frac{r^3 f_l}{6}+\frac{r^3 f_{m+l}}{6}-\frac{(Q+r)^3 f_{m+l}}{6}\}$$
$$-2D\{(1-\frac{(Q+r)^2 f_{m+l}}{2})(k+cQ+\frac{pr^3 f_l}{6}+w(-\frac{r^3 f_{m+l}}{6}+\frac{(Q+r)^3 f_{m+l}}{6}))=0 \tag{7}$$

Expanding and further simplifying gives equation (8) and (9)

$$h\{Q+\frac{r^3 f_l}{6}+\frac{r^3 f_{m+l}}{6}-\frac{Q^3 f_{m+l}}{6}-\frac{3Q^2 r f_{m+l}}{6}-\frac{3Qr^2 f_{m+l}}{6}-\frac{r^3 f_{m+l}}{6}\}^2+$$
$$2\{(Dc+\frac{DQ^2 w f_{m+l}}{2}+\frac{D2Qrwf_{m+l}}{2}+Dr^2 w f_{m+l})(Q+\frac{r^3 f_l}{6}+\frac{r^3 f_{m+l}}{6}-\frac{Q^3 f_{m+l}}{6}-\frac{3Q^2 r f_{m+l}}{6}-\frac{3Qr^2 f_{m+l}}{6}-\frac{r^3 f_{m+l}}{6})\}$$
$$-2D\{(1-\frac{Q^2 f_{m+l}}{2}-\frac{2Qrf_{m+l}}{2}-\frac{r^2 f_{m+l}}{2})(k+cQ+\frac{pr^3 f_l}{6}-\frac{wr^3 f_{m+l}}{6}+\frac{wQ^3 f_{m+l}}{6}+\frac{3wQ^2 r f_{m+l}}{6}$$
$$+\frac{3wQr^2 f_{m+l}}{6}+\frac{wr^3 f_{m+l}}{6})\}=0 \qquad (8)$$

$$\frac{h(f_{m+l})^2}{36}Q^6+\{\frac{rh(f_{m+l})^2}{6}-\frac{Dw(f_{m+l})^2}{12}+\frac{Dw(f_{m+l})^2}{6}\}Q^5+\{h(\frac{-f_{m+l}}{3}+\frac{r^2(f_{m+l})^2}{4})+(\frac{-Dwr(f_{m+l})^2}{4}-$$
$$\frac{Dwr(f_{m+l})^2}{6})+\frac{Dwr(f_{m+l})^2}{2}\}Q^4+\{h(-rf_{m+l}-\frac{r^3 f_l f_{m+l}}{18}+\frac{r^3(f_{m+l})^2}{2})+(\frac{-Dcf_{m+l}}{6}+\frac{Dwf_{m+l}}{2}-3Dr^2 w(f_{m+l})^2-$$
$$\frac{Dr^2 w(f_{m+l})^2}{6}-Dr^2 w(f_{m+l})^2)+(\frac{-Dwf_{m+l}}{3}Dcf_{m+l}+\frac{Dwr^2(f_{m+l})^2}{2}+Dr^2 w(f_{m+l})^2+\frac{Dwr^2(f_{m+l})^2}{6}\}Q^3+$$
$$\{h(1-r^2 f_{m+l}-\frac{r^4 f_l f_{m+l}}{6}+\frac{r^4(f_{m+l})^2}{4})+(-Dwrf_{m+l}+Dkf_{m+l}+\frac{Dpr^3 f_l f_{m+l}}{6}+Drcf_{m+l}+\frac{Dr^3 w(f_{m+l})^2}{2}+$$
$$(\frac{-Dcrf_{m+l}}{2}+\frac{Dwr^3 f_l f_{m+l}}{12}Drwf_{m+l}-\frac{Dr^3 w(f_{m+l})^2}{2}-\frac{Dr^3 w(f_{m+l})^2}{2}\}Q^2+\{h(\frac{r^3 f_l}{3}+\frac{r^3 f_l f_{m+l}}{12}-\frac{r^5 f_l fm+l}{12})+$$
$$(-2Dc-Dwr^2 f_{m+l}+2Drkf_{m+l}+\frac{Dr^4 p f_l f_{m+l}}{3}+Dcr^2 f_{m+l}+\frac{Dwr^4(f_{m+l})^2}{2})+(Dc-\frac{Dcr^2 f_{m+l}}{2}+2Dwr^4 f_l f_{m+l}+$$
$$Dr^2 w f_{m+l}-\frac{Dwr^4(f_{m+l})^2}{6})\}Q+\{\frac{r^6 h(f_l)^2}{36}+\frac{Dwr^5 f_l f_{m+l}}{6}-2Dk-\frac{Dpr^3 f_l}{3}+Dkr^2 f_{m+l}+\frac{Dpr^5 f_l f_{m+l}}{6}\}=0 \qquad (9)$$

We use mathematical 8 to solve equation (9), table4 shows values of Q obtained for some values of r.

Table 4: Values of Q when $r \neq 0$. Constant parameters c=h=d=1.

s/n	w	m	k	r	Q
1	1	5	1	10	0.3108
2				1	1.9872
3				5	1.9417
4	1	10	1	10	1.9981
5				1	1.9999
6				5	2.0001
7	5	5	1	10	11.5588
8				1	1.9762
9				5	1.7264
10	5	10	1	10	1.9978
11				1	1.9998
12				5	2.0000
13				1	0.4734
14				5	5.7674
15	10	5	1	10	11.2302
16				1	1.9628
17				5	1.4717
18	10	10	1	10	1.9976
19				1	1.9997
20				5	1.9999

Where there are no past records to determine the value of r, we differentiate (1) with respect to the reorder point and obtain an equation for the reorder point.

$$\frac{\partial EAC(Q,r)}{\partial r} = h + \frac{p(-\frac{1}{2}\inf inity^2 f_l + \frac{r^2 f_l}{2}) + w(-\frac{1}{2}r^2 f_{m+l} + \frac{1}{2}(Q+r)^2 f_{m+l})}{Q + \inf inity^3 \frac{f_l}{3} - \frac{1}{2}\inf inity^2 r f_l + \frac{r^3 f_l}{6} + D\{\frac{r^3 f_{m+l}}{6} + \frac{(Q+r)^3}{6} f_{m+l}} -$$

$$\frac{(-\frac{1}{2}\inf inity^2 f_l + \frac{r^2 f_l}{2} + D\{\frac{r^2 f_{m+l}}{2} + \frac{(Q+r)^2 f_{m+l}}{2}\})(k + cQ + p(\frac{\inf inity^3 f_l}{3} - \frac{\inf inity^2 r f_l}{2} + \frac{r^3 f_l}{6}) + w(\frac{-r^3 f_{m+l}}{6} + (Q+r)^3 f_{m+l}}{(Q + \inf inity^2 \frac{f_l}{3} - \frac{1}{2}\inf inity^2 r f_l + \frac{r^3 f_l}{6} + D\{\frac{r^3 f_{m+l}}{6} + \frac{(Q+r)^3 f_{m+l}}{6})^2} \quad (10)$$

Which gives

$$h + \frac{p(\frac{r^2 f_l}{2}) + w(\frac{-1}{2}r^2 f_{m+l} + \frac{1}{2}(Q+r)^2 f_{m+l})}{Q + \frac{r^3 f_l}{6} + D\{\frac{r^3 f_{m+l}}{6} + \frac{(Q+r)^3 f_{m+l}}{6}\}} -$$

$$\frac{\frac{r^2 f_l}{2} + D\{\frac{r^2 f_{m+l}}{2} + \frac{(Q+r)^2 f_{m+l}}{2}\}(k + cQ + p(\frac{r^3 f_l}{6})) + w(\frac{-r^3 f_{m+l}}{6} + (Q+r)^3 f_{m+l}}{(Q + \frac{r^3 f_l}{6} + D\{\frac{r^3 f_{m+l}}{6} + \frac{(Q+r)^3 f_{m+l}}{6})^2} = 0 \quad (11)$$

Exapanding and simplifying (11) yields (12)

$$(\frac{hQ^2 f^2}{4} - \frac{DQf^2 p}{4} + \frac{Df^2 Qp}{6})r^4 + (\frac{hQ^3 f^2}{4} + \frac{hQ^3 f^2}{4} - \frac{DQ^2 f^2 p}{12} - \frac{Df^2 wQ^2}{2} + \frac{DQ^2 f^2 p}{12})r^3$$
$$+ (\frac{hQ^4 f^2}{12} - \frac{hQ^2 f}{2} + \frac{hQ^4 f^2}{4} - \frac{hQ^2 f}{2} + \frac{hf^2 Q^4}{12} + \frac{DfpQ}{2} - \frac{Dpf^2 Q^3}{12} - \frac{Dwf^2 Q^3}{4} - \frac{Dwf^2 Q^3}{2} +$$
$$\frac{wfQ}{2})r^2 + (\frac{hQ^5 f^2}{12} - \frac{2hfQ^3}{2} + \frac{hf^2 Q^5}{12} - \frac{Dwf^2 Q^4}{4} + Q^2 Dfw - \frac{Dwf^2 Q^4}{6} + fQDk + DcfQ^2 +$$
$$\frac{wfQ^2}{2})r + hQ^2 - \frac{2hfQ^4}{6} + \frac{hf^2 Q^6}{36} + \frac{DwfQ^3}{2} - \frac{Dwf^2 Q^5}{12} + \frac{DQ^2 fk}{2} + \frac{DcQ^3 f}{2} + \frac{wfQ^3}{6} = 0 \quad (12)$$

We can use equations (9) and (12) to obtain values of Q and r

1. First assume r =0 and use (9) to obtain Q.
2. The value of Q obtained is then used in (12) to obtain the value of r.

Observe that for the minimization problem, the resulting polynomial have a number of roots. Deciding which of the roots is the order quantity, we follow the steps below.

Step 1: Eliminate all the complex roots, as the order quantity cannot be represented by a Complex number, but by a real number.

Step 2: Eliminate all negative roots, since the order quantity cannot be a negative number.

Step 3: When there is only one positive root, its taken as the value.

Step 4: If there are more than one positive root, we pick the minimum of the positive roots as our order quantity, since higher values may increase the outdating quantity.

Step 5: If zero is the only non-negative and non-complex root of the polynomial, we use the aspiration Level criterion to determine the order quantity. The aspiration level is the degree/level of Performance (determined by past experience) that a firm desires to attain or feel it can achieve. For details on aspiration level criterion see Vahid *et al*. (1992).

Conclusion

With simple minimization, the model obtained by Chiu can be solved. This will reduce the uncertainty associated with Chiu's algorithm. The results obtained using the modify method, shows that the total cost for the model is generally lower for the redefined form when compared with results obtained by Chiu, the reason for this is because the outdate quantities are lower for the redefined form. The ordered quantities when $r \neq 0$ were also obtained for some values of r.

Reference

1. Chiu HN. An approximation to the continuous review inventory model with perishable items and lead times. European journal of operational research 1995;87:93-108.
2. Vahid *et al*. An aspiration level interactive model for multiple criteria decision marking. Journal of Computers Operations Research 1992;19(7):671-681.
3. Schmidt CP, Nahmias S. (S-1, S) Policies for Perishable Inventory. Management Science 1985;31(6):719-728.
4. Omosigho SE. Determination of Outdate and Shortage Quantities in the Inventory Problem with fixed Lifetime. International Journal Computer Math 2002;79(11):1169-1177.
5. Liu L, Lian Z. (s,S) Continuous Review Models for Products with Fixed Lifetime. Operations Research 1999;47(1):150-158.
6. Olsson F, Tydesjo P. Inventory Problems with Perishable items: Fixed Lifetimes and Backlogging. European Journal of Operational Research 2010:202:131-137.
7. Shen Z, Dessouky M, Ordonez F. Perishable inventory management system with a minimum volume constraint. Journal of the Operational Research Society 2011;62:2063-2082.