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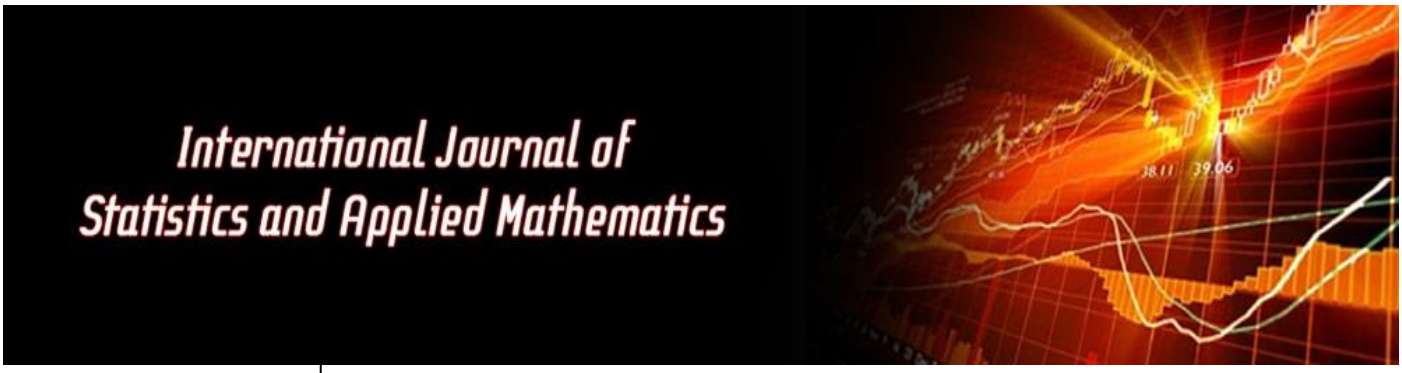

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A stochastic approach to determine seroconversion time of HIV infected under alertness using exponentiated modified weibull distribution

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Abstract
This study is based on a stochastic model for estimating the expected time to seroconversion of HIV infected using preventive strategy. The use of preventive strategy gives rise to the concept of alertness on the part of the individuals who has sexual contact with unknown partner. The effectiveness of the adoption preventive strategy in the elongation of the expected time to seroconversion is studied by introducing the stochastic model under the assumption that the threshold level of antigenic diversity is a random variable which follows exponentiated modified Weibull distribution. Numerical illustration is provided using simulated data.

Keywords: Acquired Immuno Deficiency Syndrome, Human Immuno-deficiency Virus, Seroconversion, Antigenic Diversity Threshold and Preventive Strategy

1. Introduction
Human Immuno-deficiency Virus (HIV) / Acquired Immuno Deficiency Virus (AIDS) research include all medical research that attempt to prevent, treat are cure HIV/AIDS, as well as fundamental research about the nature of HIV has an infectious agent and AIDS has the disease cost of by HIV. The HIV virus is commonly transmitted via unprotected sexual activity, blood transfusions, hypodermic needles and from mother to child. The transmission of more and more HIV from infected persons to the uninfected; the antigenic variations would be increase. The antigenic diversity threshold which means the antigenic diversity crosses a particular level, then the human immunity system collapses and seroconversion takes place immediately. The antigenic diversity threshold and its estimation has been discussed by Stilianakis *et al.* (1994) [7] and Nowak and May (1991) [6].

An individual who resort to sexual contacts with unknown partners may go in for preventive strategy to avoid the possible contraction of HIV. A universally recommended strategy to avoid the possible infection is the use of condemns by any person. It gives rise to the concept of alertness on the part of the person under risk. In deriving this model it is assumed that a person with alert uses the prevention strategy and if the person is inalert then there is a risk of transmission. The effectiveness of the adoption preventive strategies in the elongation of the expected time to seroconversion is studied by introducing a stochastic model.

A stochastic model for the HIV transmission under alertness, under the assumption that the threshold level of antigenic diversity is a random variable which follows a gamma distribution, mixed exponential distribution, exponentiated exponential distribution and exponential-geometric distribution has been discussed by Kannan *et al.* (2007, 2011, 2012 and 2015) [2]. In this paper using the concept of alertness and preventive strategy the stochastic model for the estimation of expected time to seroconversion and its variance are derived under the assumption that the threshold level of antigenic diversity is a random variable which follows an exponentiated modified Weibull distribution. In developing such a stochastic model, the concept of shock model and cumulative damage process developed by Esary *et al.* (1973) [1] is used. In this study the theoretical results are widely substantiated using numerical data simulated.

1.1 Model

A person may be alert in any contact with probability ' p ' and inalert with probability ' q ', so that $p+q=1$. It is a medically accepted fact that in any particular contact during which a person is inalert need not result in the transmission of HIV with unit probability. In other words, transmission at a contact with inalertness need not be a sure event. Hence, it is assumed that the probability of transmission or infection during a contact occurs with probability $\beta<1$.
Hence one can visualize the following possibilities.

i) A person is alert with probability ' p ' and during that contact invasion does not take place.

ii) A person is inalert with probability ' q ' and transmission occurs with probability β.

iii) A person is inalert with probability ' q ' and transmission does not occurs and the probability of this event is $(1-\beta)$.

1.2 Assumption of the model

- Human Immunodeficiency Virus transmits only through sexual contact.
- The transmission of HIV is a sure event during any contact in which a person is in-alert.
- A person is alert in a single contact with probability p, and inalert with probability q, so that $p+q=1$.
- The damage caused to the immune system due to the antigenic diversity is linear and additive.
- The total damage caused when exceeds the threshold level Y which itself is a random variable, the seroconversion occurs and a person is recognized as infected.

1.3 Notation

- $X_i =$ a random variable denoting the increase in the antigenic diversity arising due to the HIV transmitted during the i[th] contact $X_1, X_2, X_3, ..., X_k$ are continuous i.i.d. random variables, with p.d.f. g(.) and c.d.f. G(.).
- $Y =$ a random variable representing antigenic diversity threshold and follows exponentiated modified Weibull distribution with parameters α and γ, the p.d.f being h(.) and c.d.f H(.)
- $U_i =$ a continuous random variable denoting the interarrival times between successive contacts with p.d.f.f(.) and c.d.f. F(.).
- $Z =$ the random variable representing the time between damages.
- $g_k(.) =$ the p.d.f of random variable $\sum_{i=1}^{k} X_i$
- $F_k(.) =$ the k[th] convolution of F(.).
- $T =$ a continuous random variable denoting the time of to seroconversion with p.d.f. l(.) and c.d.f. L(.).
- $V_k(.) =$ probability of exactly k contacts in (o, t].
- $l^*(s) =$ is the Laplace stieltje's transform of l(t).
- $f^*(s) =$ is the Laplace stieltje's transform of f (t).

2. Result

The probability density function of exponentiated modified Weibull distribution is,

$h(y)=\alpha(\theta+\gamma\beta y^{\beta-1})e^{-(\theta y+\gamma y^{\beta})}[1-e^{-(\theta y+\gamma y^{\beta})}]^{\alpha-1}, y>0$ and $\theta,\beta,\alpha,\gamma>0.$

and its distribution function is

$$H(y)=\left[1-e^{-(\theta y+\gamma y^{\beta})}\right]^{\alpha}, y>0.$$

then corresponding survival function is

$$\overline{H}(y)=1-\left[1-e^{(\theta y+\gamma y^{\beta})}\right]^{\alpha}$$

Since Y is taken to exponentiated modified Weibull distribution (θ,γ)
It can be shown that

$$P\left[\sum_{i=1}^{k} X_i < Y\right] = \int_0^{\infty} g_k(x)\,\overline{H}(x)\,dx \qquad (1)$$

Where $\overline{H}(x) = 1 - H(x)$

Put α = 2 and β = 1 in $\overline{H}(x)$, then it becomes

$$\overline{H}(y) = \left[2e^{-y(\theta+\gamma)} - e^{-2y(\theta+\gamma)}\right] \quad (2)$$

Substituting equation (2) in (1), we get

$$P\left[\sum_{i=1}^{k} X_i < Y\right] = \int_0^{\infty} g_k(x)\left[2e^{-x(\theta+\gamma)} - e^{-2x(\theta+\gamma)}\right]dx$$

$$= \left[2g^*(\theta+\gamma)\right]^k - \left[g^*2(\theta+\gamma)\right]^k \text{ On simplification}$$

S(t) = P[T > t]

$$= \sum_{k=0}^{\infty} \Pr\{\text{there are exactly } k \text{ contacts in } (0,t]\}$$

* Pr{the cumulative total of antigenic diversity < Y}

$$\therefore S(t) = \sum_{K=0}^{\infty} V_k(t) P\left[\sum_{i=1}^{k} X_i < Y\right]$$

$$= \sum_{k=0}^{\infty}\left[F_k(t) - F_{k+1}(t)\right] \left[\left[2g^*(\theta+\gamma)\right]^k - \left[g^*2(\theta+\gamma)\right]^k\right]$$

$L(t) = 1 - S(t)$ is called the prevalence function as mentioned in Jewell and Shiboski(1990).

$$= 1 - \left\{\sum_{k=0}^{\infty}\left[F_k(t) - F_{k+1}(t)\right] \left[\left[2g^*(\theta+\gamma)\right]^k - \left[g^*2(\theta+\gamma)\right]^k\right]\right\}$$

$$= 2\left[1 - g^*(\theta+\gamma)\right]\sum_{k=1}^{\infty} F_k(t)\left[g^*(\theta+\gamma)\right]^{k-1} - \left[1 - g^*2(\theta+\gamma)\right]\sum_{k=1}^{\infty} F_k(t)\left[g^*2(\theta+\gamma)\right]^{k-1}$$

On simplification

The p.d.f of T is

$$l(t) = 2\left[1 - g^*(\theta+\gamma)\right]\sum_{k=1}^{\infty} f_k(t)\left[g^*(\theta+\gamma)\right]^{k-1} - \left[1 - g^*2(\theta+\gamma)\right]\sum_{k=1}^{\infty} f_k(t)\left[g^*2(\theta+\gamma)\right]^{k-1}$$

Taking Laplace Stieltje's transform of $l(t)$ we get,

$$l^*(s) = 2\left[1 - g^*(\theta+\gamma)\right] \sum_{k=1}^{\infty}\left[f^*(s)\right]^k\left[g^*(\theta+\gamma)\right]^{k-1} - \left[1 - g^*2(\theta+\gamma)\right] \sum_{k=1}^{\infty}\left[f^*(s)\right]^k\left[g^*2(\theta+\gamma)\right]^{k-1}$$

$$l^*(s) = \frac{2\left[1 - g^*(\theta+\gamma)\right] f^*(s)}{\left[1 - g^*(\theta+\gamma) f^*(s)\right]} - \frac{\left[1 - g^*2(\theta+\gamma)\right] f^*(s)}{\left[1 - g^*2(\theta+\gamma) f^*(s)\right]} \text{ On simplification} \quad (3)$$

But the c.d.f of Z is given by

$$F(Z) = \beta q \sum_{n=1}^{\infty}[p + q(1-\beta)]^n \, G_{n+1}(Z)$$

Taking Laplace Stieltjes transform of F(z) is

$$F^*(s) = \int_0^\infty e^{-st}\, dF(Z)$$

$$= \beta q G^*(s) \sum_{n=0}^{\infty} \left[\left[p + q(1-\beta)\right] G_*(s)\right]^n$$

Hence,

$$F^*(s) = \frac{q\beta G^*(s)}{\left[1-\left[p+q(1-\beta)\right]G^*(s)\right]}$$

The p.d.f of $F^*(s)$ is,

$$F^*(s) = \frac{q\beta g^*(s)}{\left[1-\left[p+q(1-\beta)\right]g^*(s)\right]} \quad (4)$$

Assuming that g ~ exp(c), then

$$g^*(s) = \frac{c}{c+s},$$

$$g^{*\prime}(0) = -\frac{1}{c} \text{ and } g^{*\prime\prime}(0) = \frac{2}{c^2}. \quad (5)$$

From equation (4)

$$\frac{df^*(s)}{ds} = \frac{\left\{\beta q\, g^{*\prime}(s)\left[1-\left[p+q(1-\beta)\right]g^*(s)\right]-\left[-\left[p+q(1-\beta)\right]g^{*\prime}(s)\right]\beta q\ g^*(s)\right\}}{\left[1-\left[p+q(1-\beta)\right]g^*(s)\right]^2}$$

$$f^{*\prime}(s) = \frac{\left\{\beta q\, g^{*\prime}(s)\left[1-\left[p+q-\beta q\right]g^*(s)\right]+\left[\left[p+q-\beta q\right]g^{*\prime}(s)\right]\beta q\ g^*(s)\right\}}{\left[1-\left[p+q-\beta q\right]g^*(s)\right]^2}$$

$$\left.\frac{df^*(s)}{ds}\right|_{s=0} = \frac{\left\{\beta q\, g^{*\prime}(0)\left[1-\left[p+q-\beta q\right]g^*(0)\right]+\left[\left[p+q-\beta q\right]g^{*\prime}(0)\right]\beta q\ g^*(0)\right\}}{\left[1-\left[p+q-\beta q\right]g^*(0)\right]^2}$$

$$= \left(\frac{\beta q g^{*\prime}(0)}{(\beta q)^2}\right)$$

$$= \frac{g^{*\prime}(0)}{\beta q}$$

$$f^{*\prime}(0) = \frac{-1}{c\beta q} \quad (6)$$

Therefore

$$-\frac{dl^*(s)}{ds} = -\left\{\frac{2\left\{\begin{array}{l}\left[1-g^*(\theta+\gamma)\right]f^*(s)\left[\left[1-g^*(\theta+\gamma)\right]f^{*\prime}(s)\right] \\ \qquad -\left[1-g^*(\theta+\gamma)\right]f^*(s)\left[0-g^*(\theta+\gamma)\right]f^{*\prime}(s)\end{array}\right\}}{\left[1-g^*(\theta+\gamma)\,f^*(s)\right]^2}\right.$$

$$-\left\{\frac{\left[1-g^*2(\theta+\gamma)\right]f^*(s)\left[\left[1-g^*2(\theta+\gamma)\right]f^{*\prime}(s)\right]-\left[1-g^*2(\theta+\gamma)\right]f^*(s)\left[0-g^*2(\theta+\gamma)\right]f^{*\prime}(s)}{\left[1-g^*2(\theta+\gamma)f^*(s)\right]^2}\right\}$$

$$=-\left\{\left\{\frac{\left[2\left[1-g^*(\theta+\gamma)\right]f^{*\prime}(s)\right]}{\left[1-g^*(\theta+\gamma)f^*(s)\right]^2}\left[1-g^*(\theta+\gamma)f^*(s)+g^*(\theta+\gamma)f^*(s)\right]\right\}\right.$$

$$\left.-\left\{\frac{\left[\left[1-g^*2(\theta+\gamma)\right]f^{*\prime}(s)\right]}{\left[1-g^*2(\theta+\gamma)f^*(s)\right]^2}\left[1-g^*2(\theta+\gamma)f^*(s)+g^*2(\theta+\gamma)f^*(s)\right]\right\}\right\}$$

$$=-\left\{\frac{\left[2\left[1-g^*(\theta+\gamma)\right]f^{*\prime}(s)\right]}{\left[1-g^*(\theta+\gamma)f^*(s)\right]^2}-\frac{\left[1-g^*2(\theta+\gamma)\right]f^{*\prime}(s)}{\left[1-g^*2(\theta+\gamma)f^*(s)\right]^2}\right\}$$

On simplification

$$-\left.\frac{df^*(s)}{ds}\right|_{s=0}=-\left\{\frac{2\left[1-g^*(\theta+\gamma)\right]f^{*\prime}(0)}{\left[1-g^*(\theta+\gamma)f^*(0)\right]^2}-\frac{\left[1-g^*2(\theta+\gamma)\right]f^{*\prime}(0)}{\left[1-g^*2(\theta+\gamma)f^*(0)\right]^2}\right\} \tag{7}$$

Let $$g^*(\lambda)=\frac{\mu}{\mu+\theta+\gamma},\ g^*(2\lambda)=\frac{\mu}{\mu+2\theta+2\gamma} \tag{8}$$

Substituting equation (6) and (8) in (7) we get

$$E(T)=\frac{2\left[1-\frac{\mu}{\mu+\theta+\gamma}\right]\left[\frac{-1}{c\beta q}\right]}{\left[1-\frac{\mu}{\mu+\theta+\gamma}\right]^2}-\frac{\left[1-\frac{\mu}{\mu+2\theta+2\gamma}\right]\left[\frac{-1}{c\beta q}\right]}{\left[1-\frac{\mu}{\mu+2\theta+2\gamma}\right]^2}$$

$$\therefore\quad E(T)=\frac{3\mu+2(\theta+\gamma)}{c\beta q2(\theta+\gamma)} \tag{9}$$

$$\frac{d^2f^*(s)}{ds^2}=\left\{\frac{\left[1-\left[p+q-\beta q\right]g^*(s)\right]^2\left\{\begin{array}{l}[1-[p+q-\beta q]g^*(s)]qg^{*\prime\prime}(s)+qg^{*\prime}(s)[-[p+q-\beta q]g^{*\prime}(s)]\\ +qg^*(s)[p+q-\beta q]g^{*\prime\prime}(s)+[p+q-\beta q]g^{*\prime}(s)qg^{*\prime}(s)\end{array}\right\}}{\left[1-\left[p+q-\beta q\right]g^*(s)\right]^4}\right.$$
$$\left.\frac{-\left[1-\left[p+q-\beta q\right]g^*(s)\right]qg^{*\prime}(s)+qg^*(s)\left[p+q-\beta q\right]g^{*\prime}(s)\right]2[1-[p+q-\beta q]g^*(s)]\left[-[p+q-\beta q]g^*(s)\right]}{\left[1-\left[p+q-\beta q\right]g^*(s)\right]^4}\right\}$$

$$\left.\frac{d^2f^*(s)}{ds^2}\right|_{s=0}=\frac{g^{*\prime\prime}(0)}{\beta q}+\frac{2\left[p+q-\beta q\right]\left[g^{*\prime}(0)\right]^2}{(\beta q)^2}$$ On simplification (10)

$$g^{*\prime}(0)=\frac{-1}{c}\ \text{ and }\ g^{*\prime\prime}(0)=\frac{2}{c^2} \tag{11}$$

Substituting equation (5) in (10) we get

$$f^{*\prime\prime}(0)=\frac{2}{c^2\beta q}+\frac{2[p+q-\beta q]}{(c\beta q)^2}$$

$$E(T^2)=\left.\frac{d^2 l^*(s)}{ds^2}\right|_{S=0}$$

$$=\frac{2\left\{\begin{array}{l}\left[1-g^*(\theta+\gamma)f^*(s)\right]^2\left[1-g^*(\theta+\gamma)\right]f^{*\prime\prime}(s)- \\ 2\left[\begin{array}{c}1-g^*(\theta+\gamma)f^{*\prime}(s)\left[1-g^*(\theta+\gamma)\right]f^*(s)\\ \left[0-g^*(\theta+\gamma)f^{*\prime}(s)\right]\end{array}\right]\end{array}\right\}}{\left[1-g^*(\theta+\gamma)f^*(s)\right]^4}$$

$$-\frac{\left\{\begin{array}{l}\left[1-g^*2(\theta+\gamma)f^*(s)\right]^2\left[1-g^*2(\theta+\gamma)\right]f^{*\prime\prime}(s)- \\ 2\left[\begin{array}{c}1-g^*2(\theta+\gamma)f^{*\prime}(s)\left[1-g^*2(\theta+\gamma)\right]f^*(s)\\ \left[0-g^*2(\theta+\gamma)f^{*\prime}(s)\right]\end{array}\right]\end{array}\right\}}{\left[1-g^*2(\theta+\gamma)f^*(s)\right]^4}$$

$$\left.\frac{d^2 l^*(s)}{ds^2}\right|_{s=0}=\frac{2\left\{\begin{array}{l}\left[1-g^*(\theta+\gamma)f^*(0)\right]^2\left[1-g^*(\theta+\gamma)\right]f^{*\prime\prime}(0)+ \\ 2\left[1-g^*(\theta+\gamma)\right]^2\left[f^{*\prime}(0)\right]^2 f^*(0)g^*(\theta+\gamma)\end{array}\right\}}{\left[1-g^*(\theta+\gamma)f^*(0)\right]^4}$$

$$-\frac{\left\{\begin{array}{l}\left[1-g^*2(\theta+\gamma)f^*(0)\right]^2\left[1-g^*2(\theta+\gamma)\right]f^{*\prime\prime}(0)+ \\ 2\left[1-g^*2(\theta+\gamma)\right]^2\left[f^{*\prime}(0)\right]^2 f^*(0)g^*2(\theta+\gamma)\end{array}\right\}}{\left[1-g^*2(\theta+\gamma)f^*(0)\right]^4}$$

$$=2\left\{\frac{\begin{array}{c}\left[1-g^*(\theta+\gamma)\right]^2\left[1-g^*(\theta+\gamma)\right]\left[\frac{g^{*\prime\prime}(0)}{q}+\frac{2[1-\beta q]\left[g^{*\prime}(0)\right]^2}{q^2}\right] \\ +2[1-g^*(\theta+\gamma)]^2\left[\frac{g^{*\prime}(0)}{q}\right]g^*(\theta+\gamma)^2\end{array}}{\left[1-g^*(\theta+\gamma)\right]^4}\right\}$$

$$-\left\{\frac{\begin{array}{c}\left[1-g^*2(\theta+\gamma)\right]^2\left[1-g^*2(\theta+\gamma)\right]\left[\frac{g^{*\prime\prime}(0)}{q}+\frac{2[1-\beta q]\left[g^{*\prime}(0)\right]^2}{q^2}\right] \\ +2[1-g^*2(\theta+\gamma)]^2\left[\frac{g^{*\prime}(0)}{q}\right]g^*2(\theta+\gamma)^2\end{array}}{\left[1-g^*2(\theta+\gamma)\right]^4}\right\} \quad (12)$$

Substituting equation (8) and (11) in equation (12) we get

$$= 2\left\{\frac{\left[1-\frac{\mu}{\mu+\theta+\gamma}\right]\left[\frac{2}{\beta q c^2}+2(1-\beta q)\frac{1}{(c\beta q)^2}\right]+\frac{2}{(c\beta q)^2}\left[\frac{\mu}{\mu+\theta+\gamma}\right]}{\left(\frac{\theta+\gamma}{\mu+\theta+\gamma}\right)^2}\right\}$$

$$-\left\{\frac{\left[1-\frac{\mu}{\mu+2\theta+2\gamma}\right]\left[\frac{2}{\beta q c^2}+2(1-\beta q)\frac{1}{(\beta q c)^2}+\frac{2}{(\beta q c)^2}\left[\frac{2\mu}{\mu+2\theta+2\gamma}\right]\right]}{\left(\frac{2(\theta+\gamma)}{\mu+2(\theta+\gamma)}\right)^2}\right\}$$

$$= \frac{8(\theta+\gamma)^2+14\mu^2+24\mu(\theta+\gamma)+8\theta\gamma}{c^2\beta^2q^2 4(\theta+\gamma)^2}$$ On simplification

The variance time to seroconversion is V (T) = E (T^2) – [E(T)]^2

$$V(T) = \frac{8(\theta+\gamma)^2+14\mu^2+24\mu(\theta+\gamma)+8\theta\gamma}{c^2\beta^2q^2 4(\theta+\gamma)^2} - \left[\frac{3\mu+2(\theta+\gamma)}{c\beta q 2(\theta+\gamma)}\right]^2$$

$$V(T) = \frac{5\mu^2+4(\theta+\gamma)^2+12\mu(\theta+\gamma)+8\theta\gamma}{c^2\beta^2q^2 4(\theta+\gamma)^2} \quad (13)$$

2.1 Special case

When there is no alertness, then $q=1, \quad \beta=1$, so

$$\mu_t = \frac{3\mu+2(\theta+\gamma)}{c2(\theta+\gamma)}$$

In case in-alertness

$$\mu_{ta} = \frac{3\mu+2(\theta+\gamma)}{c\beta q 2(\theta+\gamma)}$$

Therefore $\mu_{ta} > \mu_t$ and this implies that the mean time to seroconversion is larger in the case of alertness, which is inversely proportional to the probability of non alertness $'q'$ which is an interesting result.

$$\sigma^2_{ta} = \frac{5\mu^2+4(\theta+\gamma)^2+12\mu(\theta+\gamma)+8\theta\gamma}{c^2\beta^2q^2 4(\theta+\gamma)^2}$$

When $q=1, \quad \beta=1$, there is no alertness and variance in the case is given by,

$$\sigma^2_t = \frac{5\mu^2+4(\theta+\gamma)^2+12\mu(\theta+\gamma)+8\theta\gamma}{c^2 4(\theta+\gamma)^2}$$

Which are the results obtained by Kannan *et al.* (2015) [5].

2.2 Numerical Illustrations

Table 1

c	γ = 0.5, θ = 0.2, μ=0.2, q = 0.5 β=0.4	
	E(T)	V(T)
1	7.142857	59.18367
2	3.571429	14.79592
3	2.380952	6.575964
4	1.785714	3.698980
5	1.428571	2.367347
6	1.190476	1.643991
7	1.020408	1.207830
8	0.892857	0.924745
9	0.793651	0.730663
10	0.714286	0.591837

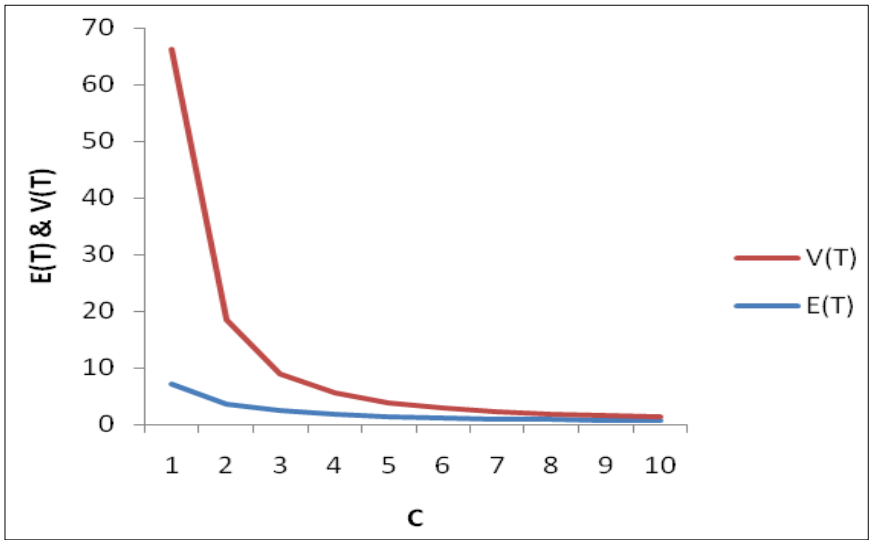

Fig 1

Table 2

μ	γ = 0.5, θ = 0.2, c=2, q = 0.5 β=0.4	
	E(T)	V(T)
0.1	3.035714	11.63903
0.2	3.571429	14.79592
0.3	4.107143	18.27168
0.4	4.642857	22.06633
0.5	5.178571	26.17985
0.6	5.714286	30.61224
0.7	6.250000	35.36352
0.8	6.785714	40.43367
0.9	7.321429	45.82270
1	7.857143	51.53061

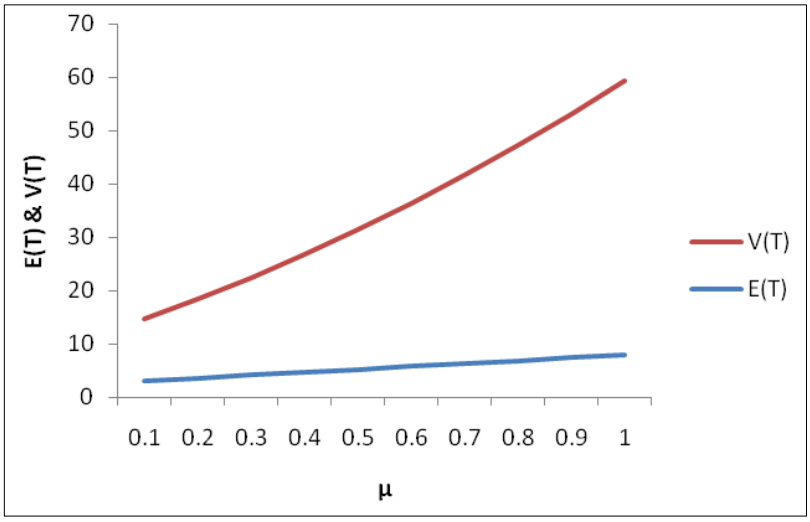

Fig 2

Table 3

θ	γ = 0.5, c = 2, μ=0.2, q = 0.5 β=0.4	
	E(T)	V(T)
0.1	3.750000	15.10417
0.2	3.571429	14.79592
0.3	3.437500	14.35547
0.4	3.333333	13.88889
0.5	3.250000	13.43750
0.6	3.181818	13.01653
0.7	3.125000	12.63021
0.8	3.076923	12.27811
0.9	3.035714	11.95791
1	3.000000	11.66667

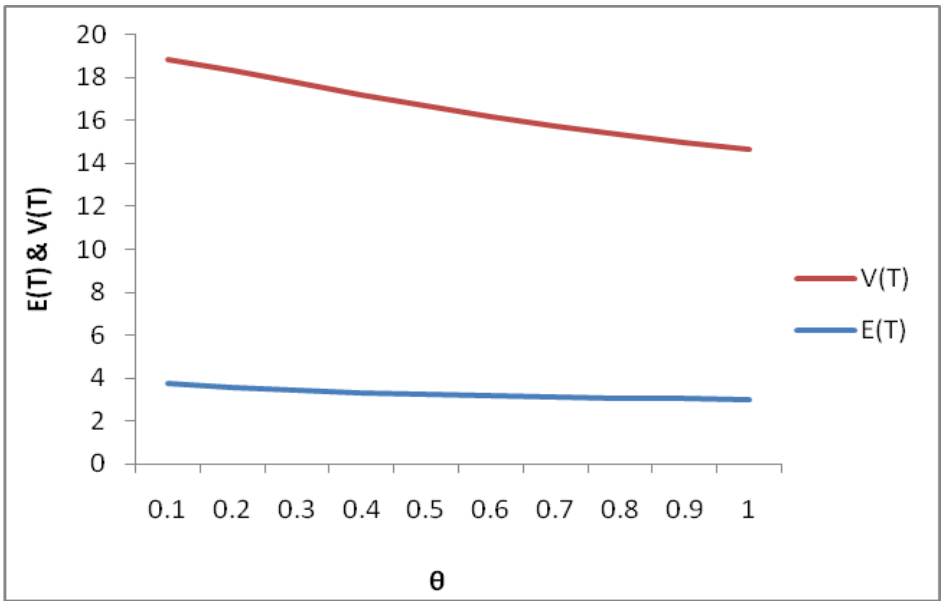

Fig 3

Table 4

γ	**θ= 0.5, c = 2, μ=0.2, q = 0.5 β=0.4**	
	E(T)	**V(T)**
0.1	5.000000	25.00000
0.2	4.375000	20.70313
0.3	4.000000	18.00000
0.4	3.750000	16.14583
0.5	3.571429	14.79592
0.6	3.437500	13.76953
0.7	3.333333	12.96296
0.8	3.250000	12.31250
0.9	3.181818	11.77686
1	3.125000	11.32813

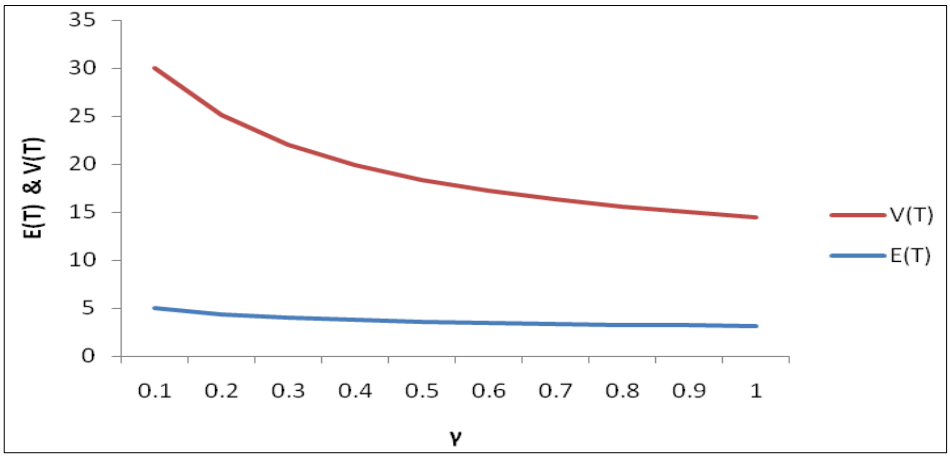

Fig 4

Table 5

q	γ = 0.5, θ = 0.2, μ=0.2, c=2					
	β=0.2		β=0.5		β=0.8	
	E(T)	V(T)	E(T)	V(T)	E(T)	V(T)
0.1	35.71429	1479.592	14.28571	236.7347	8.928571	92.47449
0.2	17.85714	369.8980	7.142857	59.18367	4.464286	23.11862
0.3	11.90476	164.3991	4.761905	26.30385	2.976190	10.27494
0.4	8.928571	92.47449	3.571429	14.79592	2.232143	5.779656
0.5	7.142857	59.18367	2.857143	9.469388	1.785714	3.698980
0.6	5.952381	41.09977	2.380952	6.575964	1.488095	2.568736
0.7	5.102041	30.19575	2.040816	4.831320	1.275510	1.887234
0.8	4.464286	23.11862	1.785714	3.698980	1.116071	1.444914
0.9	3.968254	18.26657	1.587302	2.922651	0.992063	1.141660
1	3.571429	14.79592	1.428571	2.367347	0.892857	0.924745

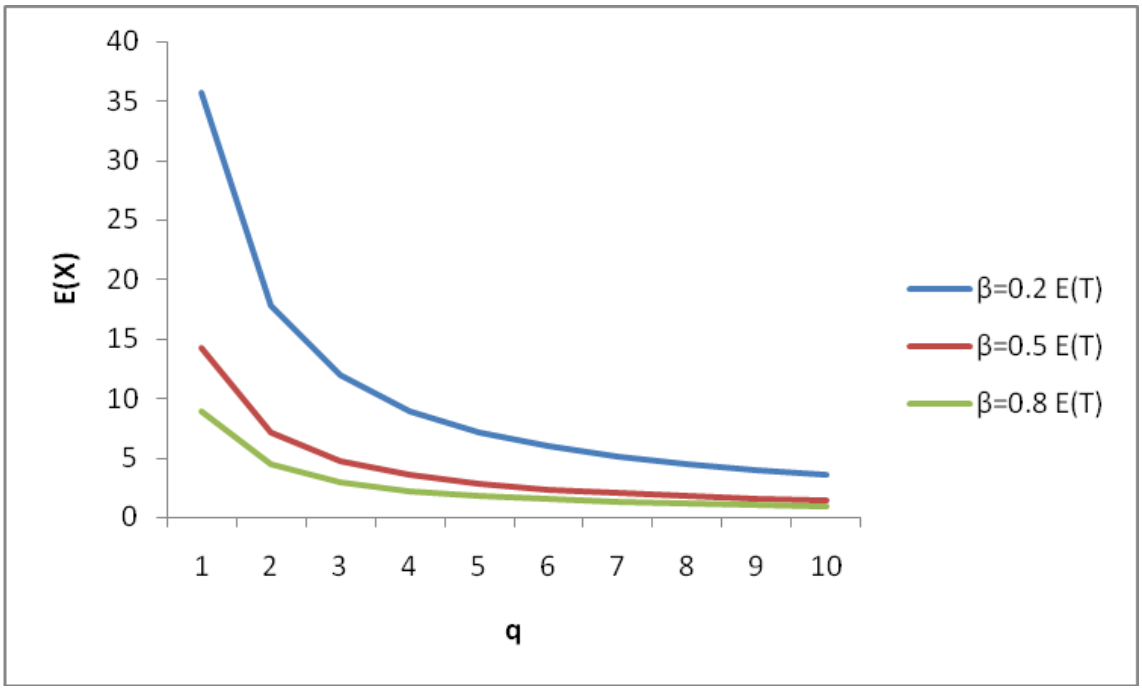

Fig 5(a)

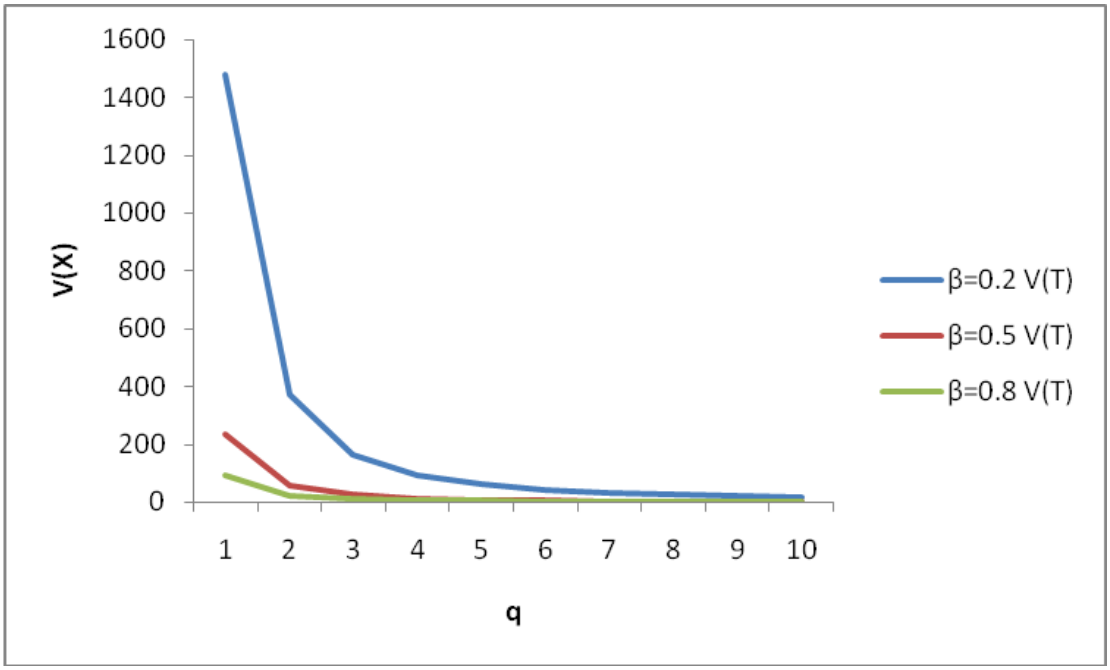

Fig 5(b)

3. Conclusion

1. In Table 1 shows the value of expected time to seroconversion corresponding to the variation in c the parameter of the distribution of interarrival time when q, γ, θ, μ and β are kept fixed. As c increases, the value of $\frac{1}{c}$ decreases that means the interarrival time between contacts become smaller and so there is a corresponding decrease in expected time to seroconversion and also its variance. The graph is also plotted in Fig.1.
2. It is observed from the contribution to the antigenic diversity threshold parameter 'μ' which increases then expected time to seroconversion increases. This implies the fact that g (.) is the distribution of $X_{(i)}$, the magnitude of contribution to antigenic diversity. Since $E(X) = \frac{1}{\mu}$, as μ increase there is a decrease in the contribution of antigenic diversity. Hence, the expected time to seroconversion and also its variance for time to seroconversion increase.
3. From Table 3, as the value of θ is the parameter of the exponentiated modified Weibull distribution of the threshold increases then expected time to seroconversion and variance of seroconversion decreases.
4. From the fixed values of θ, μ and c when threshold parameter 'γ' is allowed to increase then expected time to seroconversion and variance of seroconversion decrease as indicated in Table 4 and Figure 4.
5. From table 5, we observe that the value of q which is the probability of inalertness increases than the mean time to seroconversion decreases is quite plausible. It also quite reasonable has regard the variance it could be seen the value of the q increases the variance is decreases. It can be easily observed that when the value of β increases, there is a decrease in the expected duration of seroconversion irrespective of variation in q values. It is also quite reasonable as regards the variance for a fixed value.

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