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Variations in ground temperature in the presence of radiative heat flux and spatial-dependent soil thermophysical property

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Abstract
This research investigated the variations in ground temperature mechanism with time-dependent suction velocity under the influence of a varying thermal conductivity of the soil. It is assumed that all parameters depend only on the depth (z) since the flow considered is solely along the vertical direction, thereby making the problem one-dimensional in that direction. Considering also, the flow of the radiative heat flux through it, the surface of the ground is assumed to be an optically thin environment. The governing equations were reduced to non-dimensional form by the use of some dimensionless parameters resulting into a non-linear partial differential equation. The equation was linearized using time-dependent perturbation method and then solved using the method of undetermined coefficients. The results were displayed graphically and compared to various works in literature. The result obtained is able to address most of the critical issues affecting ground temperature distribution due to variation of various parameters affecting ground thermal properties.

Keywords: Radiative heat flux, thermal conductivity, ground temperature, suction velocity

1. Introduction

Ground temperature plays a dominant role and dictates the type or species of crops that can be planted in a particular region. It has been established that significant relationship exists between soil temperature, agricultural yield and time requirement for germination. Fahim and Ghulam [2] examined that the quality and quantity of agricultural products depends upon factors that include ground temperature which significantly affects the budding and growth rates of plants. With increase in ground temperature, chemical reactions speed up and cause seeds to germinate and regulate rate of plant development and growth.

Information on ground temperature is also necessary for construction projects. These include design of airports, road pavements, excavation of foundations and the design and construction of underground space for buildings. Engineers and Architects need the knowledge of the factors that determine the ground temperature and how the quality varies with time and depth from the surface as reported by Williams and Gold [11].

Williams and Gold [11] and Daniel [1] investigated that the ground temperature varies depending on several factors which can be grouped into three general categories: meteorological, terrain and subsurface variables. The meteorological elements include solar radiation, air temperature and rain. Examples of some thermo-physical properties of the soil that affect the temperature of the ground are specific heat capacity, thermal conductivity and thermal diffusivity.

In an investigation conducted on the problem of ground temperature with suction velocity and radiation by Nwaigwe [7], the results revealed that increase in Prandtl number or radiation decreases the ground temperature. However, the author imposed among other conditions that the thermal conductivity is constant and also assumed that the temperature of the surface is maintained at a constant temperature, $T' = T'_w$.

Daniel [1] in his work revealed that the thermal conductivity of the ground is seldom uniform. It varies with the depth of the ground, time and temperature of soil.

More so, the surface temperature is periodic and can be expressed as a function of time. Similar results were reported by Nofziger and Wu [6].

Ishikawa [4] studied the thermal heat transfer in the soil and confirmed the significant of surface temperature for understanding the temperature phenomena in the soil. By their findings, the surface temperature corresponds to the temperature at the top active layer of the ground. It is measured at the uppermost centimeters of the ground in order to avoid the radiation influences and focuses on the seasonal changes of the ground temperature.

Gao, *et al.* [3] reported that surface temperature varies canopy layer and soil water content. Changes in soil porosity and thermal conductivity have little or no effect on it.

In this research work, effort is therefore made to present a mathematical model on ground temperature variation that combines some soil parameters such as spatial dependent thermal conductivity, prandtl number and Radiation with the boundary conditions involving a periodic surface temperature.

1.1 Formulation of the problem

Consider the heat flow into the ground through the surface in the vertical direction (z-axis). It is assumed that the surface temperature T' of the ground oscillates as a sinusoidal or harmonic function of time around an average temperature T'_w. It is therefore expressed as a function of time. It is also assumed that the ground temperature is high so that the radiative heat transfer is significant. The thermal conductivity of the soil depends on the depth z. However, the density ρ is constant and does not depend on z.

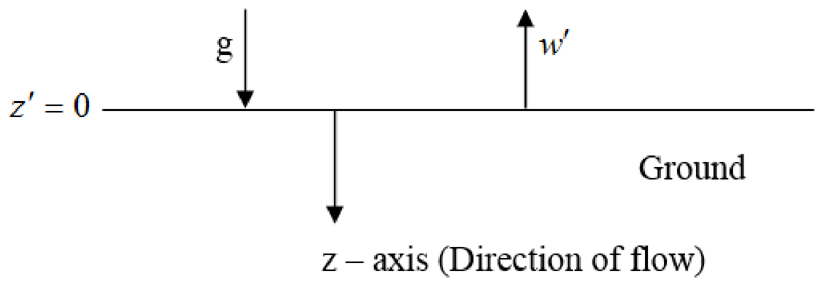

Fig 1: Physical model

1.2 Governing Equation

Based on the above assumptions, the flow is governed by the following equations.

Continuity Equation

$$\frac{\partial p}{\partial t'} + \frac{\partial(\rho w')}{\partial z'} = 0$$

$$\frac{\partial w'}{\partial z'} = 0 \tag{1}$$

Energy Equation

$$\frac{\partial T'}{\partial t'} + \frac{w'\partial T'}{\partial z'} = \frac{1}{\rho C_p}\frac{\partial}{\partial z'}\left[k\frac{\partial T'}{\partial z'}\right] - \frac{1}{\rho C_p}\frac{\partial q'}{\partial z'} \tag{2}$$

Where

$\frac{\partial q'}{\partial z'}$ is the radiative heat flux and w' is the velocity of the fluid flow.

Equation (2) is subject to the boundary conditions

$T'(z,t) = T'_w + \lambda' \cos w't'$ on $z' = 0$ and $T' \to 0$ as $z' \to \infty$

From equation (1), w' is not a function of z'. It is either a constant or a function of time.

Thus, $w' = -w'_0$ or $w' = -w'_0(1 + \varepsilon A e^{i\omega' t'})$ (3a)

Where, w' is the initial suction velocity.

Following Israel-Cookey et al [5] and for the purpose of this work, w' is taken to be

$w' = -w'_0(1 + \varepsilon A e^{i\omega' t'})$.

A and ε are very small parameters such that $\varepsilon A << 1$.

It is assumed that the ground is an optically thin environment. As reported by Tarkher *et al* [10], radiative flux takes the form

$$\frac{\partial q'}{\partial z'} = 4\alpha^2 (T' - T_\infty) \tag{3b}$$

Where α is the absorption coefficient.
Since the thermal conductivity k of the ground is assumed to be spatial dependent, and then set to be

$$k = k_0 (1 + \delta z)^n \tag{3c}$$

Where, k_0 is a constant thermal conductivity and δ is a very small parameter such that $\delta << 1$.
Substituting equations (3a) – (3b) into equation (2), it gave

$$\frac{\partial T'}{\partial t'} - w_0'(1 + \varepsilon A e^{i\omega' t'})\frac{\partial T'}{\partial z'} = \frac{k_0}{\rho C_p}\frac{\partial}{\partial z'}\left[(1+\delta z)^n \frac{\partial T'}{\partial z'}\right] + \frac{4\alpha^2}{\rho C_p}(T' - T_\infty) \tag{4}$$

Subject to

$T'(z, t) = T_w' + \lambda' \cos\omega' t'$ on $z' = 0$ and $T'(z,t) \to 0$ as $z' \to \infty$

Introducing the following non-dimensional parameters as used in Nwaigwe [7] and Okedayo [8]

$$t = \frac{w_0' t'}{4v},\quad z = \frac{w_0' z'}{v},\quad \omega = \frac{4v}{w_0'}\omega',\quad w = \frac{w'}{w_0'},\quad \theta = \frac{T' - T_\infty}{T_w' - T_\infty},\quad v = \frac{\mu}{\rho},\quad P_r = \frac{\mu C_p}{k_0},\quad R^2 = \frac{4\alpha^2}{\rho C_p k_0 w_0'^2},\quad A_0 = \frac{\lambda'}{T_w' - T_\infty}$$

Equation (4) became

$$\frac{1}{4}P_r\frac{\partial \theta}{\partial t} - P_r(1 + \varepsilon A e^{i\omega t})\frac{\partial \theta}{\partial z} = \frac{\partial}{\partial z}\left[(1+\delta z)^n \frac{\partial \theta}{\partial z}\right] - R^2\theta \tag{5}$$

Subject to

$$\theta(z,t) = 1 + A_0\cos\omega t \quad \text{on } z = 0 \text{ and } \theta(z,t) \to 0 \text{ as } z \to \infty \tag{6}$$

Where, R = radiative parameter, P_r = Prandtl number, A = suction parameter, and A_0 = amplitude of variation.

2. Method of Solution

Since equation (6) is a partial differential equation (p.d.e.) and ε is small, a time-dependent perturbation method is adopted for the flow variable. A solution is assumed in the form of

$$\theta(z,t) = \theta_0(z) + \theta_1(z)\varepsilon e^{i\omega t} \tag{7}$$

Where,

$$\theta_0(z) = \theta_{00}(z) + \delta\theta_{01}(z) \tag{8a}$$

$$\theta_1(z) = \theta_{10}(z) + \delta\theta_{11}(z) \tag{8b}$$

Substituting equation (7) into equations (5) and (6) and simplifying the results, the following coefficients of the orders of ε are collected:

Order ε^0

$$-P_r\frac{d\theta_0}{dz} = \frac{d}{dz}\left[(1+\delta z)^n \frac{d\theta_0}{dz}\right] - R^2\theta_0 \tag{9}$$

Subject to

$$\theta_0 = 1 + A_0\cos\omega t \text{ on } z = 0 \text{ and } \theta_0 \to 0 \text{ as } z \to \infty \tag{10}$$

Order ε^1

$$\frac{i\omega}{4}P_r\theta_1 - P_r\frac{d\theta_1}{dz} - P_r A\frac{d\theta_0}{dz} = \frac{d}{dz}\left[(1+\delta z)^n \frac{d\theta_1}{dz}\right] - R^2\theta_1 \tag{11}$$

Subject to

$$\theta_1 = 0 \text{ on } z = 0 \text{ and } \theta_1 \to 0 \text{ as } z \to \infty \tag{12}$$

Putting equations (8a) and (8b) into equations (9) – (12) and compiling the orders of δ with their corresponding boundary conditions, it yielded:

$$\frac{d^2\theta_{00}}{dz^2} + P_r \frac{d\theta_{00}}{dz} - R^2\theta_{00} = 0 \tag{13}$$

Subject to

$$\theta_{00} = 1 + A_0 \cos\omega t \text{ on } z = 0 \text{ and } \theta_{00} \to 0 \text{ as } z \to \infty \tag{14}$$

$$\frac{d^2\theta_{10}}{dz^2} + P_r \frac{d\theta_{10}}{dz} - \left(\frac{i\omega}{4} P_r + R^2\right)\theta_{10} = -P_r A \frac{d\theta_{00}}{dz} \tag{15}$$

Subject to

$$\theta_{10} = 0 \text{ on } z = 0 \text{ and } \theta_{10} \to 0 \text{ as } z \to \infty \tag{16}$$

$$\frac{d^2\theta_{01}}{dz^2} + P_r \frac{d\theta_{01}}{dz} - R^2\theta_{01} = nz\frac{d^2\theta_{00}}{dz^2} - n\frac{d\theta_{00}}{dz} \tag{17}$$

Subject to

$$\theta_{01} = 0 \text{ on } z = 0 \text{ and } \theta_{01} \to 0 \text{ as } z \to \infty \tag{18}$$

$$\frac{d^2\theta_{11}}{dz^2} + P_r \frac{d\theta_{11}}{dz} - \left(\frac{i\omega}{4} P_r + R^2\right)\theta_{11} = -nz\frac{d^2\theta_{10}}{dz^2} - n\frac{d\theta_{10}}{dz} - P_r A\frac{d\theta_{01}}{dz} \tag{19}$$

Subject to

$$\theta_{11} = 0 \text{ on } z = 0 \text{ and } \theta_{11} \to 0 \text{ as } z \to \infty \tag{20}$$

Solving the system (13 – 14) and putting the result into (15 – 17) and substituting the solution of (17 – 18) into (19 – 20), it gave

$$\theta_{00}(z) = B_1 e^{m_2 z}$$

$$\theta_{01}(z) = B_2 z e^{m_2 z}$$

$$\theta_{10}(z) = N_1 H$$

$$\theta_{11}(z) = N_{10} z^2 e^{m_2 z} + N_{11} z e^{m_2 z} + N_{12} H + N_{13} z^2 e^{m_4 z} + N_{14} z e^{m_4 z}$$

Using equations (7), (8a) and (8b),

$$\theta_0(z) = A_1 e^{m_2 z}$$

$$\theta_1(z) = N_1 H_1 + \delta B_3$$

Hence,

$$\theta(z,t) = A_1 e^{m_2 z} + A_2 \varepsilon e^{i\omega t} \tag{21}$$

3. Results and Discussion

In the preceding section, the problem of variation in ground temperature under the influence of spatial dependent thermal conductivity and radiation was formulated and solved by means of perturbation method. The expression for the ground temperature was obtained. To illustrate the variations in this quantity, its numerical value was computed with respect to the variations in the governing parameters namely, the Prandtl number (P_r), radiation term (R), frequency of oscillation (ω), depth (z) and thermal conductivity (k). The graphical results are also presented.

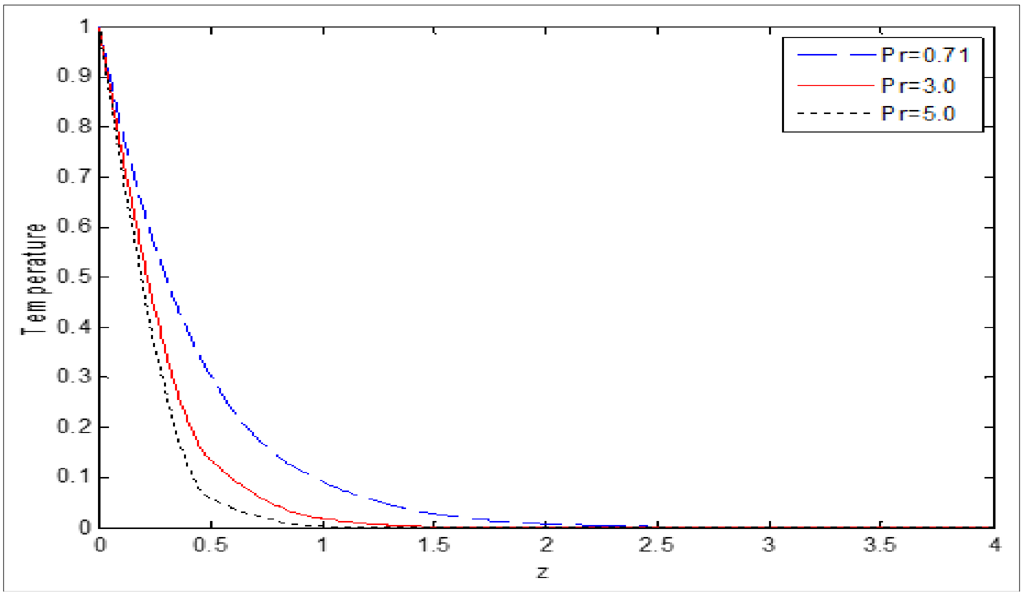

Fig 2: Temperature profiles for $\varepsilon = 0.01, \delta = 0.01\ \ n = 2, A_0 = 1, z = 0.01,\ \omega = \pi/2, t = 1, A = 1, R = 2$ at different values of P_r.

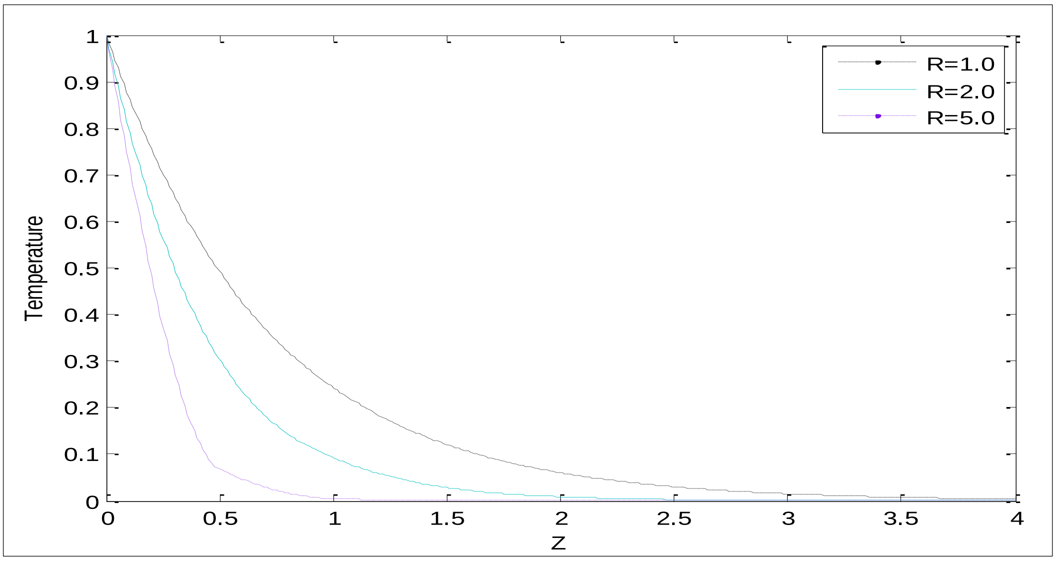

Fig 3: Temperature profiles for $\varepsilon = 0.01, \delta = 0.01\ \ n = 2, A_0 = 1, z = 0.01\ \omega = \pi/2, t = 1, A = 1, P_r = 0.71$ at different values of R.

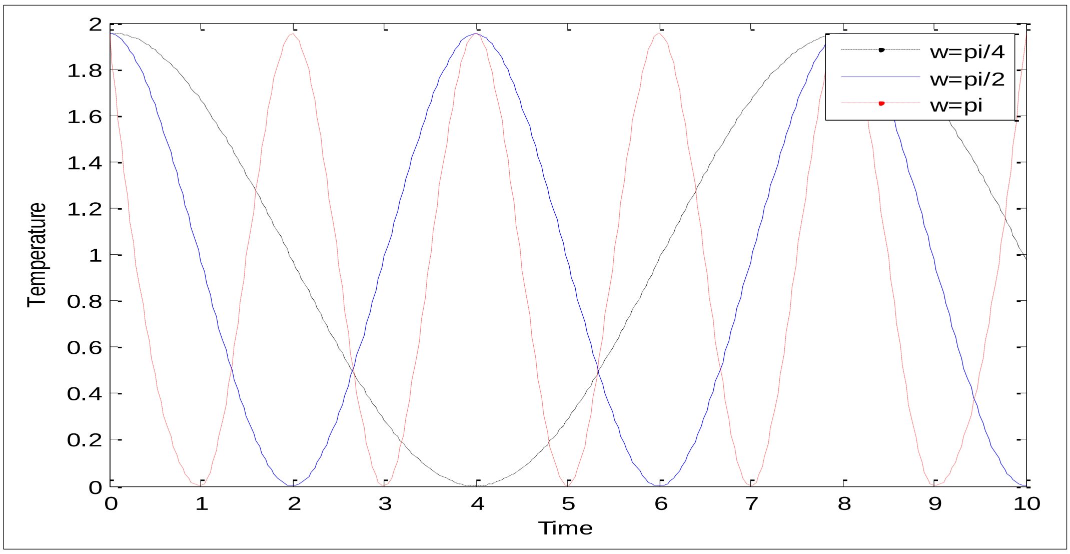

Fig 4: Temperature profiles for $\varepsilon = 0.01, \delta = 0.01,\ \ n = 2\ A_0 = 1, z = 0.01,\ A = 1, R = 2, P_r = 0.71$ at different values of ω.

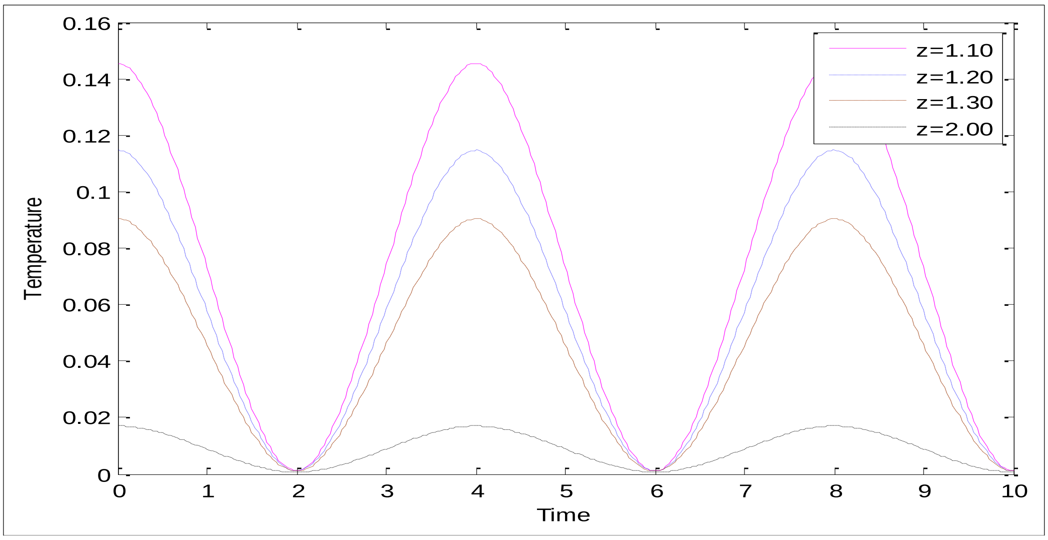

Fig 5: Temperature profiles for $\varepsilon = 0.01, \delta = 0.01$, $n = 2, A_0 = 1,\ \omega = \pi/2, A = 1, R = 2, P_r = 0.71$, at different values of z.

Table 1: Influence of the thermal conductivity on the temperature

z	E	δ	n	A	ω	R	P_r	T	k_0	$k = k_0(1+\delta z)$	θ
1.0	0.01	0.01	2.0	1.0	$\pi/2$	2.0	0.71	1.0	0.7	0.707	0.0927
2.0										0.714	0.0089
3.0										0.721	8.6911e-004

In figures 2 – 5 and table 1, the variations in the ground temperature for different values of the flow parameters mentioned above are presented.

Figures 2 and 3 indicated that increase in Prandtl number or Radiation term resulted in decrease in the ground temperature respectively. These results correspond to that of Israel-Cookey *et al.* [5] and Nwaigwe [7].

Figure 4 revealed that increase in the frequency of oscillation decreased the period or cycle of oscillation. This corresponds to the study of Stroud [9].

Figure 5 showed that as the depth of the ground increased the amplitude or the ground temperature decreased.

Table 1 revealed that increase in the thermal conductivity decreased the ground temperature.

4. Conclusion

Variation in ground temperature under the influence of spatial dependent soil thermal conductivity and Radiation has been studied. The non-dimensional governing equation was solved using perturbation method. It was observed that the ground temperature decreased with increase in Prandtl number, Radiation term and soil thermal conductivity. These quantities speed up the rate of heat diffusion and thereby decreased the temperature. Ground temperature undergoes cycle with increase in the depth and frequency of oscillation. These changes are confined to near the surface of the ground in the case of the change in depth of the soil.

5. Appendices

$A_1 = B_1 + \delta B_2 z$, $A_2 = N_1 H + \delta B_3$, $B_1 = 1 + A_0 \cos \omega t$, $B_2 = zN_2 + N_3$

$B_3 = N_{10} z^2 e^{m_2 z} + N_{11} z e^{m_2 z} + N_{12} H + N_{13} z^2 e^{m_4 z} + N_{14} z e^{m_4 z}$, $H = e^{m_2 z} - e^{m_4 z}$, $m_2 = -\frac{1}{2}\left\{P_r + \sqrt{Q_1}\right\}$

$m_4 = -\frac{1}{2}\left\{P_r + \sqrt{Q_1 + 4Q_{00}}\right\}$, $Q_1 = P_r^2 + 4R^2$, $Q_{00} = \frac{1}{4} i\omega P_r$, $N_1 = \frac{P_r A m_2}{Q_{00}}$,

$N_2 = \frac{-nm_2^2}{2\Delta_4}$, $N_3 = \frac{-nm_2(P_r + m_2)}{\Delta_4^2}$, $N_4 = \frac{-nm_2^2}{\Delta_4^3}$, $N_5 = -P_r A N_2 m_2$

$N_6 = -\left(nN_1 m_2^2 + 2P_r A N_2 m_2 - P_r A N_3 m_2\right)$, $N_7 = N_9 m_4^2$, $N_8 = -(P_r A N_3 + nN_1 m_2)$, $N_9 = -nN_1 m_4$

$N_{10} = \Delta_2 N_5$, $N_{11} = \Delta_2 (N_6 - 2\Delta_3 N_5)$, $N_{12} = \Delta_2 (2\Delta_3 N_5 - 2N_{10} - \Delta_3 N_6 + N_8)$, $N_{13} = \frac{1}{2}\Delta_5 N_7$

$N_{14} = -2N_{13}\Delta_5$, $\Delta_1 = -Q_{00}$, $\Delta_2 = \frac{1}{\Delta_1}$, $\Delta_3 = \Delta_2 \Delta_4$, $\Delta_4 = 2m_2 + P_r$ and $\Delta_5 = \frac{1}{2m_4 + P_r}$

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