International Journal of Statistics and Applied Mathematics 2017; 2(1): 01-05

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2017; 2(1): 01-05

www.mathsjournal.com
Received: 07-11-2016
Accepted: 10-12-2016

Huma Akhtar
Research Scholar Govt. M.V.M. Bhopal, India

Geeta Modi
Department of Mathematics, Govt. M.V.M. Bhopal, India

Sushma Duraphe
Department of Mathematics, Govt. M.V.M. Bhopal, India

Correspondence
Huma Akhtar
Research Scholar Govt. M.V.M. Bhopal, India

Transforming and optimizing multi-objective quadratic fractional programming problem

Huma Akhtar, Geeta Modi and Sushma Duraphe

Abstract
In this paper, we defined average technique and new average technique by using mean & median to solve multi-objective quadratic fractional programming problem (MOQFPP) to single objective quadratic fractional programming problem (QFPP) and suggested an algorithm for it. This can be illustrate with the help of numerical example. The numerical result in this paper indicates that our technique is promising.

Keywords: MOQFPP, Chandra Sen., mean & median, average mean & median and new average mean & median techniques

1. Introduction

Quadratic fractional programming problems (i.e. ratio of objectives that have numerator and denominator) has attracted considerable research and interest since they have been utilized in production planning, financial and corporative planning, health care and hospital planning. Several methods to solve such problems are proposed in (1962) [1], their method depends on transforming this linear fractional programming problem (LFPP) to an equivalent linear program. In (1983), Chandra Sen. [2] defined the multi-objective linear programming problem, and suggested an approach to construct the multi-objective function under the limitation that the optimal values of individual problems are greater than zero. In (2013) [3], Sulaiman, and Abdul Rahim gave transformation technique to solve multi-objective linear fractional programming problem, in which they used the average mean and median & new average mean and median technique to solve multi-objective linear fractional programming problem (MOQFPP) to single objective linear fractional programming problem. In (2006), Sulaiman and Sadiq studied the multi-objective function by solving the multi-objective programming problem, using mean and median value. Again in (2008) [4], Sulaiman, Hamadameen and Abdul Qader define an optimal transformation technique to solve multi-objective linear programming problem. Also Sulaiman and Salih in (2010) studied the MOLFPP by using mean & median value [5].

In order to extend this work we have defined a MOQFPP and investigated the algorithm to solve quadratic fractional programming problem for multi-objective function, irrespective of the number of objectives with less computational burden and suggest an average mean & median technique and new average mean & median technique to generate the best optimal solution.

2. Quadratic Programming

If the optimization problem assumes the form

max. z (or min. z) = $\delta + C^T X + \frac{1}{2} X^T G X$

Subject to:

$$AX \begin{pmatrix} \leq \\ \geq \\ = \end{pmatrix} b$$

$$X \geq 0$$

Where $A = (a_{ij})_{m \times n}$ is a matrix of coefficients,

$\forall\ i = 1,2,3, \dots \dots .. m$ and j= 1,2,3,……..n, b = $(b_{1,} b_{2,\dots\dots} b_m)^T$, X = $(x_{1,} x_{2,\dots\dots\dots} x_n)^T$, $C^T = (c_{1,} c_{2,\dots\dots\dots} c_n)^T$

and $G = (g_{ij})_{n\times n}$ is a positive definite(negative definite) or positive semi definite(negative semi definite) symmetric square matrix and δ is scalar, moreover the constraints are linear and the objective function is quadratic. Such optimization problem is sad to be a quadratic programming problem (QPP).

3. Mathematical form of QFPP

The mathematical form of QFP problem is given as follows:

$$\text{Max. } z = \frac{C^T\text{X} + \delta + \frac{1}{2}\text{X}^T G\text{X}}{c^T\text{X} + \gamma}$$

Subject to:

$$A\text{X}\begin{pmatrix}\le\\ \ge\\ =\end{pmatrix}\text{b}$$

$$\text{X} \ge 0$$

Where G are (n× n) matrix of coefficients with G are symmetric matrices, X is an n-dimensional column vector of decision variables, c, C are n-dimensional column vector of constants, A is an (m× n) matrix and b is an m-dimensional column vector of constants, γ,δ are scalars.

In this paper the problem that has objective function is tried to be solved can be represented as follows.

$$\text{Max. } z = \frac{(c_1{}^T\text{X}+\alpha)(c_2{}^T\text{X}+\beta)}{c\text{X}+\gamma}$$

Subject to:

$$A\text{X}\begin{pmatrix}\le\\ \ge\\ =\end{pmatrix}\text{b}$$

$$\text{X} \ge 0$$

A is an m× n matrix, all vectors are assumed to be column vectors unless transposed(T). Where X is an n--dimensional column vector of decision variables, C_1, C_2, c are the n-dimensional column vector of constants, b is an m-dimensional vector of constants, α, β, γ are scalars.

4. Multi-Objective Quadratic Fractional Programming Problem

Multi-Objective functions are the ratio of two objective functions that have quadratic objective function in numerator and linear objective function in denominator, this is said to be MOQFPP then can be defined

$$\left.\begin{array}{l}
\text{Max. } z_1 = \dfrac{(c_{11}{}^T\text{X}+\alpha_1)(c_{21}{}^T\text{X}+\beta_1)}{\text{C}_1{}^T\text{X}+\gamma_r}\\
\text{Max. } z_2 = \dfrac{(c_{12}{}^T\text{X}+\alpha_2)(c_{22}{}^T\text{X}+\beta_2)}{\text{C}_2{}^T\text{X}+\gamma_2}\\
\quad\vdots\\
\text{Max. } z_r = \dfrac{(c_{1r}{}^T\text{X}+\alpha_r)(c_{2r}{}^T\text{X}+\beta_r)}{\text{C}_r{}^T\text{X}+\gamma_r}\\
\text{Min. } z_{r+1} = \dfrac{(c_{1r+1}{}^T\text{X}+\alpha_{r+1})(c_{2r+1}{}^T\text{X}+\beta_{r+1})}{\text{C}_{r+1}{}^T\text{X}+\gamma_{r+1}}\\
\quad\vdots\\
\text{Min. } z_s = \dfrac{(c_{1s}{}^T\text{X}+\alpha_s)(c_{2s}{}^T\text{X}+\beta_s)}{\text{C}_s{}^T\text{X}+\gamma_s}
\end{array}\right] \quad (4.1)$$

Subject to:

$$A\text{X} = \text{b} \quad (4.2)$$

$$\text{X} \ge 0 \quad (4.3)$$

Where b is an m-dimensional vector of constants, X is an n-dimensional column vector of decision variables, r is number of objective functions to be maximized, s is the number of objective functions to be maximized and minimized and (s-r) is the number of objective functions that is minimized. A is an m× n matrix of constants, all vectors are assumed to be column vectors unless transposed(T). c_{1i}, c_{2i}, C_i (where i = 1,2,…,s) are n-dimensional vectors of constants, $\alpha_i, \beta_i, \gamma_i$ (where i = 1,2,…,s) are scalars.

5. Solving MOQFPP by Using the Following Technique

5.1. Chandra Sen. Technique

The same approach which was taken by Sen. (1983) [2] is followed here to formulated the constraint objective function for the MOQFPP. Suppose we obtain a single value corresponding to each of the objective functions of the MOQFPP of equation (4.1). They are being optimized individually subject to the constraints (4.2) and (4.3) as follows:

$$\left.\begin{array}{c} Max.\, z_1 = \varphi_1 \\ Max.\, z_2 = \varphi_2 \\ . \\ . \\ . \\ Max.\, z_r = \varphi_r \\ Min.\, z_{r+1} = \varphi_{r+1} \\ . \\ . \\ . \\ Min.\, z_s = \varphi_s \end{array}\right] \quad (5.1.1)$$

Where $\varphi_1, \varphi_2, \ldots, \varphi_s$ are values of the objective functions, the level of the decision variable may not necessarily be the same for all optimal solutions in presence of conflict among objectives. But we require the common set of decision variables to be the best compromising optimal solution that we can determine for the common set of the decision variables from the following combined objective function, which formulate the MOQFPP given in following equation.

Max. Z = $\sum_{i=1}^{r} \frac{z_i}{|\varphi_i|}$ - $\sum_{i=r+1}^{s} \frac{z_i}{|\varphi_i|}$ (5.1.2)

Where $\varphi_i \neq 0$, i = 1, 2,…,s. Subject to constraints(4.2) and (4.3), the optimum value of the objective functions φ_i, i = 1, 2,…, s may be positive or negative.

5.2 Mean and Median Technique

We formulate the combined objective function as follows to determine the common set of decision variables, for solving the MOQFPP.

Max. Z = $\sum_{i=1}^{r} \frac{z_i}{mean(|\varphi_i|)}$ - $\sum_{i=r+1}^{s} \frac{z_i}{mean(|\varphi_i|)}$ (5.2.1)

Subject to the same constraints (4.2) and (4.3)

Where Max. Z = $\sum_{i=1}^{r} \frac{z_i}{median(|\varphi_i|)}$ - $\sum_{i=r+1}^{s} \frac{z_i}{median(|\varphi_i|)}$ (5.2.2)

Where $\varphi_i, i = 1, 2, \ldots, s$ is the optimum value of the objective function.

5.3 Average Technique (By using Mean & Median)

Max. Z = $\frac{(\sum_{i=1}^{r} z_i - \sum_{i=r+1}^{s} z_i)}{VM_2}$ (5.3.1)

Max. Z = $\frac{(\sum_{i=1}^{r} z_i - \sum_{i=r+1}^{s} z_i)}{WM_2}$ (5.3.2)

Where, $VM_2 = \frac{VM+VN}{2}$ (Average mean)

$WM_2 = \frac{WM+WN}{2}$ (Average median)

VM = $mean(|\varphi_i|)$, ∀ i = 1, 2,…,r

VN = $mean(|\varphi_i|)$, ∀ i = $\mathrm{r}+1, \mathrm{r}+2, \ldots, \mathrm{s}$

WM = $median(|\varphi_i|)$, ∀ i = 1, 2,…,r

WN = $median(|\varphi_i|)$, ∀ $i = \mathrm{r}+1, \mathrm{r}+2, \ldots, \mathrm{s}$

An algorithm for obtaining the optimal solution for the MOQFPP is as follows

Step1: First we solve each objective function by the modified simplex method, for Quadratic fractional programming problem.

Step2: Check the feasibility of the solution obtained in step1, if it is feasible then go to step3, otherwise use dual simplex method to remove infeasibility.

Step3: Assign a name to the optimum value of each objective function z_i, sayφ_i.

Step4: Find the maximum mean & median of φ_i, i = 1,2,…,r and find the minimum mean & median of φ_i, i = $\mathrm{r}+1, \mathrm{r}+2, \ldots, s$.

Step5: Construct the combined objective function which has formula (5.3.1) or (5.3.2).

Step6: Optimize the combined objective function under the same constraints (4.2) and (4.3).

5.4 New Average Technique (By using Mean & Median)

Max. Z = $\frac{(\sum_{i=1}^{r} z_i - \sum_{i=r+1}^{s} z_i)}{VM_s}$ (5.4.1)

Max. Z = $\frac{(\sum_{i=1}^{r} z_i - \sum_{i=r+1}^{s} z_i)}{WM_s}$ (5.4.2)

Where, $VM_s = \frac{VM+VN}{s}$ (New average mean)

$WM_s = \frac{WM+WN}{s}$ (New average median)

An algorithm for obtaining the optimal solution for the MOQFPP is as follows

Step1, Step2, Step3, Step4 are the same as given in algorithm (5.3).

Step6: Construct the combined objective function which has formula (5.4.1) or (5.4.2).

Step7: Optimize the combined objective function under the same constraints (4.2) and (4.3).

6. Numerical Example

Example 6.1.

Max. $z_1 = \frac{(2x_1+x_2+1)(2x_1+x_2+2)}{(2x_1+2x_2+2)}$

Max. $z_2 = \frac{(4x_1+2x_2+2)(6x_1+3x_2+6)}{(3x_1+3x_2+3)}$

Max. $z_3 = \frac{(4x_1+2x_2+2)(6x_1+3x_2+6)}{(6x_1+6x_2+6)}$

Min. $z_4 = \frac{(-8x_1-4x_2-4)(6x_1+3x_2+6)}{(5x_1+5x_2+5)}$

Min. $z_5 = \frac{(-4x_1-2x_2-2)(10x_1+5+10)}{(2x_1+2x_2+2)}$

Subject to:

$$x_1 + 2x_2 \leq 4, 3x_1 + x_2 \leq 6$$
$$x_1, x_2 \geq 0$$

Solution: After finding the value of each of individual objective functions by modified simplex method, the results are given below in table 1:

Table 1

i	z_i	x_i	φ_i
1	5	(2,0)	5
2	20	(2,0)	20
3	10	(2,0)	10
4	-24	(2,0)	-24
5	-50	(2,0)	-50

i. Chandra Sen. Technique

Max. Z = $\sum_{i=1}^{r} \frac{z_i}{|\varphi_i|}$ - $\sum_{i=r+1}^{s} \frac{z_i}{|\varphi_i|}$

$\sum_{i=1}^{r} \frac{z_i}{|\varphi_i|} = \frac{(2x_1+x_2+1)(6x_1+3x_2+6)}{(10x_1+10x_2+10)}$ and $\sum_{i=r+1}^{s} \frac{z_i}{|\varphi_i|} = \frac{(-4x_1-2x_2-2)(2x_1+x_2+2)}{(10x_1+10x_2+10)}$

Max. Z = $\frac{(2x_1+x_2+1)(6x_1+3x_2+6)}{(10x_1+10x_2+10)}$ - $\frac{(-4x_1-2x_2-2)(2x_1+x_2+2)}{(10x_1+10x_2+10)}$

Max. Z = $\frac{(2x_1+x_2+1)(2x_1+x_2+2)}{(2x_1+2x_2+2)}$

Subject to:

$$x_1 + 2x_2 \leq 4, 3x_1 + x_2 \leq 6$$
$$x_1, x_2 \geq 0$$

Hence the optimal solution is:

Max. Z = 5, x_1 = 2, $x_2 = 0$.

ii. Mean Technique:-

Max. Z = $\sum_{i=1}^{r} \frac{z_i}{mean(|\varphi_i|)}$ - $\sum_{i=r+1}^{s} \frac{z_i}{mean(|\varphi_i|)}$

$\sum_{i=1}^{r} \frac{z_i}{mean(|\varphi_i|)} = \frac{(2x_1+x_2+1)(6x_1+3x_2+6)}{(10x_1+10x_2+10)}$ and $\sum_{i=r+1}^{s} \frac{z_i}{mean(|\varphi_i|)} = \frac{(-2x_1-x_2-1)(2x_1+x_2+2)}{(5x_1+5x_2+5)}$

Max. Z = $\frac{(2x_1+x_2+1)(6x_1+3x_2+6)}{(10x_1+10x_2+10)}$ - $\frac{(-2x_1-x_2-1)(2x_1+x_2+2)}{(5x_1+5x_2+5)}$

Max. Z = $\frac{(2x_1+x_2+1)(2x_1+x_2+2)}{(2x_1+2x_2+2)}$

After solving the Max. Z by given subject to the same constraints as before, we find the optimal solution:

Max. Z = 5, x_1 = 2, $x_2 = 0$.

iii. Median Technique

Max. Z = $\sum_{i=1}^{r} \frac{z_i}{median(|\varphi_i|)}$ - $\sum_{i=r+1}^{s} \frac{z_i}{median(|\varphi_i|)}$

$\sum_{i=1}^{r} \frac{z_i}{median(|\varphi_i|)} = \frac{(14x_1+7x_2+7)(2x_1+x_2+2)}{(20x_1+20x_2+20)}$ and $\sum_{i=r+1}^{s} \frac{z_i}{median(|\varphi_i|)} = \frac{(2x_1+x_2+1)(2x_1+x_2+2)}{(5x_1+5x_2+5)}$

Max. Z = $\frac{(14x_1+7x_2+7)(2x_1+x_2+2)}{(20x_1+20x_2+20)}$ - $\frac{(2x_1+x_2+1)(2x_1+x_2+2)}{(5x_1+5x_2+5)}$

Max. Z = $\frac{(22x_1+11x_2+11)(2x_1+x_2+2)}{(20x_1+20x_2+20)}$

After solving the Max. Z by given subject to the same constraints as before, we find the optimal solution:

Max. Z = 5.5, x_1 = 2, $x_2 = 0$.

iv. Average Mean Technique

Max. Z = $\frac{(\sum_{i=1}^{r} z_i - \sum_{i=r+1}^{s} z_i)}{VM_2}$

Max. Z = $\frac{(218x_1+109x_2+109)(6x_1+3x_2+6)}{(730x_1+730x_2+730)}$

After solving the Max. Z by given subject to the same constraints as before, we find the optimal solution:

Max. Z = 4.479, x_1 = 2, $x_2 = 0$.

v. Average Median Technique

Max. Z = $\frac{(\sum_{i=1}^{r} z_i - \sum_{i=r+1}^{s} z_i)}{WM_2}$

Max. Z = $\frac{(218x_1+109x_2+109)(4x_1+2x_2+4)}{(470x_1+470x_2+470)}$

After solving the Max. Z by given subject to the same constraints as before, we find the optimal solution:
Max. Z = 4.638, x_1 = 2, $x_2 = 0$.

vi. New Average Mean Technique

Max. Z = $\frac{(\sum_{i=1}^{r} z_i - \sum_{i=r+1}^{s} z_i)}{VM_S}$

Max. Z = $\frac{(218x_1+109x_2+109)(6x_1+3x_2+6)}{(292x_1+292x_2+292)}$

After solving the Max. Z by given subject to the same constraints as before, we find the optimal solution:
Max. Z = 11.199, x_1 = 2, $x_2 = 0$.

vii. New Average Median Technique

Max. Z = $\frac{(\sum_{i=1}^{r} z_i - \sum_{i=r+1}^{s} z_i)}{WM_S}$

Max. Z = $\frac{(218x_1+109x_2+109)(2x_1+x_2+2)}{(94x_1+94x_2+94)}$

After solving the Max. Z by given subject to the same constraints as before, we find the optimal solution:
Max. Z = 11.596, x_1 = 2, $x_2 = 0$.

7. Comparison of the Numerical Results

Comparison of the numerical results which are obtained from the example 6.1 is shown in the following table2:

Table 2

Techniques		Example 6.1
Chandra Sen. Technique		5
Mean Technique		5
Median Technique		5.5
Average Technique	Mean Technique	4.479
	Median Technique	4.638
New Average Technique	Mean Technique	11.199
	Median Technique	11.596

In the above table**,** it is clear that the results obtained in example 6.1 when using new average technique are better than other results.

In this paper, we have defined and discussed a number of techniques, the comparison of these techniques are based on the value of the objective function. After solving the numerical example, we found that max. z which obtained by our technique(New average mean & median technique) is better than other techniques(Chandra Sen., mean & median, average mean & median techniques).

8. References

1. Carnes, Cooper WW. Programming with linear fractional functional, Naval Research Logistics Quarterly, 1962; 9:181-186.
2. Sen. Chandra, A new approach for multi-objective rural development planning, The Indian Economic Journal. 1983; 30(4):91-96.
3. Sulaiman NA, Abdulrahim Basiya K. Using Transformation technique to solve multi-objective linear fractional programming problem, IJRRAS. 2013; 14(3).
4. Sulaiman NA, Hamadameen abdul-Qader O. Optimal transformation technique to solve multi-objective linear programming problem (MOLPP), Journal of Kirkuk University-Scientific Studies, 2008; 3(2).
5. Sulaiman NA, salih AD. Using mean and median values to solve linear fractional multi-objective programming problem, Zanco journal for pure and applied science, Salahaddin-Erbil University, 2010; 22(5).