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**Ping Kang**
Department of Mathematics, Tianjin Polytechnic University, Tianjin, People's Republic of China

**Correspondence**
**Ping Kang**
Department of Mathematics, Tianjin Polytechnic University, Tianjin, People's Republic of China

# An extension of a theorem of Ito on conjugacy class sizes

## Ping Kang

**Abstract**

Let G be a finite group and let $G^*$ be the set of elements of prime power order of G. In this paper we show that, if no conjugacy class size of $G^*$ is divisible by the product pq, then G is p-nilpotent with abelian Sylow p-subgroup or G is q-nilpotent with abelian Sylow q-subgroup, where p, q are distinct primes.

**Keywords:** Finite group, conjugacy class sizes, elements of prime power orders

## 1. Introduction

A well-established research area in finite group theory consists in exploring the relationship between the structure of a group G and certain sets of positive integers, which are naturally associated to G. One of those sets, denoted by cs(G), is the set of conjugacy class sizes of the elements of G.

A classical remark concerning the influence of cs(G) on the group structure of G is the following: if p is a prime number which does not divide any element of cs(G), then G has a central Sylow p-subgroup (see [1, Theorem 33.4]). In [2], Ito proves the following well-known theorem: if no conjugacy class size of G is divisible by the product pq, then G is p-nilpotent with abelian Sylow p-subgroup or G is q-nilpotent with abelian Sylow q-subgroup, where p, q are distinct primes (see [2,Proposition 5.1]). In view of that, one can ask whether particular subsets of cs(G) still encode nontrivial information on the structure of G. For instance, cs ($G^*$ ), which is the set of conjugacy class sizes of the elements of prime power order of G. In [3] Kong and Guo obtain a complete extension of the above former result: if p is a prime number which does not divide any element of cs($G^*$), then G has a central Sylow p-subgroup (see [3,Lemma 2.4]).

In this paper, we will continue to focus our attention on cs($G^*$) and obtain a complete extension of above resul of Itot. Our main result is the following:

Theorem A. Let G be a group and p, q distinct primes. If no conjugacy class size of $G^*$ is divisible by the product pq, then G is p-nilpotent with abelian Sylow p-subgroup or G is q-nilpotent with abelian Sylow q-subgroup.

## 2. Proof of the Main Theorem

**Proof of Theorem A.** Let P and Q be a Sylow p-subgroup and a Sylow q-subgroup of G, respectively. Now we write $M = C_G(P)$ and $N = C_G(Q)$. By assumption every element of prime power order of G centralizes some conjugate of either P or Q. Thus every element of G centralizes some conjugate of either P or Q. It follows that $G = \bigcup_{x \in G} M^x \cup \bigcup_{y \in G} N^y$.

We can assume that M and N are proper subgroups of G, as otherwise we are done. It follows that $|G| < |G : N_G(M)||M| + |G : N_G(N)||N|$, where the inequality is strict, since the identity of G is counted more than once. So $1 < \frac{1}{|N_G(M) : M|} + \frac{1}{|N_G(N) : N|}$ and hence, say,

$M = N_G(M)$. Since $M = C_G(P) \le N_G(P) \le N_G(M)$, it follows that $C_G(P) = N_G(P)$. Thus P is abelian and, by criterion of Burnside, G is p-nilpotent.

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