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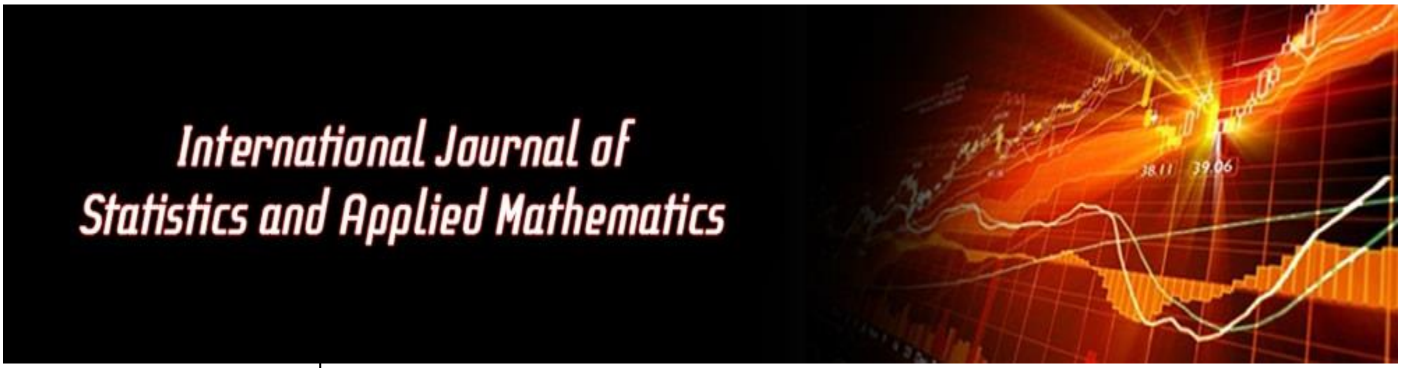

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A unified formula of a series of exact solutions for coupled Schrödinger-KdV equation

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Abstract
In this paper, a unified formula of a series of exact solutions for the coupled Schrödinger-KdV equation is obtained by using Hirota bilinear method and trial function method. These solutions contain bi-soliton solution, singular periodic amplitude solution and breather soliton solution. The results contribute to a better understanding of the structure of the solutions and non-linear phenomena for the coupled Schrödinger-KdV equation.

Keywords: The coupled Schrödinger-KdV equation. Hirota bilinear method. Trial function method. Exact solution

Introduction
Takao Yoshinaga *et al.* studied nonlinear interaction between long and short waves from the following model, which was called coupled Schrödinger-KdV equation [1].

$$\begin{cases} iu_t \pm u_{xx} = uv, \\ v_t + \alpha v v_x + \beta v_{xxx} = (|u|^2)_x, \end{cases} \qquad (1)$$

where $u = u(x,t)$ and $v = v(x,t)$ denote, respectively, the complex amplitude of the short wave and the long wave, and x and t are the normalized space-time coordinates, while α and β are control parameters. The positive or negative sign in front of u_{xx} should be adopted depending upon the property of the medium. Their study results exhibit recurrence or chaotic motion, depending upon the magnitude of the control parameters involved in the governing equations (1).

Many researchers have investigated the special case of Eqs.(1), that is the following form [2-5].

$$\begin{cases} iu_t = u_{xx} + uv, \\ v_t + 6vv_x + v_{xxx} = (|u|^2)_x. \end{cases} \qquad (2)$$

Such as Dogan Kaya and Salah M. El-Sayed obtained the exact and approximate traveling-wave solutions for Eqs.(2) by using the Adomian's Decomposition Method [2]. Ardeshir Ahmadi Siahdareh and his colleagues found analytical solutions of Eqs.(2) through a hybrid of Fourier Transform and Adomian Decomposition Method [3]. A number of Jacobi-elliptic function solutions, soliton-like solutions and trigonometric-function solutions were also obtained by A. Filiz *et al.* by using F-expansion Method [4]. While numerical analysis of Eqs.(2) is studied by S. Kücükarslan through the Homotopy Perturbation Method [5].

To our knowledge, breather soliton solutions of Eqs.(2) have not been reported. The aim of the present paper will be to investigate Eqs.(2) by using Hirota bilinear method and trial function method, and to obtain a unified formula of a series of exact solutions including soliton solution, periodic amplitude solution and breather soliton solution [6, 7].

2 Exact solutions to Schrödinger-KdV equation

First, we find

$$u_0(x,t)=(a+k^2)e^{(-i(kx+at))},v_0(x,t)=a+k^2 \tag{3}$$

is a solution of Eqs.(2).

Second, we set

$$u(x,t)=\frac{G(x,t)}{F(x,t)},v(x,t)=v_0+2(\ln F)_{xx}, \tag{4}$$

where $v_0=a+k^2$, then Eqs.(2) can be changed into the following bilinear equations with respect to F and G

$$\begin{aligned}&(iD_t-D_x{}^2-v_0)G\cdot F=0,\\&(D_tD_x+D_x{}^4+6v_0D_x{}^2+C)F\cdot F-|G|^2=0,\end{aligned} \tag{5}$$

where C is a integration constant. In the calculation, Eqs.(5) is written as another form

$$\begin{aligned}&i(FG_t-F_tG)-(FG_{xx}-2F_xG_x+F_{xx}G)-v_0GF=0,\\&2(FF_{xt}-F_xF_t)+2(FF_{xxx}-4F_{xxxx}F_x+3F_{xx}{}^2)+12v_0(FF_{xx}-F_x{}^2)+CF^2-GG^*=0,\end{aligned} \tag{6}$$

where the asterisk denotes complex conjugate. Now, assuming that

$$\begin{aligned}F\left(x,t\right)&=a_1e^{Px+Qt}+a_2e^{-(Px+Qt)}+a_3\cos\left(Kx+Lt\right),\\G\left(x,t\right)&=v_0e^{-i(kx+at)}\left(b_1e^{Px+Qt}+b_2e^{-(Px+Qt)}+b_3\cos\left(Kx+Lt\right)+b_4\sin\left(Kx+Lt\right)\right),\end{aligned} \tag{7}$$

where $a_i(i=1,2,3),b_j(j=1,2,3,4)$, P,Q,K and L are constants to be determined, we substitute Eqs.(7) into Eqs.(6), some equations with respect to $a_i(i=1,2,3)$, $b_j(j=1,2,3,4)$, P,Q,K and L can be obtained (the tedious calculation process is omitted). Solving these equations, we obtain the following results

$$\begin{aligned}&b_1=-\frac{\sqrt{2}}{2}\frac{a_1(3a+3k^2-k)(4a_1a_2-a_3{}^2)}{(a+k^2)(4a_1a_2+a_3{}^2)},b_2=-\frac{\sqrt{2}}{2}\frac{a_2(3a+3k^2-k)(4a_1a_2-a_3{}^2)}{(a+k^2)(4a_1a_2+a_3{}^2)},\\&b_3=\frac{\sqrt{2}}{2}\frac{a_3(3a+3k^2-k)(4a_1a_2-a_3{}^2)}{(a+k^2)(4a_1a_2+a_3{}^2)},b_4=0,C=\frac{1}{2}\left(\frac{(4a_1a_2-a_3{}^2)(3a+3k^2-k)}{4a_1a_2+a_3{}^2}\right)^2,\\&P=\sqrt{-\frac{3a+3k^2-k}{4a_1a_2+a_3{}^2}}\alpha_3,Q=-2k\sqrt{-\frac{3a+3k^2-k}{4a_1a_2+a_3{}^2}}a_3,K=2\sqrt{\frac{a_1a_2(3a+3k^2-k)}{4a_1a_2+a_3{}^2}},\\&L=-4k\sqrt{\frac{a_1a_2(3a+3k^2-k)}{4a_1a_2+a_3{}^2}}\end{aligned} \tag{8}$$

where a_1,a_2,a_3,a and k are arbitrary constants. Then, we have

$$F(x,t)=a_1 e^{\sqrt{-\frac{3a+3k^2-k}{4a_1a_2+a_3^2}}a_3(x-2kt)}+a_2 e^{-\sqrt{-\frac{3a+3k^2-k}{4a_1a_2+a_3^2}}a_3(x-2kt)}+a_3\cos(2\sqrt{\frac{a_1a_2(3a+3k^2-k)}{4a_1a_2+a_3^2}}(x-2kt)),$$

$$G(x,t)=-\frac{\sqrt{2}}{2}\frac{(4a_1a_2-a_3^2)(3a+3k^2-k)}{4a_1a_2+a_3^2}e^{-i(kx+at)}\left(a_1 e^{\sqrt{-\frac{3a+3k^2-k}{4a_1a_2+a_3^2}}a_3(x-2kt)}+a_2 e^{-\sqrt{-\frac{3a+3k^2-k}{4a_1a_2+a_3^2}}a_3(x-2kt)}\right.$$
$$\left.-a_3\cos(2\sqrt{\frac{a_1a_2(3a+3k^2-k)}{4a_1a_2+a_3^2}}(x-2kt))\right), \quad (9)$$

Substituting Eqs.(9) into Eqs.(4), we can obtain solutions of Schrödinger-KdV equation(2) as follows

$$u(x,t)=$$
$$-\frac{\sqrt{2}}{2}\frac{(4a_1a_2-a_3^2)(3a+3k^2-k)}{4a_1a_2+a_3^2}e^{-i(kx+at)}(a_1 e^{\sqrt{-\frac{3a+3k^2-k}{4a_1a_2+a_3^2}}a_3(x-2kt)}+a_2 e^{-\sqrt{-\frac{3a+3k^2-k}{4a_1a_2+a_3^2}}a_3(x-2kt)}$$
$$-a_3\cos(2\sqrt{\frac{a_1a_2(3a+3k^2-k)}{4a_1a_2+a_3^2}}(x-2kt)))\Big/(a_1 e^{\sqrt{-\frac{3a+3k^2-k}{4a_1a_2+a_3^2}}a_3(x-2kt)}+a_2 e^{-\sqrt{-\frac{3a+3k^2-k}{4a_1a_2+a_3^2}}a_3(x-2kt)}$$
$$+a_3\cos(2\sqrt{\frac{a_1a_2(3a+3k^2-k)}{4a_1a_2+a_3^2}}(x-2kt))),$$

$$v(x,t)=a+k^2-\frac{2a_3(3a+3k^2-k)}{4a_1a_2+a_3^2}(a_1a_3 e^{\sqrt{-\frac{3a+3k^2-k}{4a_1a_2+a_3^2}}a_3(x-2kt)}+a_2a_3 e^{-\sqrt{-\frac{3a+3k^2-k}{4a_1a_2+a_3^2}}a_3(x-2kt)}$$
$$+4a_1a_2\cos(2\sqrt{\frac{a_1a_2(3a+3k^2-k)}{4a_1a_2+a_3^2}}(x-2kt)))\Big/(a_1 e^{\sqrt{-\frac{3a+3k^2-k}{4a_1a_2+a_3^2}}a_3(x-2kt)}+a_2 e^{-\sqrt{-\frac{3a+3k^2-k}{4a_1a_2+a_3^2}}a_3(x-2kt)}$$
$$+a_3\cos(2\sqrt{\frac{a_1a_2(3a+3k^2-k)}{4a_1a_2+a_3^2}}(x-2kt)))-2a_3^2(\sqrt{-\frac{3a+3k^2-k}{4a_1a_2+a_3^2}}(a_1 e^{\sqrt{-\frac{3a+3k^2-k}{4a_1a_2+a_3^2}}a_3(x-2kt)}$$
$$-a_2 e^{-\sqrt{-\frac{3a+3k^2-k}{4a_1a_2+a_3^2}}a_3(x-2kt)})-2\sqrt{\frac{a_1a_2(3a+3k^2-k)}{4a_1a_2+a_3^2}}\sin(2\sqrt{\frac{a_1a_2(3a+3k^2-k)}{4a_1a_2+a_3^2}}(x-2kt)))^2$$
$$\Big/((a_1 e^{\sqrt{-\frac{3a+3k^2-k}{4a_1a_2+a_3^2}}a_3(x-2kt)}+a_2 e^{-\sqrt{-\frac{3a+3k^2-k}{4a_1a_2+a_3^2}}a_3(x-2kt)}+a_3\cos(2\sqrt{\frac{a_1a_2(3a+3k^2-k)}{4a_1a_2+a_3^2}}(x-2kt))))^2 \quad (10)$$

This is a unified solution set which contains soliton solution, periodic amplitude solution, breather soliton solution.
Now, we choose properly constans a_1, a_2, a_3 and a in Eqs.(10), different solutions can be obtained.
Case I Setting $a_1=2a_3, a_2=2a_3$, and $a=-2k^2$ in Eqs.(10), then, a soliton solution for Eqs.(2) can be expressed as

$$u(x,t)=\frac{15\sqrt{2k}(3k+1)}{34}\cdot \frac{4\cosh(\frac{1}{17}\sqrt{17}\sqrt{k(3k+1)}(x-2kt))-\cosh(\frac{4}{17}\sqrt{17}\sqrt{k(3k+1)}(x-2kt))e^{ik(x-2kt)}}{4\cosh(\frac{1}{17}\sqrt{17}\sqrt{k(3k+1)}(x-2kt))+\cosh(\frac{4}{17}\sqrt{17}\sqrt{k(3k+1)}(x-2kt))},$$

$$v(x,t)=-k^2+\frac{8}{17}\frac{k(3k+1)(\cosh(\frac{1}{17}\sqrt{17}\sqrt{k(3k+1)}(x-2kt))+4\cosh(\frac{4}{17}\sqrt{17}\sqrt{k(3k+1)}(x-2kt)))}{4\cosh(\frac{1}{17}\sqrt{17}\sqrt{k(3k+1)}(x-2kt))+\cosh(\frac{4}{17}\sqrt{17}\sqrt{k(3k+1)}(x-2kt))}-\frac{32}{17}\frac{k(3k+1)(\sinh(\frac{1}{17}\sqrt{17}\sqrt{k(3k+1)}(x-2kt))+\sinh(\frac{4}{17}\sqrt{17}\sqrt{k(3k+1)}(x-2kt)))^2}{(4\cosh(\frac{1}{17}\sqrt{17}\sqrt{k(3k+1)}(x-2kt))+\cosh(\frac{4}{17}\sqrt{17}\sqrt{k(3k+1)}(x-2kt)))^2}. \tag{11}$$

where k is a constant and satisfies $k(3k+1)>0$.

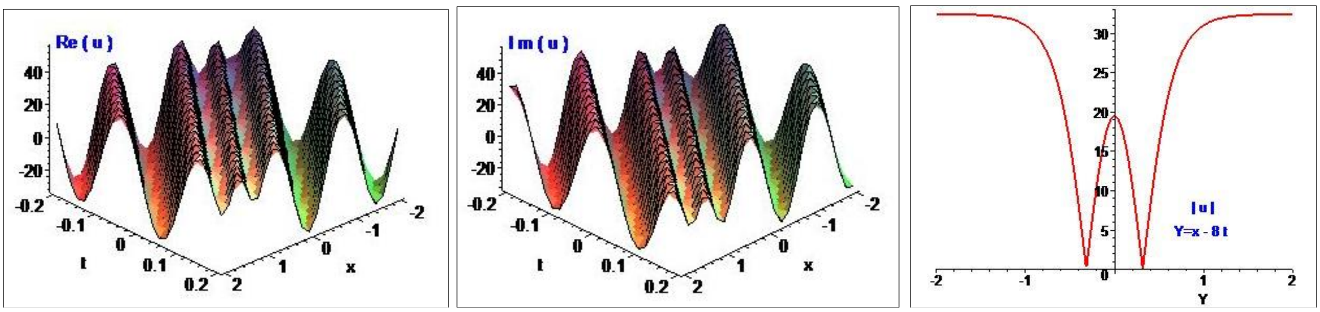

Fig 1: The profiles to *Re*(u), *Im*(u) and $|u|$ for $u(x,t)$ with $k=4,$ and $Y=x-8t$ in Eqs.(11).

From Fig.1, we clearly see that $u(x,t)$ is a bi-soliton solution.

Case II Setting $a_1=a_3, a_2=a_3$, and $a=k^2$ in Eqs.(10), then, a solution with periodic amplitude for Eqs.(2) can be written as

$$u(x,t)=-\frac{3}{10}\frac{\sqrt{2}\text{k}(6k\text{-}1)(2\cos(\frac{1}{5}\sqrt{5}\sqrt{k(6k-1)}(x-2kt))-\cos(\frac{2}{5}\sqrt{5}\sqrt{k(6k-1)}(x-2kt)))e^{-ik(x+kt)}}{2\cos(\frac{1}{5}\sqrt{5}\sqrt{k(6k-1)}(x-2kt))+\cos(\frac{2}{5}\sqrt{5}\sqrt{k(6k-1)}(x-2kt))},$$

$$v(x,t)=2k^2-\frac{4}{5}\frac{k(6k-1)(\cos(\frac{1}{5}\sqrt{5}\sqrt{k(6k-1)}(x-2kt))+2\cos(\frac{2}{5}\sqrt{5}\sqrt{k(6k-1)}(x-2kt)))}{2\cos(\frac{1}{5}\sqrt{5}\sqrt{k(6k-1)}(x-2kt))+\cos(\frac{2}{5}\sqrt{5}\sqrt{k(6k-1)}(x-2kt))}+\frac{4}{5}\frac{k(6k-1)(\sin(\frac{1}{5}\sqrt{5}\sqrt{k(6k-1)}(x-2kt))+\sin(\frac{2}{5}\sqrt{5}\sqrt{k(6k-1)}(x-2kt)))^2}{(2\cos(\frac{1}{5}\sqrt{5}\sqrt{k(6k-1)}(x-2kt))+\cos(\frac{2}{5}\sqrt{5}\sqrt{k(6k-1)}(x-2kt)))^2}. \tag{12}$$

where k is a constant and satisfies $k(6k-1)>0$.

Similarly, Fig.2 shows that $u(x,t)$ is a singular solution with periodic amplitude (unbounded).

Case III Setting $a_1=2a_3, a_2=-2a_3$, and $a=2k^2$ in Eqs.(10), then, a breather soliton solution for Eqs.(2) can be obtained.

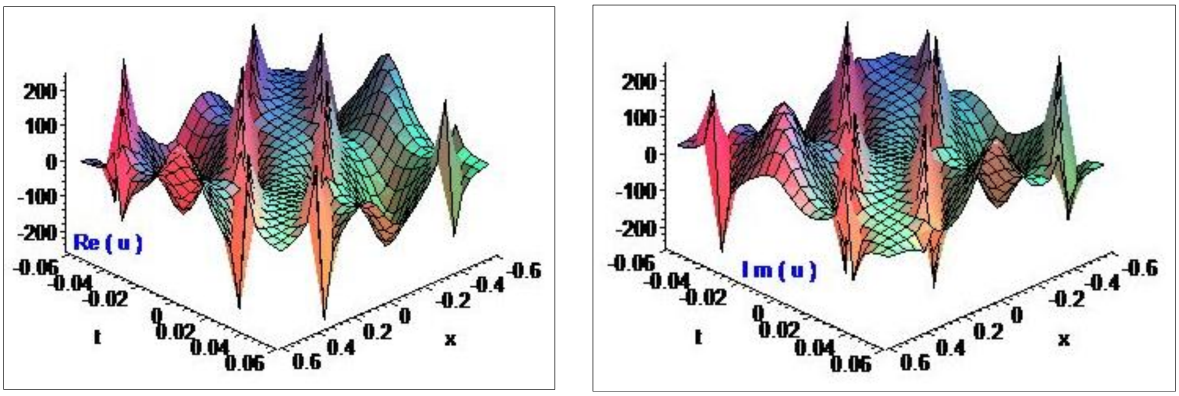

Fig 2: The profiles to *Re*(u) and *Im*(u) for $u(x,t)$ in Eqs.(12) with $k=5$.

$$u(x,t)=$$

$$-\frac{17}{30}\frac{\sqrt{2}k(9k-1)(4\sinh(\frac{1}{15}\sqrt{15}\sqrt{k(9k-1)}(x-2kt))-\cos(\frac{4}{15}\sqrt{15}\sqrt{k(9k-1)}(x-2kt)))e^{-ik(x-2kt)}}{4\sinh(\frac{1}{15}\sqrt{15}\sqrt{k(9k-1)}(x-2kt))+\cos(\frac{4}{15}\sqrt{15}\sqrt{k(9k-1)}(x-2kt))},$$

$$v(x,t)=3k^2+$$

$$\frac{8k(9k-1)}{15}\frac{\sinh(\frac{1}{15}\sqrt{15}\sqrt{k(9k-1)}(x-2kt))-\cos(\frac{4}{15}\sqrt{15}\sqrt{k(9k-1)}(x-2kt))}{4\sinh(\frac{1}{15}\sqrt{15}\sqrt{k(9k-1)}(x-2kt))+\cos(\frac{4}{15}\sqrt{15}\sqrt{k(9k-1)}(x-2kt))}$$

$$-\frac{32k(9k-1)}{15}\frac{(\cosh(\frac{1}{15}\sqrt{15}\sqrt{k(9k-1)}(x-2kt))-\sin(\frac{4}{15}\sqrt{15}\sqrt{k(9k-1)}(x-2kt)))^2}{(4\sinh(\frac{1}{15}\sqrt{15}\sqrt{k(9k-1)}(x-2kt))+\cos(\frac{4}{15}\sqrt{15}\sqrt{k(9k-1)}(x-2kt)))^2}. \quad (13)$$

where k is a constant and satisfies $k(9k-1)>0$.

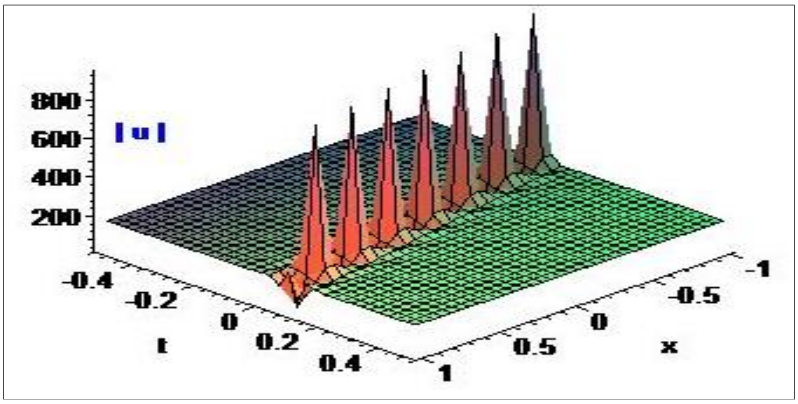

Fig 3: The profile of $|u(x,t)|$ in Eqs.(13) with $k=5$.

Fig 3: shows that $u(x,t)$ is a breather soliton solution.

3 Conclusion

In this paper, the coupled Schrödinger-KdV equation is transformed into the bilinear equations and which are solved by using trial function method. A unified formula of a series of exact solutions which includes bi-soliton solution, periodic amplitude solution and breather soliton solution is obtained. Three structures of exact solutions are displayed. These results greatly enriched the diversity of wave structures for the coupled Schrödinger-KdV equation.

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