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6.5 Temperatures in the prism involving A- function of several variables

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Abstract
The aim of this section is to obtain the temperatures in the prism involving A- function of several variables.

Keywords: A-function of several variables, temperature and prism

1. Introduction

Gautam and Goyal (1981, 82) characterized the multivariable A-function, which is a generalization of multivariable H - function of Shrivastava and Panda (1976).The definition of multivariable A-function runs as follows:

$$A\,[Z_1, \ldots, Z_r] = A^{m,n;m_1,n_1;\ldots;m_r,n_r}_{p,q;p_1,q_1;\ldots;p_r,q_r}$$

$$\left[\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix}\left|\begin{matrix}(a_j;\ A_j', \ldots, A_j^{(r)})_{1,p}\ ;\ (c_j';C_j'')_{1,p_1}; \ldots ; (c_j^{(r)}, C_j^{(r)})_{1,p_r} \\ (b_j\ ;\ B_j', \ldots, B_j^{(r)})_{1,q};\ (d_j';D_j'')_{1,q_1}; \ldots ; (d_j^{(r)}, D_j^{(r)})_{1,q_r}\end{matrix}\right.\right]$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1} \ldots \int_{L_r} \emptyset_1(s_1) \ldots \emptyset_r(s_r)\emptyset(s_1, \ldots, s_r) z_1^{s_1} \ldots z_r^{s_r}\, ds_1 \ldots ds_r, \qquad (1.1)$$

Where ω = √ −1;

$$\theta_i(s_i) = \frac{\prod_{j=1}^{m_i} \Gamma\left(d_j^{(i)} - D_j^{(i)} s_i\right) \prod_{j=1}^{n_i} \Gamma\left(1 - c_j^{(i)} + C_j^{(i)} s_i\right)}{\prod_{j=m_i+1}^{q_i} \Gamma\left(1 - d_j^{(i)} + D_j^{(i)} s_i\right) \prod_{j=n_i+1}^{p_i} \Gamma\left(c_j^{(i)} - C_j^{(i)} s_i\right)}; \forall\ i\{1, \ldots, r\} \qquad (1.2)$$

$$\Phi\,(s_1, \ldots, s_r) = \frac{\prod_{j=1}^{n} \Gamma(1 - a_j + \sum_{i=1}^{r} A_j^{(i)} s_i) \prod_{j=1}^{m} \Gamma(b_j - \sum_{i=1}^{r} B_j^{(i)} s_i)}{\prod_{j=n+1}^{p} \Gamma(a_j - \sum_{i=1}^{r} A_j^{(i)} s_i) \prod_{j=1}^{q} \Gamma(1 - b_j + \sum_{i=1}^{r} B_j^{(i)} s_i)}\,. \qquad (1.3)$$

Here m , n , p , q , m_i , n_i , p_i , and q_i (i=1,….,r) are non-negative integers and all a'_js, b'_js, $d_j^{(i)}$s, $c_j^{(i)}$s, $A_j^{(i)'}$s, $B_j^{(i)'}$s are complex numbers.The multiple integral defining the A-function of r - variables converges absolutely if

$$|\arg\,(\gamma_i)\, z_k| < \frac{\pi}{2}\eta_i,\ \xi_i^* = 0\,, \eta_i > 0 \qquad (1.4)$$

$$\gamma_i = \sum_{j=1}^{p}\{A_j^{(i)}\}^{A_j^{(i)}} \sum_{j=1}^{q}\{B_j^{(i)}\}^{-B_j^{(i)}} \sum_{j=1}^{q_i}\{D_j^{(i)}\}^{D_j^{(i)}} \sum_{j=1}^{p_i}\{C_j^{(i)}\}^{-C_j^{(i)}},$$
$$\forall i \in (i = 1,\ldots,r\}; \qquad (1.5)$$

$$\delta_i^* = I_m\left[\sum_{j=1}^{p} A_j^{(i)} - \sum_{j=1}^{q} B_j^{(i)} + \sum_{j=1}^{p_i} D_j^{(i)} - \sum_{j=1}^{q_i} C_j^{(i)}\right], \forall i \in (i = 1,\ldots,r\}; \qquad (1.6)$$

$\eta_i = Re\left[\sum_{j=1}^{n} A_j^{(i)} - \sum_{j=n+1}^{p} A_j^{(i)} + \sum_{j=1}^{m} B_j^{(i)} - \sum_{j=m+1}^{q} B_j^{(i)} + \sum_{j=1}^{m_i} D_j^{(i)} - \sum_{j=m_i+1}^{q_i} D_j^{(i)} + \sum_{j=1}^{n_i} C_j^{(i)} - \sum_{j=n_i+1}^{p_i} C_j^{(i)}\right], \forall i \in$ (i = 1,…,r}; (1.7)

If we take A_j's, B_j's, C_j's and D_j's as real and positive and m = 0, the A-function reduces to multivariable H -function of Shrivastava and Panda (1976),We are utilizing the multivariable A-function characterized in the accompanying brief structure all through the content.

$$A[z_1,\ldots,z_r] = A_{p,q;N_r}^{m,n;M_r}\left[\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix}\middle| \begin{matrix} P:P_r^{(r)} \\ Q:Q_r^{(r)} \end{matrix}\right] = \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r} \emptyset_1(s_1)\ldots\emptyset_r(s_r)\emptyset(s_1,\ldots,s_r) z_1^{s_1}\ldots z_r^{s_r}\, ds_1\ldots ds_r \quad (1.8)$$

Where $\omega = \sqrt{-1}$;

From Gradshteyn

$$\int_0^L (\sin\pi x/L)^{\omega-1} \sin\pi x/L \, dx = \frac{L \sin 1/2n\pi\, \Gamma(\omega)}{2^{\omega-1}\Gamma\{1/2(\omega \pm n+1)\}}, \quad (1.9)$$

Where n is any integer and $\omega > 0$.

2. Formulation of the problem

All four faces of an infinitely long rectangular prism, formed by the planes x = 0, x =ρ, y = 0 and y =σ, are kept at temperature zero. Let the initial temperature distribution be p(x, y), and derive this expression for the temperature c (x, y, t) in the prism is given by [1] as follows:

$$C(x, y, t) = \sum_{r=1}^{\infty}\sum_{s=1}^{\infty} A_{rs} \exp[-\pi^2 kt (\tfrac{s^2}{p^2} + \tfrac{r^2}{\sigma^2})] \sin\frac{r\pi x}{\rho} \sin\frac{s\pi y}{\sigma}, \quad (2.1)$$

Where

$$A_{r,s} = \frac{4}{\rho\sigma}\int_0^{\sigma}\int_0^{\rho} p(x,y) \sin\frac{r\pi x}{\rho} \sin\frac{s\pi y}{\sigma}\, dx\, dy. \quad (2.2)$$

3. Solution in terms of A- function

Consider

$$P(x, y) = (\sin\frac{r\pi x}{\rho})^{\gamma-1}(\sin\frac{s\pi y}{\sigma})^{\delta-1} A_{p,q:(p_1,q_1);\ldots\ldots;(p_r,q_r)}^{0,n:(m_1,n_1);\ldots\ldots;\ldots\ldots;(m_r,n_r)}\left[\begin{matrix} z_1 (\sin\frac{r\pi x}{\rho})^{\omega_1}(\sin\frac{s\pi y}{\sigma})^{\mu_1} \\ \vdots \\ z_r(\sin\frac{r\pi x}{\rho})^{\omega_r}(\sin\frac{s\pi y}{\sigma})^{\omega_r}\end{matrix}\right] \quad (3.1)$$

Where $A\left[\begin{matrix} z_1 \\ \vdots \\ z_r\end{matrix}\right]$ is the A-function of several variables.

Combining (3.1) and (2.1), making use of the definition of A- function changing the order of integration, after using the Mellin bernes integral,we arrive at

$$A_{r,s} = 2^{4-\gamma-\delta} \sin\frac{s\pi}{2} \sin\frac{r\pi}{2} A_{p+2,q+2:(p_1,q_1);\ldots\ldots;(p_r,q_r)}^{0,n+2:(m_1,n_1);\ldots\ldots;\ldots\ldots;(m_r,n_r)}\left[\begin{matrix} z_1 2^{-\omega_1-\mu_1} \\ \vdots \\ z_r 2^{-\omega_r-\mu_r}\end{matrix}\middle| \begin{matrix}(1-\gamma,\omega_1,\ldots\ldots,\omega_r),(1-\delta,\mu_1,\ldots\ldots,\mu_r),\ldots\ldots,\ldots\ldots \\ \ldots\ldots,(\frac{1}{2}-\frac{\gamma}{2}\pm\frac{s}{2},\frac{\omega}{2})(\frac{1}{2}-\frac{\delta}{2}\pm\frac{r}{2},\frac{\mu}{2}):\ldots\ldots\ldots\ldots\end{matrix}\right], \quad (3.2)$$

provided that $\omega \geq 0, \mu \geq 0, Re(\gamma) > 0, Re(\delta) > 0$

Putting the value of $A_{r,s}$ from (3.2) in (2.1), we get following required solution of the problem:

$$C(x,y,t) = \sum_{r=1}^{\infty}\sum_{s=1}^{\infty} 2^{4-\gamma-\delta} \exp[-\pi^2 kt (\tfrac{s^2}{p^2} + \tfrac{r^2}{\sigma^2})] \sin\frac{r\pi x}{\rho} \sin\frac{s\pi y}{\sigma},$$

$$A_{p+2,q+2:(p_1,q_1);\ldots\ldots;(p_r,q_r)}^{0,n+2:(m_1,n_1);\ldots\ldots;\ldots\ldots;(m_r,n_r)}$$

$$\left[\begin{matrix} z_1 2^{-\omega_1-\mu_1} \\ \vdots \\ z_r 2^{-\omega_r-\mu_r}\end{matrix}\middle| \begin{matrix}(1-\gamma,\omega_1,\ldots\ldots,\omega_r),(1-\delta,\mu_1,\ldots\ldots,\mu_r),\ldots\ldots,\ldots\ldots \\ \ldots\ldots,(\frac{1}{2}-\frac{\gamma}{2}\pm\frac{s}{2},\frac{\omega}{2})(\frac{1}{2}-\frac{\delta}{2}\pm\frac{r}{2},\frac{\mu}{2}):\ldots\ldots\ldots\ldots\end{matrix}\right] \sin\frac{r\pi x}{\rho} \sin\frac{s\pi y}{\sigma}, \quad (3.3)$$

provided that $\omega \geq 0, \mu \geq 0, Re(\gamma) > 0, Re(\delta) > 0$

4. Special Cases

On specializing the parameters in (3.3), we get the following result in terms of A – function, which is a result given by shrivastava [69, p. 71(6)]:

C(x,y,t) = $\sum_{r=1}^{\infty}\sum_{s=1}^{\infty} 2^{4-\gamma-\delta}$ exp[-π^2 kt $(\frac{s^2}{p^2}+\frac{r^2}{\sigma^2})$] sin $\frac{r\pi x}{\rho}$ sin $\frac{s\pi y}{\sigma}$, $H^{m,n+2:(m_1,n_1);\ldots\ldots;\ldots\ldots;(m_r,n_r)}_{p+2,q+4:(p_1,q_1);\ldots\ldots;(p_r,q_r)}$

$$\left[\begin{bmatrix} z_1 2^{-\omega_1-\mu_1} \\ \vdots \\ z_r 2^{-\omega_r-\mu_r} \end{bmatrix} \left| \begin{matrix} (1-\gamma,\omega_1,\ldots\ldots,\omega_r),(1-\delta,\mu_1,\ldots\ldots,\mu_r),(a_j,\alpha_j)_{1,p}\ldots\ldots,\ldots\ldots \\ \ldots\ldots,(b_j,\beta_j)_{1,q},(\frac{1}{2}-\frac{\gamma}{2}\pm\frac{s}{2},\frac{\omega}{2})(\frac{1}{2}-\frac{\delta}{2}\pm\frac{r}{2},\frac{\mu}{2}):\ldots\ldots, \end{matrix} \right.\right] \sin\frac{r\pi x}{\rho}\sin\frac{s\pi y}{\sigma}, \quad (4.1)$$

Provided that |arg z| $< (\frac{1}{2})\pi A, \omega \geq 0, \mu \geq O, Re(\gamma) > 0, Re(\delta) > 0, where$ A is given as:

$$\sum_{j=1}^{p} A_j^{(i)} - \sum_{j=n+1}^{q} B_j^{(i)} + \sum_{j=1}^{p_i} D_j^{(i)} - \sum_{j=m+1}^{q_i} C_j^{(i)} = A > 0,$$

5. Conclusion

Specializing the parameters of the multivariable I – function, we can obtain a large number of results involving various special functions of one and several variables useful in Mathematics analysis, Applied Mathematics, Physics and Mechanics . The result derived in this paper is of general character and may prove to be useful in several interesting situations appearing in the literature of sciences.

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