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Umesh Kumar Gupta
Assistant Professor, Department of Mathematics, MGPG College, Gorakhpur, Uttar Pradesh, India

Subhash Kumar Sharma
Assistant Professor, Department Of Physics, MGPG College, Gorakhpur, Uttar Pradesh, India

Corresponding Author:
Umesh Kumar Gupta
Assistant Professor, Department of Mathematics, MGPG College, Gorakhpur, Uttar Pradesh, India

Determination of eigenvalues of linear electrical circuits with characteristics equation

Umesh Kumar Gupta and Subhash Kumar Sharma

Abstract
In this paper eigenvalues in the time-domain, we can say, indirectly the poles of system functions in the s-domain, are used to determine the dynamics of the systems in circuit analysis of linear electrical circuit. This paper proposes an efficient method to determine the eigenvalues of linear electrical circuits. Which is based on using the characteristic equation obtained by applying nodal and mesh analysis, conventional methods whose applications are simpler than the state-space representation. Here the examples of active and passive electrical circuits are given to describe the method. It has been shown that the characteristic equation of the linear electrical circuit is independent of the method used for its analysis. Three well-known methods of the analysis of linear electrical circuits: state space method, mesh method and node method have been analysed.

Keywords: Eigenvalues; linear electrical circuits; mesh analysis; nodal analysis

1. Introduction

We know that the eigenvalues are important parameters in both circuit analysis and control systems courses. They refer to the transient-response of circuits and determine the dynamics of circuits. The eigenvalues also refer to poles of transfer functions of circuits, in the Laplace (s) domain. In electrical engineering education, the eigenvalues, indirectly system poles, are particularly useful in the design of feedback systems in which relative stability and other complete response characteristics are important functions. The stability concept is necessary for the dynamics of circuits. For stability, all eigenvalues of circuits must be located on the left hand side of the complex s plane. In general, the eigenvalues of circuits are obtained from the coefficient matrix relating to state equations, in the time-domain, in circuit analysis courses. But the state-space representation has some restrictions in obtaining the system equations. Moreover, it is not very suitable for computer applications. In this paper, the eigenvalues are determined according to nodal and mesh analysis methods in the Laplace domain, in which the system equations are easily obtained. Haley [1] introduced a modifi cation-decomposition (MD) method to compute linear system transfer function poles and zeros. Hauksdottir and Hjaltadottir [2] gave closed form expressions, for real and/or complex eigenvalues, of transfer function responses. Hagiwara [3] used the eigenvalue approach to calculate the zeros of the system. A global feedback controller was designed to assign all parallel D-spectrum eigenvalues (PD eigenvalues) of the closed-loop Takage-Sugeno (T-S) fuzzy system in [4].

A dynamical system is called positive if its trajectory starting from any nonnegative initial state remains forever in the positive orthant for all nonnegative inputs. An overview of state of the art in positive theory is given in the monographs [3, 8]. Variety of models having positive behaviour can be found in engineering, especially in electrical circuits [15], economics, social sciences, biology and medicine, etc. [3, 9]. The analysis of linear systems and electrical circuits has been addressed in [1, 2, 15-18]. The positive electrical circuits have been analysed in [4-7, 15]. The constructability and observability of standard and positive electrical circuits has been addressed in [5], the decoupling zeros in [6] and minimal-phase positive electrical circuits. A new class of normal positive linear electrical circuits has been introduced in [7]. Positive linear systems with different fractional orders in [10, 11] and positive unstable electrical circuits in [12]. Zeroing of state variables in descriptor electrical circuits has been addressed in [13].

In this paper the problem of calculation of the characteristic equations of the standard, positive and descriptor linear electrical circuits will be analyzed.

The following notation will be used: $\Re$ - the set of real numbers, $\Re^{nxm}$ - the set of n× m real matrices, $\Re^{nxm}$ - the set of n× m real matrices with nonnegative entries and

$\Re_+^n = \Re_+^{n\times 1}$, M_n - the set of $n \times n$ Metzler matrices

M_n - the set of n × n Metzler matrices (real matrices with nonnegative off-diagonal entries),

I_n - the n× n identity matrix.

2. Characteristic Equations of the Electrical Circuits

2.1 State Space Method

Consider the linear continuous-time electrical circuit described by the state equation

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

Where $x(t)\in\Re^n$, $u(t)\in\Re^m$ are the state and input vectors and $A \in\Re^{nxn}$, $B\in\Re^{nxm}$. It is well-known [1, 2, 16-20] that any standard linear electrical circuit composed of resistors, coils, capacitors and voltage (current) sources can be described by the equation (1). Usually as the state variables $x_1(t)\ldots x_n(t)$ (the components of the vector x(t)) the currents in the coils and voltages on the capacitors are chosen.

Here the electrical Circuit equation (1) is called (internally) positive if

$x(t) \in \Re_+^n$, for any initial condition $x(0) \in \Re_+^n$ and every $u(t) \in \Re_+^m$, $t \in [0,+\infty)$.

The electrical circuit Equation (1) is positive if and only if

$$A \in M_n,\ B \in \Re_+^{n\times m}. \tag{2}$$

The positive electrical circuit Equation (1) for u(t) = 0 is called asymptotically stable if

$$\lim_{t\to\infty} x(t) = 0 \text{ for all } x(0) \in \Re_+^n \tag{3}$$

The positive electrical circuit equation (1) is asymptotically stable if and only if

$$\operatorname{Re}\lambda_k < 0 \text{ for } k = 1,\ldots,n,$$

where λ_k is the eigenvalue of the matrix $A \in M_n$ and

$$\det[I_n\lambda - A] = (\lambda-\lambda_1)(\lambda-\lambda_2)\ldots(\lambda-\lambda_n). \tag{4}$$

2.2 Mesh Method

Any linear electrical circuit composed of resistors, coils, capacitors and voltage (current) sources in transient states can be also analysed by the use of the mesh method [2, 15]. Using the mesh method and the Laplace transform we can describe the electrical circuit in transient states by the equation

$$Z(s)\,X(s) = E(s) \tag{5(a)}$$

where $X(s) = \mathcal{L}[x(t)] = \int_0^\infty x(t)e^{-st}dt$ ($\mathcal{L}$ is the Laplace operator),

$$Z(s) = \begin{bmatrix} Z_{11}(s) & Z_{12}(s) & \cdots & Z_{1n}(s) \\ Z_{21}(s) & Z_{22}(s) & \cdots & Z_{2n}(s) \\ \vdots & \vdots & \ddots & \vdots \\ Z_{n1}(s) & Z_{n2}(s) & \cdots & Z_{nn}(s) \end{bmatrix},\ E(s) = \begin{bmatrix} E_1(s) \\ E_2(s) \\ \vdots \\ E_n(s) \end{bmatrix}. \tag{5(b)}$$

For example for the electrical circuit with given resistances R1, R2, R3, inductances L1, L2 and voltage sources e_1, e_2 shown in Fig(1) using the mesh method we obtain the following

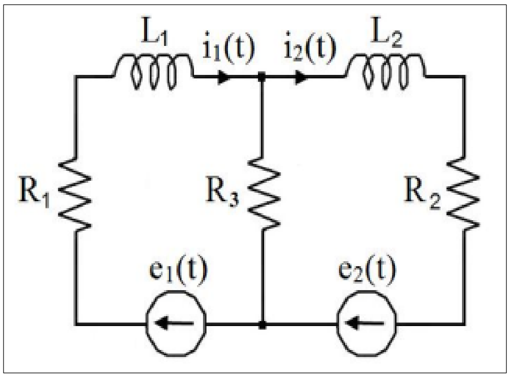

Fig 1: Electrical Circuit

Using the Kirchhoff's laws for the electrical circuit we obtain the equations

$$e_1 = R_1 i_1 + L_1 \frac{di_1}{dt} + R_3(i_1 - i_2)$$
$$e_2 = R_2 i_2 + L_2 \frac{di_2}{dt} + R_3(i_2 - i_1) \quad (6)$$

Which can be written in the form

$$\frac{d}{dt}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = A_1 \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} + B_1 \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \quad 7(a)$$

Where

$$A_1 = \begin{bmatrix} -\frac{R_1 + R_3}{L_1} & \frac{R_3}{L_1} \\ \frac{R_3}{L_2} & -\frac{R_2 + R_3}{L_2} \end{bmatrix}, B_1 = \begin{bmatrix} \frac{1}{L_1} & 0 \\ 0 & \frac{1}{L_2} \end{bmatrix}. \quad 7(b)$$

The electrical circuit is positive since $A1 \in M_2$ and $B_1 \in \Re^{2x2}$. The characteristic equation of the electrical circuit has the form

$$\det[I_2 s - A_1] = \begin{vmatrix} s + \frac{R_1 + R_3}{L_1} & -\frac{R_3}{L_1} \\ -\frac{R_3}{L_2} & s + \frac{R_2 + R_3}{L_2} \end{vmatrix}$$
$$= s^2 + \left(\frac{R_1 + R_3}{L_1} + \frac{R_2 + R_3}{L_2}\right)s + \frac{R_1(R_2 + R_3) + R_2 R_3}{L_1 L_2} = 0 \quad (8)$$

Using the mesh method and the Laplace transform we obtain

$$\begin{bmatrix} R_1 + R_3 + sL_1 & -R_3 \\ -R_3 & R_2 + R_3 + sL_2 \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} E_1(s) \\ E_2(s) \end{bmatrix} \quad 9(a)$$

where $I_k(s) = \mathcal{L}[i_k(t)]$, $E_k(s) = \mathcal{L}[e_k(t)]$, $k = 1,2$

In this case we have

$$Z(s)=\begin{bmatrix} R_1+R_3+sL_1 & -R_3 \\ -R_3 & R_2+R_3+sL_2 \end{bmatrix},\ X(s)=\begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix},\ E(s)=\begin{bmatrix} E_1(s) \\ E_2(s) \end{bmatrix} \quad 9(b)$$

Note that

$$\det Z(s)=\begin{vmatrix} R_1+R_3+sL_1 & -R_3 \\ -R_3 & R_2+R_3+sL_2 \end{vmatrix}$$
$$=L_1L_2s^2+[(R_1+R_3)L_2+(R_2+R_3)L_1]s+R_1(R_2+R_3)+R_2R_3 \quad (10)$$

And after multiplication by 1 /L_1 L_2 we obtain

$$\det Z(s)=L_1L_2\det[I_2s-A_1] \quad (11)$$

From (11) we have the following conclusion

It means we can say that in state space method the characteristic equation (8) of the electrical circuit can be also obtained by computation of the determinant of the matrix Z(s) in the mesh method.

2.3 Node method

Any linear electrical circuit composed of resistors, coils, capacitors and voltage (current) sources in transient states can be also analyzed by the use of the node method. Using the node method and the Laplace transform we can describe the electrical circuit in transient states by the equation [2, 15].

Y (s)V(s) =I_z(s) 12(a)

Where

$$Y(s)=\begin{bmatrix} Y_{11}(s) & Y_{12}(s) & \cdots & Y_{1q}(s) \\ Y_{21}(s) & Y_{22}(s) & \cdots & Y_{2q}(s) \\ \vdots & \vdots & \ddots & \vdots \\ Y_{q1}(s) & Y_{q2}(s) & \cdots & Y_{qq}(s) \end{bmatrix},\ V(s)=\begin{bmatrix} V_1(s) \\ V_2(s) \\ \vdots \\ V_q(s) \end{bmatrix},\ I_z(s)=\begin{bmatrix} I_{z1}(s) \\ I_{z2}(s) \\ \vdots \\ I_{zq}(s) \end{bmatrix}, \quad 12(b)$$

q is the number of linearly independent nodes, Y_{ij} (s) and V_i(s), i, j =1,...,q are Laplace transforms of conductances and current sources of the electrical circuit, respectively

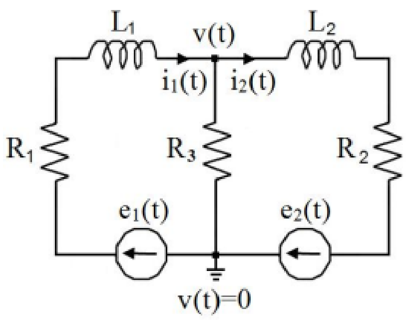

Fig 2: Electrical Circuit

For example for the electrical circuit shown in Fig.(2) using the node method we obtain

$$Y(s)V(s)=I_z(s) \quad 13(a)$$

$V(s)=\mathcal{L}[v(t)]$, $E_k(s)=\mathcal{L}[e_k(t)]$, $k=1,2$ and

$$Y(s)=Y_{11}(s)=\frac{1}{R_1+sL_1}+\frac{1}{R_2+sL_2}+\frac{1}{R_3}, \quad 13(b)$$

$$I_z(s) = -\frac{E_1(s)}{R_1 + sL_1} + \frac{E_2(s)}{R_2 + sL_2}.$$

Note that

$$\det Y(s) = Y(s) = \frac{1}{R_1 + sL_1} + \frac{1}{R_2 + sL_2} + \frac{1}{R_3}$$
$$= \frac{L_1 L_2 s^2 + [(R_1 + R_3)L_2 + (R_2 + R_3)L_1]s + R_1(R_2 + R_3) + R_2 R_3}{(R_1 + sL_1)(R_2 + sL_2)R_3} \quad (14)$$

And after multiplication by

$$\frac{(R_1 + sL_1)(R_2 + sL_2)R_3}{L_1 L_2}$$ We obtain

$$\det Y(s) = \frac{L_1 L_2}{(R_1 + sL_1)(R_2 + sL_2)R_3} \det[I_2 s - A_1]. \quad (15)$$

From equation (15) we have the following conclusion.
It means we can say that in Mesh Method the characteristic equation (8) of the electrical circuit can be also obtained by computation of the determinant of the matrix Y (s) in the node method.

3. System Equations
In general, in the time-domain, the eigenvalues are determined from the system matrix in the state-space representation of circuit equations.5–7 Although the state space method, based on the graph theoretical approach, has minimum variables, It involves an intensive mathematical process and has major limitations in the formulation of circuit equations. Some of these limitations arise because the state variables are capacitor voltages and inductor currents. Not every circuit element can be easily included into the state equations. It has a structure of differential equations. Especially, there are some restrictions in the analysis of active circuits. Therefore, students of electrical engineering generally have these difficulties in obtaining the state-space representation of system equations. It is not always suitable to use this method for obtaining both eigenvalues and transfer functions. In this study, it is shown that the eigenvalues can be easily determined according to nodal and mesh equations in the s-domain, through more efficient and understandable analysis methods in circuit analysis courses. It has a structure of algebraical equations. There are no restrictions in the formulation of circuit equations.
The system equations in the algebraical structure, in the Laplace domain, obtained by using nodal or mesh analysis method, relating to a linear circuit are

AX(s)= BU(s) (1)

Where, A, B are coefficient matrices, U(s) is the source vector, X(s) is the unknown vector. The frequency-dependent elements (inductor, capacitor) can be entered in the form having
s(sL, sC) or 1/s(1/sL, 1/sC) into the system equations. Matrix A is also called the characteristic matrix. By taking the inverse of matrix A, solutions of the system equations are given as in equation (2), as follows

$$\mathbf{X}(s) = \mathbf{A}^{-1}\mathbf{B}\mathbf{U}(s) = \left(\frac{1}{\det(\mathbf{A})}\mathrm{Adj}(\mathbf{A})\right)\mathbf{B}\mathbf{U}(s) \quad (2)$$

Where det(A) denotes the determinant of matrix A, and adj are notes the adjoint matrix. It is obvious that solutions of equation (2) are fractional. The determinant of the characteristic matrix (A) has also fractional and polynomial form as in equation (3). Q(s) represents the numerator of the determinant and R(s) represents the denominator of the determinant.

$$\det(\mathbf{A}) = \frac{Q(s)}{R(s)} \quad (3)$$

The determinant expression in equation (3) is substituted into equation (2)

$$\mathbf{X}(s)=\left(\frac{1}{\frac{Q(s)}{R(s)}}\text{Adj}(\mathbf{A})\right)\mathbf{BU}(s)$$

$$\mathbf{X}(s)=\frac{\text{Adj}(\mathbf{A})*\mathbf{B}*R(s)}{Q(s)}\mathbf{U}(s) \quad (4)$$

According to Equation (4), all variables relating to any circuit have the same denominator, Q(s). It is the numerator of determinant of the coefficient matrix (A) in equation (3).

Therefore, Q(s) polynomial is also called the characteristic equation. The eigenvalues, indirectly poles, of any circuit are obtained from the roots of the characteristic polynomial, Q(s) = 0. In the proposed approach, the system equations in the form of equation(1) are obtained algebraically by nodal or mesh analysis in the Laplace (s) domain. The characteristic equation, Q(s), is determined in terms of the numerator of determinant of the coefficient matrix (A) relating to system equations. Later, the eigenvalues of the circuit are easily obtained from the roots of the characteristic equation.

4. Applications

Three application examples are given in this section to present the method. The system equations and the eigenvalues are obtained by mesh analysis in the first example and nodal analysis in the second and third example.

Example 1. Consider the circuit shown in Fig.(3). Element values are R = 2 Ω, C1 = C2 = 3 F, L = 5 H. Mesh equations in the s-domain are as follows. The variables of the method are I_1, I_2 mesh currents.

$$1 \rightarrow R(I_1)+\frac{1}{sC_1}(I_1-I_2)=U(s)$$

$$2 \rightarrow \frac{1}{sC_1}(I_2-I_1)+sL(I_2)+\frac{1}{sC_2}(I_2)=0$$

Let's rearrange the system equations in matrix form.

$$\mathbf{AX}(s)=\mathbf{BU}(s)$$

$$\begin{bmatrix} R+\frac{1}{sC_1} & -\frac{1}{sC_1} \\ -\frac{1}{sC_1} & sL+\frac{1}{sC_1}+\frac{1}{sC_2} \end{bmatrix}\begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix}=\begin{bmatrix} 1 \\ 0 \end{bmatrix}\mathbf{U}(s) \quad (5)$$

After substituting the element values into the system equations, the determinant of the coefficient matrix (A) is obtained as follows.

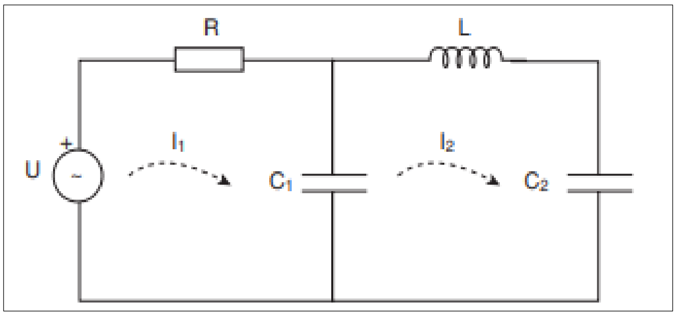

Fig 3: Circuit for example 1

$$\det(\mathbf{A})=\frac{Q(s)}{R(s)}=\frac{90s^3+15s^2+12s+1}{9s^2} \quad (6)$$

The characteristic equation: $Q(s)=90s^3+15s^2+12s+1$

After solving the system equations in (5), the circuit variables, mesh currents, are obtained as follows.

$$\begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} \dfrac{45s^3+6s}{90s^3+15s^2+12s+1} \\ \dfrac{3s}{90s^3+15s^2+12s+1} \end{bmatrix} \text{U(s)} \tag{7}$$

It can be easily seen that all variables relating to the circuit have the same denominator, the characteristic equation (Q(s)). The roots of characteristic equation (Q(s)) give the eigenvalues of the third order circuit: α1 ≅ −0.0394 + 0.3534i, α2 ≅ −0.0394 − 0.3534i, α3 = −0.0879. The circuit is stable because all eigenvalues are located on the left-hand side of the complex s-plane.

Example 2. Consider the circuit shown in Fig.(4). Element values are $R_1 = ½\ \Omega$, $R_2 = 4\ \Omega$, $R_3 = 2\ \Omega$, C = 1 F, L = 2 H. Nodal equations in the s-domain are as follows. Since Ua = U and Ud = Uk, the variables of the method are Ub, Uc nodal voltages.

$$b \rightarrow \frac{1}{sL}(U_b - U_c) + sCU_b - \frac{1}{R_1}(U_a - U_b) = 0$$
$$c \rightarrow \frac{1}{R_2}(U_c - U_d) - \frac{1}{sL}(U_b - U_c) - J(s) = 0$$

Additional equations:

$$U_a = U(s), \quad U_d = U_k = \frac{1}{2} I_{R1} = \frac{1}{2}\frac{1}{R_1}(U_a - U_b) = (U_a - U_b)$$

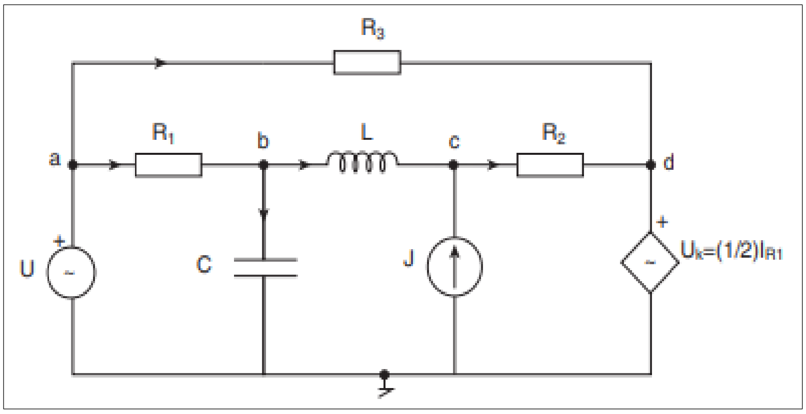

Fig 4: Circuit for example 2

Let's rearrange the system equations in matrix form.

$$\mathbf{AX}(s) = \mathbf{BU}(s)$$
$$\begin{bmatrix} \dfrac{1}{R_1} + sC + \dfrac{1}{sL} & -\dfrac{1}{sL} \\ -\dfrac{1}{sL} + \dfrac{1}{R_2} & \dfrac{1}{R_2} + \dfrac{1}{sL} \end{bmatrix} \begin{bmatrix} U_b(s) \\ U_c(s) \end{bmatrix} = \begin{bmatrix} \dfrac{1}{R_1} & 0 \\ \dfrac{1}{R_2} & 1 \end{bmatrix} \begin{bmatrix} U(s) \\ J(s) \end{bmatrix} \tag{8}$$

After substituting the element values into the system equations, the determinant of the coefficient matrix (A) is obtained as follows:

$$\det(\mathbf{A}) = \frac{Q(s)}{R(s)} = \frac{s^2+4s+5}{4s} \tag{9}$$

The characteristic equation: Q(s) = s 2 + 4s+5 after solving the system equations in (8), the circuit variables, nodal voltages, are obtained as follows.

$$\begin{bmatrix} U_b(s) \\ U_c(s) \end{bmatrix} = \begin{bmatrix} \dfrac{2s+9/2}{s^2+4s+5} & \dfrac{2}{s^2+4s+5} \\ \dfrac{s^2+9/2}{s^2+4s+5} & \dfrac{2s^2+8s+2}{s^2+4s+5} \end{bmatrix} \begin{bmatrix} U(s) \\ J(s) \end{bmatrix} \tag{10}$$

It can be easily seen that all variables relating to the circuit have the same denominator, the characteristic equation (Q(s)). The roots of characteristic equation (Q(s)) give the eigenvalues of the second order circuit: $\alpha1 = -2 + i$, $\alpha2 = -2 - i$. The circuit is stable because all eigenvalues are located on the left hand of the complex s plane.
Example 3. Consider the op−amp circuit shown in Fig. (5). Element values are $R_1 = 4\ \Omega$, $R_2 = 5\ \Omega$, $C_1 = 1$ F, $C_2 = 2$ F. Node p is chosen as a reference, Up = 0. The voltage and current constraints of the ideal op-amp are Ip = 0, In = 0, Up − Un = 0.

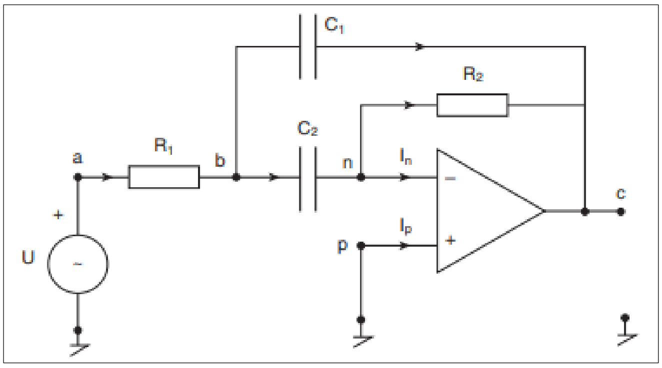

Fig 5: Circuit for example 3

The input terminals of the op-amp are simultaneously short circuit and open circuit. It is an interesting property of the op-amp. It is not necessary to write a nodal equation at the output node of the op-amp (node c) because its output voltage is determined by other nodal voltages.5, 6 Nodal equations in the s-domain are as follows.

$$b \rightarrow sC_1(U_b - U_c) + sC_2(U_b - U_n) - \frac{1}{R_1}(U_a - U_b) = 0$$

$$n \rightarrow \frac{1}{R_2}(U_n - U_c) - sC_2(U_b - U_n) + I_n = 0$$

Additional equations:

$$U_a = U(s), \quad U_p = U_n = 0$$

Let's rearrange the system equations in matrix form.

$$\mathbf{AX}(s) = \mathbf{BU}(s)$$

$$\begin{bmatrix} \dfrac{1}{R_1} + sC_1 + sC_2 & -sC_1 \\ -sC_2 & -\dfrac{1}{R_2} \end{bmatrix} \begin{bmatrix} U_b(s) \\ U_c(s) \end{bmatrix} = \begin{bmatrix} \dfrac{1}{R_1} \\ 0 \end{bmatrix} U(s) \tag{11}$$

After substituting the element values into the system equations, the determinant of the coefficient matrix (A) is obtained as follows:

$$\det(\mathbf{A}) = \frac{Q(s)}{R(s)} = \frac{40s^2 + 12s + 1}{20} \tag{12}$$

The characteristic equation: Q(s) = 40s 2 + 12s + 1 after solving the system equations in (11), the circuit variables, nodal voltages, are obtained as follows.

$$\begin{bmatrix} U_b(s) \\ U_c(s) \end{bmatrix} = \begin{bmatrix} \frac{1}{40s^2+12s+1} \\ \frac{-10s}{40s^2+12s+1} \end{bmatrix} U(s) \quad (13)$$

All variables relating to the circuit have the same denominator, the characteristic equation (Q(s)). The roots of the characteristic equation (Q(s)) give the eigenvalues of the second order circuit: $\alpha 1 = -0.15 + 0.05i$, $\alpha 2 = -0.15 - 0.05i$. The circuit is stable because all eigenvalues are located on the left hand side of the complex s plane.

5. Conclusion

The difficulty of determining the eigenvalues in circuit analysis courses depends on obtaining the system equations. In general, the eigenvalues are determined from a state variables method having a structure of differential equations and some restrictions in obtaining the system equations. In this paper, it is shown how the eigenvalues of linear electrical circuits can be determined according to the characteristic equation created by nodal and mesh analysis, in algebraical form. In terms of complexity, the proposed method is simpler and more understandable than the commonly used form of state equations. The method is general and can be easily applied to all possible active and passive circuits. It has no restrictions. The main advantages of the method are efficient, systematic, and understandable in terms of advances in student learning. Students can easily determine the eigenvalues and, moreover, can write a computer program about computation of the eigenvalues by employing the presented method and also the problem of calculation of the characteristic equations of the standard positive and linear electrical circuits.

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