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Backward divided difference: Representation of numerical data by a polynomial curve

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Abstract
Due to the necessity of a formula for representing a given set of numerical data on a pair of variables by a suitable polynomial, in interpolation by the approach which consists of the representation of numerical data by a suitable polynomial and then to compute the value of the dependent variable from the polynomial corresponding to any given value of the independent variable, one such formula has been derived from Newton's divided difference interpolation formula. This paper describes the derivation of the formula with numerical example as its application.

Keywords: Interpolation, divided difference formula, polynomial curve, representation of numerical data

1. Introduction
There exist a number of interpolation formulae such as Newton's Forward Interpolation formula, Newton's Backward Interpolation formula, Lagrange's Interpolation formula, Newton's Divided Difference Interpolation formula, Newton's Central Difference Interpolation formula, Stirlings formula, Bessel's formula etc. in the literature of numerical analysis {Bathe & Wilson (1976), Jan (1930), Hummel (1947) *et al*} for handling the problems of interpolation {Hummel (1947), Erdos & Turan (1938) *et al*}.
In the existing approach of interpolation {Hummel (1947), Erdos & Turan (1938) *et al*}, where a number of interpolation formulae are available {Bathe & Wilson (1976), Jan (1930), Hummel (1947) *et al*}, if it is wanted to interpolate the values of the dependent variable corresponding to a number of values of the independent variable by a suitable existing interpolation formula then it is required to apply the formula for each value separately and thus the numerical computation of the value of the dependent variable based on the given data are to be performed in each of the cases. In order to get rid of these repeated numerical computations from the given data, one can think of an approach which consists of the representation of the given numerical data by a suitable polynomial and then to compute the value of the dependent variable from the polynomial corresponding to any given value of the independent variable. However, a method is necessary for representing a given set of numerical data on a pair of variables by a suitable polynomial. Das & Chakrabarty (2016*a*, 2016*b*, 2016*c* & 2016*d*) [2-5] derived four formulae for representing numerical data on a pair of variables by a polynomial curve. They derived the formulae from Lagranges Interpolation Formula, Newton's Divided Difference Interpolation Formula, Newton's Forward Interpolation Formula and Newton's Backward Interpolation respectively. In this study, another formula has been derived from Newton's Divided Difference formula applied in reverse order. This paper describes the derivation of the formula with numerical example in order to show the application of the formula to numerical data.

2. Backward Divided Difference Interpolation Formula:
Let
$$y_0 = f(x_0), y_1 = f(x_1), y_2 = f(x_2), …………, y_{n-1} = f(x_{n-1}), y_n = f(x_n)$$
be the values of the function $y = f(x)$ corresponding to the values
$x_0, x_1, x_2, …………, x_{n-1}, x_n$ of the independent variable x. Then Newton's Divided Difference uses interpolation formula is given by

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$$f(x) = f(x_0) + (x - x_0)f(x_0, x_1) + (x - x_0)(x - x_1) f(x_0, x_1, x_2) + (x - x_0)(x - x_1)(x - x_2)$$
$$f(x_0, x_1, x_2, x_3) + \ldots\ldots + (x - x_0)(x - x_1)(x - x_2) \ldots (x - x_{i-1}) f(x_0, x_1, x_2, x_3 \ldots x_{i-1})$$
$$+ \ldots\ldots + (x - x_0)(x - x_1)(x - x_2) \ldots\ldots\ldots (x - x_{n-1}) f(x_0, x_1, x_2, x_3 \ldots x_n)$$

where

$$f(x_n, x_{n-1}, \ldots\ldots, x_2, x_1, x_0) = \frac{f(x_{n-1}, x_{n-2}, \ldots\ldots, x_1, x_0) - f(x_n, x_{n-1}, \ldots\ldots, x_2, x_1)}{x_0 - x_n}$$

for n = 1, 2, 3, …………., n.

Now, applying the formula to the above values of x & f(x) arranged in reverse order
i.e. applying the formula to the values

$$y_n = f(x_n), y_{n-1} = f(x_{n-1}), y_{n-2} = f(x_{n-2}), \ldots, y_2 = f(x_2), y_1 = f(x_1), y_0 = f(x_0)$$

of the function y = f(x) corresponding to the values

$$x_n, x_{n-1}, x_{n-2}, \ldots\ldots\ldots\ldots\ldots, x_2, x_1, x_0$$

of the independent variable x,

it is obtained that

$$f(x) = f(x_n) + (x - x_n)f(x_n, x_{n-1}) + (x - x_n)(x - x_{n-1}) f(x_n, x_{n-1}, x_{n-2}) + (x - x_n)(x - x_{n-1})(x - x_{n-2}) f$$
$$(x_n, x_{n-1}, x_{n-2}, x_{n-3}) + \ldots\ldots\ldots\ldots\ldots + (x - x_n)(x - x_{n-1})(x - x_{n-2}) \ldots (x - x_{n-i}) f(x_n, x_{n-1}, x_{n-2}, x_{n-3} \ldots x_{n-i})$$
$$+ \ldots\ldots + (x - x_n)(x - x_{n-1})(x - x_{n-2}) \ldots (x - x_1) f(x_n, x_{n-1}, x_{n-2}, x_{n-3}, \ldots, x_1, x_0) \to (1)$$

3. Representation of Numerical Data by Polynomial Curve:

By algebraic expansion, one can obtain that

$$(x - x_n)(x - x_{n-1}) = x^2 - x x_{n-1} - x x_n + x_{n-1} x_n$$
$$= x^2 - (x_{n-1} + x_n) x + x_{n-1} x_n$$
$$= x^2 - (\sum_{i=n-1}^{n} x_i) x + x_{n-1} x_n$$

$$(x - x_n)(x - x_{n-1})(x - x_{n-2}) = [x^2 - (x_{n-1} + x_n) x + x_{n-1} x_n](x - x_{n-2})$$
$$= x^3 - x^2(x_{n-1} + x_n) + x x_{n-1} x_n - x^2 x_{n-2} + x x_{n-2}(x_{n-1} + x_n) - x_{n-2} x_{n-1} x_n$$
$$= x^3 - (x_{n-2} + x_{n-1} + x_n) x^2 + (x_n x_{n-1} + x_n x_{n-2} + x_{n-1} x_{n-2}) x - x_{n-2} x_{n-1} x_n$$
$$= x^3 - (\sum_{i=n-2}^{n} x_i) x^2 + (\sum_{i=n}^{n-1} \sum_{j=n-1}^{n-2} x_i x_j) x - x_{n-2} x_{n-1} x_n$$

$$(x - x_n)(x - x_{n-1})(x - x_{n-2})(x - x_{n-3})$$
$$= [x^3 - (x_{n-2} + x_{n-1} + x_n) x^2 + (x_n x_{n-1} + x_n x_{n-2} + x_{n-1} x_{n-2}) x - x_{n-2} x_{n-1} x_n](x - x_{n-3})$$
$$= x^4 - (x_{n-2} + x_{n-1} + x_n) x^3 + (x_n x_{n-1} + x_n x_{n-2} + x_{n-1} x_{n-2}) x^2 - x x_{n-2} x_{n-1} x_n - x_{n-3} x^3$$
$$+ (x_{n-2} + x_{n-1} + x_n) x^2 x_{n-3} - (x_n x_{n-1} + x_n x_{n-2} + x_{n-1} x_{n-2}) x x_{n-3} + x_{n-3} x_{n-2} x_{n-1} x_n$$
$$= x^4 - (x_{n-3} + x_{n-2} + x_{n-1} + x_n) x^3 + (x_n x_{n-1} + x_n x_{n-2} + x_{n-1} x_{n-2} + x_{n-2} x_{n-3} + x_{n-1} x_{n-3} + x_n x_{n-3}) x^2$$
$$- (x_{n-3} x_{n-1} x_n + x_{n-3} x_{n-2} x_n + x_{n-3} x_{n-2} x_{n-1} + x_{n-2} x_{n-1} x_n) x$$
$$+ x_{n-3} x_{n-2} x_{n-1} x_n$$
$$= x^4 - (\sum_{i=n-3}^{n} x_i) x^3 + (\sum_{i=n-2}^{n} \sum_{j=n-3}^{n-1} x_i x_j) x^2 - (\sum_{i=n-1}^{n} \sum_{j=n-2}^{n-1} \sum_{k=n-3}^{n-2} x_i x_j x_k) x + x_{n-3} x_{n-2} x_{n-1} x_n$$

In general,

$$(x - x_n)(x - x_{n-1})(x - x_{n-2}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (x - x_1)(x - x_0)$$
$$= x^n - (\sum_{i=1}^{n} x_i) x^{n-1} + (\sum_{i=2}^{n} \sum_{j=1}^{n-1} x_i x_j) x^{n-2} - (\sum_{j=3}^{n} \sum_{j=2}^{n-1} \sum_{k=1}^{n-2} x_i x_j x_k) x^{n-3} +$$
$$+ \ldots\ldots\ldots\ldots + (-1)^n (x_n x_{n-1} x_{n-2} x_{n-3} \ldots\ldots\ldots x_1 x_0)$$

Now, Backward divided difference interpolation formula, described by equation (1) can be expressed as

$$f(x) = C_n + C_{n-1}(x - x_n) + C_{n-2}\{x^2 - (\sum_{i=n-1}^{n} x_i) x + x_{n-1} x_n\} + C_{n-3}\{x^3 - (\sum_{i=n-2}^{n} x_i) x^2 +$$
$$(\sum_{i=n}^{n-1} \sum_{j=n-1}^{n-2} x_i x_j) x - x_{n-2} x_{n-1} x_n\} + C_{n-4}\{x^4 - (\sum_{i=n-3}^{n} x_i) x^3 + (\sum_{i=n-2}^{n} \sum_{j=n-3}^{n-1} x_i x_j) x^2$$
$$- (\sum_{i=n-1}^{n} \sum_{j=n-2}^{n-1} \sum_{k=n-3}^{n-2} x_i x_j x_k) x + x_{n-3} x_{n-2} x_{n-1} x_n\} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots +$$
$$C_0 \{x^n - (\sum_{i=1}^{n} x_i) x^{n-1} + (\sum_{i=2}^{n} \sum_{j=1}^{n-1} x_i x_j) x^{n-2} - (\sum_{i=3}^{n} \sum_{j=2}^{n-1} \sum_{k=1}^{n-1} x_i x_j x_k) x^{n-3}$$
$$+ \ldots\ldots\ldots\ldots + (-1)^n (x_n x_{n-1} x_{n-2} x_{n-3} \ldots\ldots\ldots x_1 x_0)\}$$

where $C_n = f(x_n)$

$C_{n-1} = f(x_n, x_{n-1})$

$C_{n-2} = f(x_n, x_{n-1}, x_{n-2})$

$C_{n-3} = f(x_n, x_{n-1}, x_{n-2}, x_{n-3})$

…………………
…………………

$C_{n-i} = f(x_n, x_{n-1}, x_{n-2}, x_{n-3}, \ldots\ldots\ldots\ldots, x_{n-i})$

…………………
…………………

$C_0 = f(x_n, x_{n-1}, x_{n-2}, x_{n-3}, \ldots\ldots\ldots\ldots, x_0)$

Now, we have

Constant term $= C_n - C_{n-1} x_n + C_{n-2} x_{n-1} x_n - C_{n-3} x_{n-2} x_{n-1} x_n + C_{n-4} x_{n-3} x_{n-2} x_{n-1} x_n -$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots + (-1)^n C_0(x_n x_{n-1} x_{n-2} x_{n-3} \ldots\ldots\ldots x_1 x_0)$$

Coefficient of x $= C_{n-1} - C_{n-2} (\sum_{i=n-1}^{n} x_i) + C_{n-3} (\sum_{i=n}^{n-1} \sum_{j=n-1}^{n-2} x_i x_j) -$
$$C_{n-4}(\sum_{i=n-1}^{n} \sum_{j=n-2}^{n-1} \sum_{k=n-3}^{n-2} x_i x_j x_k) + \ldots\ldots$$

$$+ (-1)^n C_0 (x_n x_{n-1} x_{n-2} x_{n-3} \ldots\ldots\ldots\ldots x_1 x_2 + x_n x_{n-1} x_{n-2} x_{n-3} \ldots\ldots x_1 x_0)$$

Coefficient of $x^2 = C_{n-2} - C_{n-3} \left(\sum_{i=n-2}^{n} x_i \right) + C_{n-4} \left(\sum_{i=n-2}^{n} \sum_{j=n-3}^{n-1} x_i x_j \right) -$

$$\ldots\ldots\ldots + (-1)^n C_0 (x_n x_{n-1} x_{n-2} x_{n-3} \ldots\ldots\ldots\ldots x_3 + x_n x_{n-1} x_{n-2} x_{n-3} \ldots\ldots\ldots\ldots x_2$$

$$+ x_n x_{n-1} x_{n-2} x_{n-3} \ldots\ldots\ldots\ldots x_1)$$

Coefficient of $x^3 = C_{n-3} - C_{n-4} \left(\sum_{i=n-3}^{n} x_i \right) + \ldots\ldots\ldots + (-1)^n C_0 (x_n x_{n-1} x_{n-2} x_{n-3} \ldots\ldots\ldots x_4 +$

$$x_n x_{n-1} x_{n-2} x_{n-3} \ldots\ldots\ldots x_3 + x_n x_{n-1} x_{n-2} x_{n-3} \ldots\ldots\ldots x_2 + x_n x_{n-1} x_{n-2} \ldots\ldots\ldots x_1)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

Coefficient of $x^i = C_{n-i} - C_{n-(i+1)} \left(\sum_{i=n-1}^{n} x_i \right) + \ldots\ldots\ldots\ldots + (-1)^{n-i} C_0 (x_n x_{n-1} x_{n-2} \ldots\ldots\ldots$

$$x_{n-(i+1)} + \ldots\ldots\ldots\ldots\ldots + x_n x_{n-1} x_{n-2} \ldots\ldots\ldots x_1)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

Coefficient of $x^n = C_0$

$\therefore$ The equation (2), can be expressed as

$$f(x) = A_n + A_{n-1} x + A_{n-2} x^2 + A_{n-3} x^3 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots + A_0 x^n \rightarrow (3)$$

which is the required formula for representation of numerical data by a polynomial curve

where

$A_n = C_n - C_{n-1} x_n + C_{n-2} x_{n-1} x_n - C_{n-3} x_{n-2} x_{n-1} x_n + C_{n-4} x_{n-3} x_{n-2} x_{n-1} x_n -$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots + (-1)^n C_0 (x_n x_{n-1} x_{n-2} x_{n-3} \ldots\ldots\ldots\ldots x_1 x_0)$$

$A_{n-1} = C_{n-1} - C_{n-2} \left(\sum_{i=n-1}^{n} x_i \right) + C_{n-3} \left(\sum_{i=n}^{n-1} \sum_{j=n-1}^{n-2} x_i x_j \right) -$

$C_{n-4} \left(\sum_{i=n-1}^{n} \sum_{j=n-2}^{n-1} \sum_{k=n-3}^{n-2} x_i x_j x_k \right) + \ldots\ldots$

$$+ (-1)^n C_0 (x_n x_{n-1} x_{n-2} x_{n-3} \ldots\ldots\ldots\ldots x_1 x_2 + x_n x_{n-1} x_{n-2} x_{n-3} \ldots\ldots x_1 x_0)$$

$A_{n-2} = C_{n-2} - C_{n-3} \left(\sum_{i=n-2}^{n} x_i \right) + C_{n-4} \left(\sum_{i=n-2}^{n} \sum_{j=n-3}^{n-1} x_i x_j \right) -$

$$\ldots\ldots\ldots + (-1)^n C_0 (x_n x_{n-1} x_{n-2} x_{n-3} \ldots\ldots\ldots\ldots x_3 + x_n x_{n-1} x_{n-2} x_{n-3} \ldots\ldots\ldots\ldots x_2$$

$$+ x_n x_{n-1} x_{n-2} x_{n-3} \ldots\ldots\ldots\ldots x_1)$$

$A_{n-3} = C_{n-3} - C_{n-4} \left(\sum_{i=n-3}^{n} x_i \right) + \ldots\ldots\ldots + (-1)^n C_0 (x_n x_{n-1} x_{n-2} x_{n-3} \ldots\ldots\ldots x_4$

$$+ x_n x_{n-1} x_{n-2} x_{n-3} \ldots\ldots\ldots x_3 + x_n x_{n-1} x_{n-2} x_{n-3} \ldots\ldots\ldots x_2 + x_n x_{n-1} x_{n-2} \ldots\ldots\ldots x_1)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$A_{n-i} = C_{n-i} - C_{n-(i+1)} \left(\sum_{i=n-1}^{n} x_i \right) + \ldots\ldots\ldots\ldots + (-1)^{n-i} C_0 (x_n x_{n-1} x_{n-2} \ldots\ldots\ldots$

$$x_{n-(i+1)} + \ldots\ldots\ldots\ldots\ldots + x_n x_{n-1} x_{n-2} \ldots\ldots\ldots x_1)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$A_0 = C_0$

Equation (3), with the coefficients

$A_n, A_{n-1}, A_{n-2}, A_{n-3}, \ldots\ldots\ldots\ldots\ldots\ldots.., A_0$

,as defined above, is the required formula for representing a given set of numerical data on a pair of variables by a suitable polynomial we have aimed at.

4. Example of Application of the Formula:

Example (4.1): The following table shows the data on total population of Assam corresponding to the years:

Year	2011	2001	1991	1981	1971
Total Population	31205576	26638407	22414322	18041248	14625152

Taking 1971 as origin and changing scale by 1/10, one can obtain the following table for independent variable x (representing time) and f(x) (representing total population of Assam):

Year	2011	2001	1991	1981	1971
x_i	0	1	2	3	4
$f(x_i)$	31205576	26638407	22414322	18041248	14625152

Now here $x_4 = 0, x_3 = 1, x_2 = 2, x_1 = 3, x_0 = 4$

$f(x_4) = 31205576, f(x_3) = 26638407, f(x_2) = 22414322, f(x_1) = 18041248, f(x_0) = 14625152$

Difference Table

X	f(x)	$f(x_4, x_3)$	$f(x_4, x_3, x_2)$	$f(x_4, x_3, x_2, x_1)$	$f(x_4, x_3, x_2, x_1, x_0)$
0	31205576				
		− 4567169			
1	26638407		171542		
		− 4224085		− 82012.16	
2	22414322		−74494.5		66584.99
		− 4373074		184327.83	
3	18041248		478489		
		− 3416096			
4	14625152				

Now, $C_4 = f(x_4) = 31205576$

$\qquad C_3 = f(x_4, x_3) = -4567169$

$\qquad C_2 = f(x_4, x_3, x_2) = 171542$

$\qquad C_1 = f(x_4, x_3, x_2, x_1) = -82012.16$

$\qquad C_0 = f(x_4, x_3, x_2, x_1, x_0) = 66584.99$

$\therefore$ The polynomial is

$f(x) = A_4 + A_3 x + A_2 x^2 + A_1 x^3 + A_0 x^4 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ (4)

$\qquad$ Where $A_4 = C_4 - C_3 x_4 + C_2 x_3 x_1 - C_1 x_2 x_3 x_4 + C_0 x_1 x_2 x_3 x_4 = 31205576$

$\qquad A_3 = C_3 - C_2(x_3 + x_4) + C_1(x_4 x_3 + x_3 x_2) - C_0(x_4 x_3 x_2 + x_4 x_3 x_1 + x_4 x_2 x_1 + x_3 x_2 x_1)$

$\qquad\qquad = -4567169 - 171542\,(1 + 0) - 82012.16\,(0 + 1\times2) - 66584.99\,(0 + 0 + 0 + 1\times2\times3)$

$\qquad\qquad = -5302245.3$

$\qquad A_2 = C_2 - C_1(x_4 + x_3 + x_2) + C_0(x_4 x_3 + x_4 x_2 + x_3 x_2 + x_4 x_1 + x_3 x_1 + x_2 x_1)$

$\qquad\qquad = 171542 + 82012.16\,(0 + 1 + 2) + 66584.99\,(0 + 0 + 1\times2 + 0 + 1\times3 + 2\times3)$

$\qquad\qquad = 1150013.4$

$\qquad A_1 = C_1 - C_0(x_4 + x_3 + x_2 + x_1)$

$\qquad\qquad = -82012.16 - 66584.99\,(0 + 1 + 2 + 3)$

$\qquad\qquad = -481522.1$

$\qquad A_0 = C_0 = 66584.99$

Thus, the polynomial that can represent the given numerical data is

$\therefore$ (4) $\Rightarrow$ $f(x) = 31205576 - 5302245.3\,x + 1150013.4\,x^2 - 481522.1\,x^3 + 66584.99\,x^4$

This polynomial yields the values of the function $f(x)$ corresponding to the respective observed values as follows:

$\qquad f(0) = 31205576$

$\qquad f(1) = 31205576 - 5302245.3 \times 1 + 1150013.4 \times 1 - 481522.1 \times 1 + 66584.99 \times 1$

$\qquad\qquad = 26638407$

$\qquad f(2) = 31205576 - 5302245.3 \times 2 + 1150013.4 \times 4 - 481522.1 \times 8 + 66584.99 \times 16$

$\qquad\qquad = 22414322$

$\qquad f(3) = 31205576 - 5302245.3 \times 3 + 1150013.4 \times 9 - 481522.1 \times 27 + 66584.99 \times 81$

$\qquad\qquad = 18041248$

$\qquad f(4) = 31205576 - 5302245.3 \times 3 + 1150013.4 \times 9 - 481522.1 \times 27 + 66584.99 \times 81$

$\qquad\qquad = 14625152$

Example (4.2): The following table shows the data on total population of India corresponding to the years:

Year	2011	2001	1991	1981	1971
Total Population	1210193422	1027015247	846302688	683329097	548159652

Taking 1971 as origin and changing scale by 1/10, one can obtain the following table for independent variable x (representing time) and f(x) (representing total population of Assam):

Year	2011	2001	1991	1981	1971
x_i	0	1	2	3	4
$f(x_i)$	1210193422	1027015247	846302688	683329097	548159652

Now here $x_4 = 0$, $x_3 = 1$, $x_2 = 2$, $x_1 = 3$, $x_0 = 4$

$\qquad f(x_4) = 1210193422$, $f(x_3) = 1027015247$, $f(x_2) = 846302688$, $f(x_1) = 683329097$, $f(x_0) = 548159652$

Difference Table

x	f(x)	$f(x_4, x_3)$	$f(x_4, x_3, x_2)$	$f(x_4, x_3, x_2 x_1)$	$f(x_4, x_3, x_2, x_1, x_0)$
0	1210193422				
		-183178175			
1	1027015247		1232808		
		-180712559		2545558.66	
2	846302688		8869484		-217007.25
		-162973591		1677529.66	
3	683329097		13902073		
		-135169445			
4	548159652				

Now, $C_4 = f(x_4) = 1210193422$

$\qquad C_3 = f(x_4, x_3) = -183178175$

$\qquad C_2 = f(x_4, x_3, x_2) = 1232808$

$\qquad C_1 = f(x_4, x_3, x_2, x_1) = 2545558.66$

$$C_0 = f(x_4, x_3, x_2, x_1, x_0) = -217007.25$$

$\therefore$ The polynomial is

$$f(x) = A_4 + A_3 x + A_2 x^2 + A_1 x^3 + A_0 x^4 \dots\dots\dots\dots\dots\dots\dots \text{(5)}$$

Where $A_4 = C_4 - C_3 x_4 + C_2 x_3 x_1 - C_1 x_2 x_3 x_4 + C_0 x_1 x_2 x_3 x_4 = 1210193422$

$A_3 = C_3 - C_2(x_3 + x_4) + C_1(x_4 x_3 + x_3 x_2) - C_0(x_4 x_3 x_2 + x_4 x_3 x_1 + x_4 x_2 x_1 + x_3 x_2 x_1)$
$\quad = -183178175 - 1232808(1+0) + 2545558.66(0 + 1\times2) + 217007.25(0 + 0 + 0 + 1\times2\times3)$
$\quad = -178017822.18$

$A_2 = C_2 - C_1(x_4 + x_3 + x_2) + C_0(x_4 x_3 + x_4 x_2 + x_3 x_2 + x_4 x_1 + x_3 x_1 + x_2 x_1)$
$\quad = 1232808 - 2545558.66(0 + 1 + 2) - 217007.25(0 + 0 + 1\times2 + 0 + 1\times3 + 2\times3)$
$\quad = -8790947.73$

$A_1 = C_1 - C_0(x_4 + x_3 + x_2 + x_1)$
$\quad = 2545558.66 + 217007.25(0 + 1 + 2 + 3) = 3847602.16$

$A_0 = C_0 = -217007.25$

Thus, the polynomial that can represent the given numerical data is

$\therefore (5) \Rightarrow f(x) = 1210193422 - 178017822.18\, x - 8790947.73 x^2 + 3847602.16\, x^3 - 217007.25\, x^4$

This polynomial yields the values of the function $f(x)$ corresponding to the respective observed values as follows:

$f(0) = 1210193422 - 178017822.18 \times 0 - 8790947.73 \times 0 + 3847602.16 \times 0 - 217007.25 \times 0$
$\quad = 1210193422$

$f(1) = 1210193422 - 178017822.18 \times 1 - 8790947.73 \times 1 + 3847602.16 \times 1 - 217007.25 \times 1$
$\quad = 1027015247$

$f(2) = 1210193422 - 178017822.18 \times 2 - 8790947.73 \times 4 + 3847602.16 \times 8 - 217007.25 \times 16$
$\quad = 846302688$

$f(3) = 1210193422 - 178017822.18 \times 3 - 8790947.73 \times 9 + 3847602.16 \times 27 - 217007.25 \times 81$
$\quad = 683329097$

$f(4) = 1210193422 - 178017822.18 \times 4 - 8790947.73 \times 16 + 3847602.16 \times 64 - 217007.25 \times 256$
$\quad = 548159652$

5. Conclusion

The formula described by equation (3) can be used to represent a given set of numerical data on a pair of variables, by a polynomial.

The polynomial that represents the given set of numerical data can be used for interpolation at any position of the independent variable lying within its two extreme values.

Newton's forward interpolation formula is valid for estimating the value of the dependent variable under the following two conditions:

(i) The given values of the independent variable are at equal interval.

(ii) The value of the independent variable corresponding to which the value of the dependent variable is to be estimated lies in the first half of the series of the given values of the independent variable.

However, Newton's divided difference interpolation formula is valid for estimating the value of the dependent variable beyond these two conditions. Therefore, the formula derived here is valid for representing a set of numerical data on a pair of variables by a polynomial beyond these two conditions.

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