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Exponential Renyi's entropy of 'Type (a, ß)' and new mean code-Word length

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Abstract

In this paper, we introduce a quantity which is called exponential entropy of 'type (α, β)' and discuss its some major properties corresponding to exponential entropy of concave function. Further, a new measure $L_{\beta}^{\alpha}(A)$ called average codeword length of 'type (α, β)' has been define and its relationship with a result of an exponential Reyni's entropy of 'type (α, β)'has been discussed. Using $L_{\beta}^{\alpha}(A)$ and $E_{\alpha}^{\beta}(A)$, coding theorem for discrete noiseless has been proved. At the end of the paper, we illustrate the veracity of the theorem by taking empirical data as given in the table 3.1 and 3.2.

Keywords: Renyi's entropy, Exponential entropy, Mean code word length, Holder's inequality, Huffman code, Kraft inequality convex and concave function

1. Introduction

Let $\Delta_n = \{A = (a_{1,} a_2, ..., a_n) : a_i \geq 0, i = 1, 2, ..., n, n \geq 2, \sum_{i=1}^{n} a_i = 1\}$ be a set of n-complete probability distributions. For any probability distribution $A = (a_{1,} a_2, ..., a_n) \in \Delta_n$, Shannon's entropy [13], is defined as

$$H(A) = -\sum_{i=1}^{n} a_i \log a_i \tag{1.1}$$

Various generalized entropies have been introduced in the literature, taking the Shannon entropy as basic and have found applications in various disciplines such as economics, statistics, information processing and computing etc.

Generalizations of Shannon's entropy started with Renyi's entropy [12] of order-α, given by

$$H_{\alpha}(A) = \frac{1}{1-\alpha} \log \left[\sum_{i=1}^{n} a_i^{\alpha} \right], \quad \alpha \neq 1;\ \alpha > 0 \tag{1.2}$$

Campbell [3] studied exponentials of the Shannon's and Renyi's entropies, given by

$$E(A) = e^{H(A)} \tag{1.3}$$

and $E^{\alpha}(A) = e^{H_{\alpha}(A)}$, (1.4)

where $H(A)$ and $H_{\alpha}(A)$ represent respectively the Shannon's and Ranyi's entropies. It may also be mentioned that Koski and Persson [7] studied

$$E_{(\alpha,\beta)}(A) = e^{H_{(\alpha,\beta)}(A)}, \tag{1.5}$$

exponential of Kapur's entropy [6] given by

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$$H_{(\alpha,\beta)}(A)=\frac{1}{(\beta-\alpha)}\log\frac{\sum_{i=1}^{n}a_i^{\alpha}}{\sum_{i=1}^{n}a_i^{\beta}},\quad \alpha\neq\beta;\ \alpha,\beta>0 \qquad (1.6)$$

It is interesting to notice that, in the case of discrete uniform distribution $A\in\Delta_n$, (1.3), (1.4) and (1.5) all reduce to n, just the 'size of the sample space of the distribution'.

This paper is organized as follows: Sec.II, define the exponential entropy of 'type (α,β)' and discuss its some major properties corresponding to exponential entropy of concave function. Sec. III, discuss a measure of length and discuss the relationship between the $E_{\alpha}^{\beta}(A)$ and $L_{\beta}^{\alpha}(A)$

In the next section, we define a new information measure $E_{\alpha}^{\beta}(A)$ and study its properties.

2. Exponential Entropy of 'Type (α,β)' and its Properties:

Corresponding to Renyi's entropy of 'type (α,β)', the exponential entropy of 'type (α,β)' is defined as follows:

Definition: Exponential 'type (α,β)' entropy of a discrete distribution A is given by:

$$E_{\alpha}^{\beta}(A)=\frac{\left[e^{\log_2\sum_{i=1}^{n}a_i^{\alpha}}-e^{\log_2\sum_{i=1}^{n}a_i^{\beta}}\right]}{\beta-\alpha},\quad \alpha,\beta>0;\ \alpha\neq\beta \text{ either } \alpha>1;\beta<1 \text{ or } \alpha<1;\beta>1. \qquad (2.1)$$

(i) When $\beta=1;\ \alpha\to1$ or $\alpha=1;\ \beta\to1$ measure (2.1) reduces to Shannon's entropy.

(ii) When $\beta=1$ or $\alpha=1$ (2.1) becomes exponential entropy of type α or β.

The quantity (2.1) introduced in the present section is entropy. Such a name will be justified, if it shares some major properties with Shannon's and other entropies in the literature. We study some such properties in the next theorem.

Theorem 2.1: The measure of information $E_{\alpha}^{\beta}(A)$, $\{A=(a_{1,}a_2,...,a_n), 0<a_i\leq1, \sum_{i=1}^{n}a_i=1\}$ has the following properties:

1) Symmetry:

$E_{\alpha}^{\beta}(A)=E_{\alpha}(a_{1,}a_2,...,a_n)$ is a symmetric function of $(a_{1,}a_2,...,a_n)$.

2) Non-negative:

$E_{\alpha}^{\beta}(A)>0$, for all $\beta,\alpha>0;\ \alpha\neq\beta$.

3) Expansible:

$E_{\alpha}^{\beta}(a_{1,}a_2,...,a_n;0)=E_{\alpha}^{\beta}(a_{1,}a_2,...,a_n)$.

4) Decisive:

$E_{\alpha}^{\beta}(0,1)=E_{\alpha}^{\beta}(1,0)=0$.

5) Maximality:

$$E_{\alpha}^{\beta}(a_{1,}a_2,...,a_n)\leq E_{\alpha}^{\beta}(1/n,1/n,...,1/n)=\frac{e^{(1-\alpha)\log_2 n}-e^{(1-\beta)\log_2 n}}{\beta-\alpha}.$$

6) Concavity:

The measure $E_{\alpha}^{\beta}(A)$ is a concave function of the probability distribution $A=(a_{1,}a_2,...,a_n), a_i\geq0, \sum_{i=1}^{n}a_i=1,$ when either $\alpha>1;\ \beta\leq1$ or $\alpha\leq1;\ \beta>1$.

7) Continuity:

$E_{\alpha}^{\beta}(a_{1,}a_2,...,a_n)$ is continuous in the region $a_i\geq0$ for all $\alpha,\beta>0;\ \alpha\neq\beta$.

Proof: (1), (3), (4) and (5): these properties are obvious and can be verified easily for property (7) We know that $\sum_{i=1}^{n}a_i^{\alpha}-\sum_{i=1}^{n}a_i^{\beta}$ is continuous in the region $a_i\geq0$ for all $\alpha,\beta>0$. Hence, $E_{\alpha}^{\beta}(A)$, is also continuous in the region $a_i\geq0$ for all $\alpha,\beta>0;\ \alpha\neq\beta$.

Property (2): The measure $E_{\alpha}^{\beta}(A)$ is non-negative for all $\alpha,\beta>0;\ \alpha\neq\beta$.

Proof: We consider the following cases:

Case (i): When $\alpha>1;\beta<1$

$e^{\log_2\left[\sum_{i=1}^{n} a_i^{\alpha}\right]} < 1$ and $e^{\log_2\left[\sum_{i=1}^{n} a_i^{\beta}\right]} > 1$. (2.2)

From (2.2), we get

$e^{\log_2\left[\sum_{i=1}^{n} a_i^{\alpha}\right]} - e^{\log_2\left[\sum_{i=1}^{n} a_i^{\beta}\right]} < 0$,

Since for $\alpha > 1; \beta < 1 \quad \Rightarrow \beta - \alpha < 0$.

We get $\dfrac{e^{\log_2\left[\sum_{i=1}^{n} a_i^{\alpha}\right]} - e^{\log_2\left[\sum_{i=1}^{n} a_i^{\beta}\right]}}{\beta-\alpha} > 0$. i.e., $E_{\alpha}^{\beta}(A) > 0.$

Case (ii): Similarly, for $\alpha < 1; \beta > 1;$ we get

$e^{\log_2\left[\sum_{i=1}^{n} a_i^{\alpha}\right]} - e^{\log_2\left[\sum_{i=1}^{n} a_i^{\beta}\right]} > 0$,

and $\beta - \alpha > 0$,

we get $\dfrac{e^{\log_2\left[\sum_{i=1}^{n} a_i^{\alpha}\right]} - e^{\log_2\left[\sum_{i=1}^{n} a_i^{\beta}\right]}}{\beta-\alpha} > 0$.

i.e., $E_{\alpha}^{\beta}(A) > 0.$

From case (i) and (ii), we conclude that

$E_{\alpha}^{\beta}(A) > 0$ for all $\alpha, \beta > 0; \alpha \neq \beta$ either $\alpha > 1; \beta < 1$ or $\alpha < 1; \beta > 1$.

To prove the next property, we shall use the following definition of a concave function.

Definition :(Concave Function): A function $f(.)$ over the points in a convex set $\Re$ is concave if for all $r_1, r_2 \in \Re$ and $\mu \in (0,1)$

$$\mu f(r_1) + (1-\mu) f(r_2) \le f(\mu r_1 + (1-\mu) r_2) \tag{2.3}$$

The function $f(.)$ is convex if the above inequality holds with $\geq$ in place of $\leq$.

Property (6): The measure $E_{\alpha}^{\beta}(A)$ is a concave function of the probability distribution $A = (a_1, ..., a_n), a_i \geq 0, \sum_{i=1}^{n} a_i = 1,$ for all $\alpha, \beta > 0; \alpha \neq \beta$ either $\alpha > 1; \beta \leq 1$ or $\alpha \leq 1; \beta > 1$.

Proof: Associated with the random variable $X = (x_1, x_2, ..., x_n)$, let us consider r distributions

$$A_k(X) = \{a_k(x_1), a_k(x_2), ..., a_k(x_n)\},$$

where

$$\sum_{i=1}^{n} a_k(x_i) = 1, k = 1, 2, ..., r.$$

Next, let there be r numbers $(\lambda_1, \lambda_2, ..., \lambda_r)$ such that $\lambda_k \geq 0, \sum_{k=1}^{r} \lambda_k = 1$ and define

$$A_0(X) = \{a_0(x_1), a_0(x_2), ..., a_0(x_n)\},$$

where

$$a_0(x_i) = \sum_{k=1}^{r} \lambda_k a_k(x_i), i = 1, 2, ..., n.$$

Obviously $\sum_{i=1}^{n} a_0(x_i) = 1$, and thus $A_0(X)$ is a bonafide distribution of X.

If $\alpha > 1; \beta < 1$, then we have

$$\sum_{k=1}^{r} \lambda_k E_{\beta}^{\alpha}(A_k) - E_{\beta}^{\alpha}(A_0)$$

$$= \sum_{k=1}^{r} \lambda_k E_{\alpha}^{\beta}(A_k) - \frac{\left[e^{\log_2\left[\sum_{i=1}^{r} \lambda_i a_i\right]^{\alpha}} - e^{\log_2\left[\sum_{i=1}^{r} \lambda_i a_i\right]^{\beta}} \right]}{\beta - \alpha}$$

$$\le \sum_{k=1}^{r} \lambda_k E_{\alpha}^{\beta}(A_k) - \frac{\left[e^{\log_2 \sum_{i=1}^{r} \lambda_i a_i^{\alpha}} - e^{\log_2 \sum_{i=1}^{r} \lambda_i a_i^{\beta}} \right]}{\beta - \alpha} \text{ (by Jensen inequality)}$$

$$\sum_{i=1}^{r} \lambda_i E_{\alpha}^{\beta}(A_i) \le E_{\alpha}^{\beta}(A) \tag{2.4}$$

Similarly, for $\alpha \le 1;\ \beta > 1$ (2.4) holds. Therefore $E_{\alpha}^{\beta}(A)$ is a concave function for all $\alpha, \beta > 0;\ \alpha \ne \beta$.

In the next section, we will define a new exponential mean length and prove a lower bound $E_{\alpha}^{\beta}(A)$.

3. A Measure of Length

In the ususal discussion of the coding theorem for a noiseless channel (Feinstein [4]) one choose code lengths to minimize the average code length. The minimization is done subject to the constraint that the code be uniquely decipherable. The solution of this minimization problem is that the best code length for an input symbol of probability p is $-\log p$. This solution has the disadvantage that the code length is very great if the probability of the symbol is very small.

Implicit in the use of average code length as a criterion of performance is the assumption that cost varies linearly with code length. This is not always the case. In the present paper another measure of code length is introduced which implies that the cost is an exponential function of code length. Linear dependence is a limiting case of this measure.

A coding theorem analogus to the ordinary coding theorem for a noiseless channel will be proved. The theorem states that it is possible to encode so that the measure of length is arbitrarily close to the exponential Renyi's entropy of 'type (α, β)' of the input.

Let a finite set of n input symbols $X = \{x_1, x_2, ..., x_n\}$ be encoded using alphabet of D sysmbols, then it has been shown by Feinstien [4] that there is a uniquely decipherable code with lengths $N_1, N_2, ..., N_n$ if and only if the Kraft inequality holds, that is,

$$\sum_{i=1}^{n} D^{-N_i} \le 1, \tag{3.1}$$

where D is the size of code alphabet. Furthermore, if

$$L = \sum_{i=1}^{n} N_i a_i \tag{3.2}$$

is the average codeword length, then for a code satisfying (3.1), the inequality

$$L \ge H(A) \tag{3.3}$$

is also fulfilled and equality holds if and only if

$$N_i = -\log_D(a_i) \ ;\ \forall i = 1, 2, ..., n \tag{3.4}$$

and that by suitable encoded into words of long sequences, the average length can be made arbitrarily close to $H(A)$, (see Feinstein [4]). This is Shannon's noiseless coding theorem. By considering Renyi's entropy (see e.g. [12]), a coding theorem and analogous to the above noiseless coding theorem has been established by Campbell [2] and the authors obtained bounds for it in terms of $H_{\alpha}(A)$.

Kieffer [8] defined a class rules and showed $H_{\alpha}(A)$ is the best decision rule for deciding which of the two sources can be coded with expected cost of sequences of length N when $N \to \infty$, where the cost of encoding a sequence is assumed to be a function of length only. Futher, in Jelinek [11] it is shown that coding with respect to Campbell's mean length is useful in minimizing the problem of buffer overflow which occurs when the source symbol is produced at a fixed rate and the code words are stored temporarily in a finite buffer.

There are many different codes whose lengths satisfy the constraint (3.1). To compare different codes and pick out an optimum code it is customary to examine the mean length, $\sum_{i=1}^{n} N_i a_i$, and to minimize this quantity. This is a good procedure if the cost of using a sequence of length N_i is directly proportional to N_i. However, there may be occasions when the cost is more nearly an exponential function of N_i. This could be the case, for example, if the cost of encoding and decoding equipment were an important factor. Thus, in some circumstances, it might be more appropriate to choose a code which minimizes the quantity

$$C = e^{\alpha \log_2 \sum_{i=1}^{n} a_i 2^{-N_i\left(\frac{\alpha-1}{\alpha}\right)}} - e^{\beta \log_2 \sum_{i=1}^{n} a_i 2^{-N_i\left(\frac{\beta-1}{\beta}\right)}},$$

where α, β is some parameter related to the cost. For reasons which will become evident later we prefer to minimize a monotonic function of C.

Clearly this will also minimize C. In order to make the result of this paper more directly comparable with the usual coding theorem we introduce a quanity which resembles the mean length. Let a code length of order α, β be defined by

$$L_{\beta}^{\alpha}(A) = \frac{1}{\beta - \alpha}\left[e^{\alpha \log_2 \sum_{i=1}^{n} a_i 2^{-N_i\left(\frac{\alpha-1}{\alpha}\right)}} - e^{\beta \log_2 \sum_{i=1}^{n} a_i 2^{-N_i\left(\frac{\beta-1}{\beta}\right)}}\right]$$

$$\alpha, \beta > 0;\ \alpha \ne \beta \text{ either } \alpha > 1; \beta < 1 \text{ or } \alpha < 1; \beta > 1. \tag{3.5}$$

In the following theorem, we give a lower bound for $L_\beta^\alpha(A)$ in terms of $E_\alpha^\beta(A)$.

Theorem 3.1: If $N_1, N_2, ..., N_n$, denote the length of a uniquely decipherable code satisfying (3.1), then $L_\beta^\alpha(A) \geq E_\alpha^\beta(A)$ (3.6)

Proof: By Holder's inequality,

$$\left[\sum_{i=1}^n x_i^p\right]^{\frac{1}{p}} \left[\sum_{i=1}^n y_i^q\right]^{\frac{1}{q}} \leq \sum_{i=1}^n x_i y_i \ , \qquad (3.7)$$

for all $x_i, y_i > 0;\ i = 1, 2, ..., n$ and $\frac{1}{p} + \frac{1}{q} = 1;\ p < 1(\neq 0);\ q < 0$ or $q < 1(\neq 0);\ p < 0.$

Making the substitutions, $p = \dfrac{\alpha - 1}{\alpha};\ q = 1 - \alpha;\ x_i = p_i^{\frac{\alpha}{\alpha-1}} 2^{-N_i};\ y_i = p_i^{\frac{\alpha}{1-\alpha}};$ in (3.7) and using (3.1), we get

$$\left[\sum_{i=1}^n a_i 2^{-N_i\left(\frac{\alpha-1}{\alpha}\right)}\right]^{\frac{\alpha}{\alpha-1}} \leq \left[\sum_{i=1}^n a_i^\alpha\right]^{\frac{1}{\alpha-1}} . \qquad (3.8)$$

Now, we have two possibilities:

Case 1: If $\alpha > 1;\ \beta < 1$, (3.8) gives us

$$\left[\sum_{i=1}^n a_i\, 2^{-N_i\left(\frac{\alpha-1}{\alpha}\right)}\right]^{\alpha} \leq \sum_{i=1}^n a_i^\alpha \qquad (3.9)$$

Taking $\log_2$ on both side of (3.9) and after simplification, we get

$$e^{\alpha \log_2 \sum_{i=1}^n a_i\, 2^{-N_i\left(\frac{\alpha-1}{\alpha}\right)}} \leq e^{\log_2 \sum_{i=1}^n a_i^\alpha}, \text{ and } e^{\beta \log_2 \sum_{i=1}^n a_i\, 2^{-N_i\left(\frac{\beta-1}{\beta}\right)}} \geq e^{\log_2 \sum_{i=1}^n a_i^\beta} \qquad (3.10)$$

since $\dfrac{1}{\beta - \alpha} < 0$ for $\alpha > 1;\ \beta < 1$, Therefore

$$\frac{1}{\beta-\alpha}\left[e^{\alpha \log_2 \sum_{i=1}^n a_i\, 2^{-N_i\left(\frac{\alpha-1}{\alpha}\right)}} - e^{\beta \log_2 \sum_{i=1}^n a_i\, 2^{-N_i\left(\frac{\beta-1}{\beta}\right)}}\right] \geq \frac{e^{\log_2 \sum_{i=1}^n a_i^\alpha} - e^{\log_2 \sum_{i=1}^n a_i^\beta}}{\beta - \alpha}. \qquad (3.11)$$

i.e., $L_\beta^\alpha(A) \geq E_\alpha^\beta(A)$.

Case 2: If $\alpha < 1;\ \beta > 1$, the proof follows on the same lines. But the inequality sign in (3.10) is reversed.

It is clear that the equality in (3.10) is true if $N_i = -\log a_i$. The necessity of this condition for equality in (3.6) follows from the condition for equality in Holder's inequality. In the case of the Holder's inequality given above, equality holds iff for some c,

$$x_i^p = c\, y_i^q \ . \qquad (3.12)$$

Plugging this condition into our situation, with x_i, y_i and p, q as specified, and using the fact that the $\sum_{i=1}^n a_i = 1$, the necessity is proven.

Remark:1. If $\alpha \to 1; \beta = 1$ or $\alpha = 1; \beta \to 1$, Then (3.6) becomes well known result studied by Shannon.

i.e., $\bar{L} = \sum_{i=1}^n a_i N_i \geq -\sum_{i=1}^n p_i \log p_i$. (3.13)

Remark: 2. Huffman [5] introduced a procedure for designing a variable length source code which achieves performance close to Shannon's entropy bound. For individual codeword lengths N_i, the average length $\bar{L} = \sum_{i=1}^n a_i N_i$ of Huffman code is always within one unit of Shannon's measure of entropy, i.e., $H(A) \leq \bar{L} < H(A) + 1$

Where $H(A) = \sum_{i=1}^n a_i \log_2 a_i$ is the Shannon's measure of entropy. Huffman coding scheme can also be applied to codeword length, i.e., $L_\beta^\alpha(A)$ for codeword length N_i, the average length $L^\alpha(A)$ of Huffman code satisfies

$$L_\beta^\alpha(A) \geq E_\alpha^\beta(A)$$

In the following Tables, we have developed the relation between the entropy $E_\alpha^\beta(A)$ and average codeword length $L_\beta^\alpha(A)$.

Table 3.1: for $\alpha > 1;\ \beta < 1$

Relation between $E_\alpha^\beta(A)$ and $L_\beta^\alpha(A)$.								
Length of Huffman code words (N_i)	**Huffman code words**	α	β	a_i	$L_\beta^\alpha(A)$	$E_\alpha^\beta(A)$	**Efficiency code** $\eta = \frac{E_\alpha^\beta(A)}{L_\beta^\alpha(A)}$	**Redundancy code** $\nu = 1-\eta$
2	00	3	0.5	0.3	1.3775306	1.3672434	0.992532144	0.00746785
2	10			0.25				
2	11			0.2				
3	011			0.1				
4	0100			0.1				
4	0101			0.05				

From the above Table 3.1, we can make the observation as average codeword length $L_\beta^\alpha(A)$ exceeds the exponential entropy $E_\alpha^\beta(A)$ by 0.746785%.

Table 3.2: for $\alpha < 1;\ \beta > 1$

Relation between $E_\alpha^\beta(A)$ and $L_\beta^\alpha(A)$.								
Length of Huffman code words (N_i)	**Huffman code words**	α	β	a_i	$L_\beta^\alpha(A)$	$E_\alpha^\beta(A)$	**Efficiency code** $\eta = \frac{E_\alpha^\beta(A)}{L_\beta^\alpha(A)}$	**Redundancy code** $\nu = 1-\eta$
2	00	0.5	3	0.3	1.3877006	1.3797204	0.994249335	0.00575066
2	10			0.25				
2	11			0.2				
3	011			0.1				
4	0100			0.1				
4	0101			0.05				

From the above Table 3.2, we can make the observation as average codeword length $L_\beta^\alpha(A)$ exceeds the exponential entropy $E_\alpha^\beta(A)$ by 0.575066%.

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