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Analytical expressions of a steady MHD viscous incompressible fluid flow through a porous planner channel

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Abstract
In this paper we investigate the oscillatory flow of a steady MHD viscous incompressible fluid with radiative heat transfer analysis through a porous planar channel filled with a saturated porous medium in slip condition is considered. It is guessing that the no-slip condition between the wall and the fluid remains no longer valid. The approximate analytical expressions the dimensionless axial velocity, dimensionless temperature and the slip parameters for the non-linear boundary value problem are derived. We also discuss the graphical representations of the dimensionless velocities, dimensionless temperatures are also discussed.

Keywords: Radical functional equation, Generalized Hyers-Ulam stability.

1. Introduction
Viscosity can be defined through the relation between the shear stress, and force per unit area *F/A*, needed to keep the upper plate moving with a constant velocity [1]. Applications of viscous flow are turbulent boundary layer, flat plate frictional law, application to heat transfer problems (boundary layer energy equations, similar solution for energy equation in flat plate, integral method, convective thickness, velocity and temperature distributions, Stanton number, Couette flow assumption and numerical solution of differential equations).When a viscous fluid flows over a solid surface, there is no relative motion between the fluid and the solid at the interface. Under normal circumstances, the no-slip condition provides a realistic restriction on solutions of the Navier-stokes equation ([2]). The boundary condition for a viscous fluid at a solid wall is one of no-slip, i.e. the fluid velocity matches the velocity of the solid boundary. Navier proposed a general boundary condition that incorporates the possibility of fluid slip at a solid boundary. Navier's proposed condition assumes that the velocity, $V_x$at a solid surface is proportional to the shear stress at the surface (Navier [3], Goldstein [4]).

$$V_x = \gamma\left(dv_x/dy\right) \quad (1)$$

Where, $\gamma$is the slip strength or slip coefficient. If γ=0 then the general assumed no-slip boundary condition is obtained. If γ is finite, fluid slip occurs at the wall but its effect depends upon the length scale of the flow. A fluid in direct contact with a solid sticks to the surface due to viscous effects, and there is no slip. The flow region adjacent to the wall in which the viscous effects (and thus the velocity gradients) are significant is known as boundary layer. Viscous flow region–flows in which the frictional effect is significant [5]. Fluid flow under the influence of magnetic field and heat transfer occurs in magneto-hydrodynamics accelerators, pumps and generators. This type of fluid has uses in nuclear reactors, plasma studies, geothermal energy extraction, and the boundary layer control in the field of aerodynamics [6]. By applying the perturbation technique Kumar *et al.* [8] investigated the same problem of slip-flow regime for the unsteady MHD periodic flow of viscous fluid through a planer channel.
Hayat *et al.* [7] studied heat transfer and slip flow of a second grade fluid past a stretching sheet through a porous space.

2. Mathematical formulation of the problem

Assume that the flow of an incompressible viscous and electrically conducting fluid in a channel filled with saturated porous medium under the influence of an externally applied homogeneous magnetic field and radiative heat transfer as shown in Fig.1.

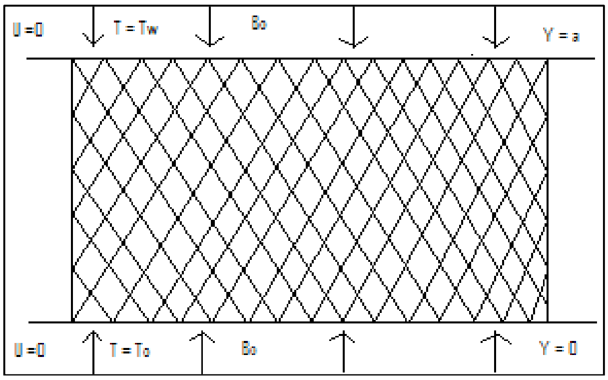

Fig 1: Geometry of the problem

It is assumed that the fluid has small electrical conductivity and the electromagnetic force produced is very small. Take a Cartesian coordinate system (x', y'), where, ox' lies along the centre of the channel, y' is the distance measured in the normal section. Then, assuming Boussinesq incompressible fluid model, the equations governing the motion are given by

$$\frac{\partial u'}{\partial t'} = -\frac{1}{\rho}\frac{\partial P'}{\partial x'} + \upsilon\frac{\partial^2 u'}{\partial y'^2} - \frac{\upsilon}{K}u' - \frac{\sigma_e B_0^2}{\rho}u' + g\beta(T - T_0) \qquad (2)$$

$$\frac{\partial T}{\partial t'} = \frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho C_p}\frac{\partial q}{\partial y'} + \frac{Q}{\rho C_p}(T - T_0) \qquad (3)$$

The corresponding boundary conditions are as follows:

$$\left.\begin{aligned} u' - a\gamma\frac{\partial u'}{\partial y'} = 0, T = T \quad on \quad y' = 0 \\ u' = 0, T = T_w \quad on \quad y' = 1 \end{aligned}\right\} \qquad (4)$$

It is assumed that both walls temperature T_0, T_w are high enough to induce radiative heat transfer. It is also assumed that the fluid is optically thin with a relatively low density and the radiative heat flux is given by

$$\frac{\partial q}{\partial y'} = 4\alpha^2(T_0 - T) \qquad (5)$$

Where α is the mean radiation absorption coefficient. We introduce the dimensionless variables and parameters are as follows:

$$\mathrm{Re} = \frac{U_a}{\upsilon}, x = \frac{x'}{a}, y = \frac{y'}{a}, u = \frac{u'}{U}, \theta = \frac{T - T_0}{T_w - T_0}, t = \frac{t'U}{a},$$
$$H^2 = \frac{a^2\sigma_e B_0^2}{\rho\upsilon}, S^2 = (1/Da), b^2 = (Qa^2/k), P = \frac{aP'}{\upsilon\rho U},$$
$$G_r = \frac{a^2 g\beta(T_w - T_0)}{\upsilon U}, Pe = \frac{Ua\rho C_p}{k}, N^2 = \frac{4\alpha^2 a^2}{k}, Da = \frac{K}{a^2} \qquad (6)$$

Where U is the flow mean velocity, the dimensionless governing equations together with the appropriate boundaries conditions, can be written as

$$\mathrm{Re}\frac{\partial u}{\partial t} = -\frac{\partial P}{\partial x} + \frac{\partial^2 u}{\partial y^2} - (S^2 + H^2)u + Gr\theta \qquad (7)$$

$$Pe\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + (N^2 + b^2)\theta \qquad (8)$$

The steady flow eqns. of (7) and (8) are as follows:

$$\lambda + \frac{d^2 u}{dy^2} - (S^2 + H^2)u + Gr\theta = 0 \qquad (9)$$

$$\frac{d^2\theta}{dy^2} + (N^2 + b^2)\theta = 0 \qquad (10)$$

The corresponding boundary conditions are as follows:

$$u - \gamma\frac{\partial u}{\partial y} = 0, \theta = 0 \quad on \quad y = 0 \qquad (11)$$

$$u = 0, \theta = 1 \quad on \quad y = 1 \qquad (12)$$

Where Gr means Grashoff number, H indicates Hartmann number, N denotes Radiation parameter, Pe represents Peclet number, Re refers Reynolds number, Da means Darcy number and S denotes porous medium shape factor parameter.

3. Solution of the non-linear boundary value problem

In recent days, a basic tool for solving non-linear problem in Homotopy analysis method (HAM) which was generated by Liao [12], is employed to solve the non-linear differential equation. The Homotopy analysis method is based on a basic concept in topology, i.e. Homotopy by Hilton [13] which is widely applied in numerical techniques as in [14-16]. Homotopy analysis method is independent of the small/large parameters not like the perturbation techniques [17]. There is a simple way to adjust and control the convergence region and rate of approximation series in Homoyopy analysis method. The Homotopy analysis method has applied in many non-linear problems such as heat transfer, viscous, non-linear oscillations [18], non-linear water waves [22], etc. Such varied successful applications of the Homotopy analysis method to conform its validity for non-linear problems in science and engineering. The auxillary parameter h is used to adjust and control the convergence of the series solution. In [27], mathematical expression is solved using the Homotopy analysis method.

In this paper, the non-linear boundary value problem which is expressed in the eqns. (9) – (12) can be solved directly. The approximate analytical expressions for the dimensionless axial velocity $u(y)$ and dimensionless temperature $\theta(y)$ are as follows:

$$u(y) = C_1\exp\left(\sqrt{M}y\right) + C_2\exp\left(-\sqrt{M}y\right) + \frac{\lambda}{M} + \frac{Gr\sin\left(\sqrt{A}y\right)}{\sin\left(\sqrt{A}\right)(A + M)} \qquad (13)$$

$$\theta(y) = \frac{\sin\left(\sqrt{A}y\right)}{\sin\sqrt{A}} \tag{14}$$

Where

$$C_1 = \frac{-C_2\exp\left(-\sqrt{M}\right) - \frac{\lambda}{M} - \frac{Gr}{A+M}}{\exp\left(\sqrt{M}\right)} \tag{15}$$

$$C_2 = \frac{\frac{-\lambda\left(\exp\sqrt{M} - 1 + \gamma\sqrt{M}\right)}{M} + \frac{Gr\left(\frac{\exp\sqrt{M}\gamma\sqrt{A}}{\sin\sqrt{A}} + 1 - \gamma\sqrt{M}\right)}{A+M}}{\exp\sqrt{M}\left(1+\gamma\sqrt{M}\right) - \exp\left(-\sqrt{M}\right)\left(1-\gamma\sqrt{M}\right)} \tag{16}$$

$$M = S^2 + H^2 \tag{17}$$

$$A = N^2 + b^2 \tag{18}$$

4. Result and Discussion

Figure 1 shows the geometry of the MHD flow problem. Figure 2 represents the dimensionless temperature $\theta(y)$ versus the dimensionless transverse distance y. From Fig. 2(a and b) it is noted that when the heat generating source parameter b and the radiation parameter N increases the corresponding dimensionless temperature profile also increases in some fixed values of other dimensionless parameters $Pe, S, H, Gr, \mathrm{Re}, \lambda$.

Figure 3 represents the axial velocity $u(y)$ versus the transverse distance y. From Fig. 3(a) it is clear that when the Grashoff number Gr increases the corresponding axial velocity profile increases in some fixed values of the other parameters. From Fig. 3(b) it is inferred that when the Hartmann number H increases the corresponding axial velocity profile decreases in some fixed values of the other parameters. From Fig. 3(c) it is depict that when the porous medium shape factor S increases the corresponding axial velocity profile decreases in some fixed values of the other parameters. From Fig. 3(d) it is clear that when the heat generating source parameter b increases the corresponding axial velocity profile increases in some fixed values of the other parameters. From Fig. 3(e) it is clear that when the radiation parameter N increases the corresponding axial velocity profile increases in some fixed values of the other parameters. From Fig. 3(f) it is clear that when the slip parameter γ increases the corresponding axial velocity profile increases in some fixed values of the other parameters.

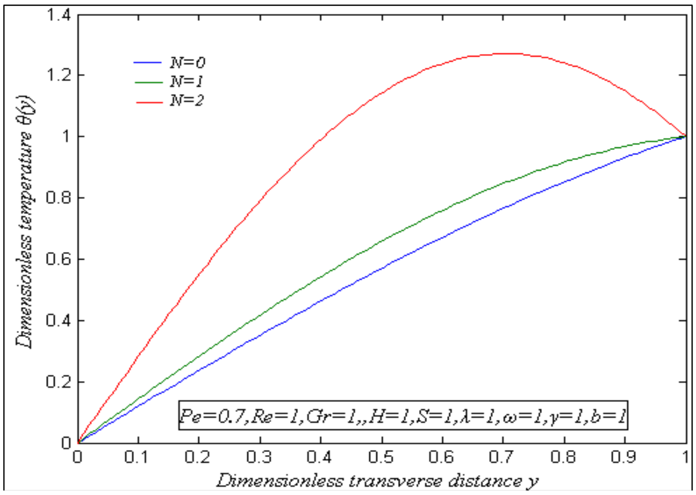

Fig 2a

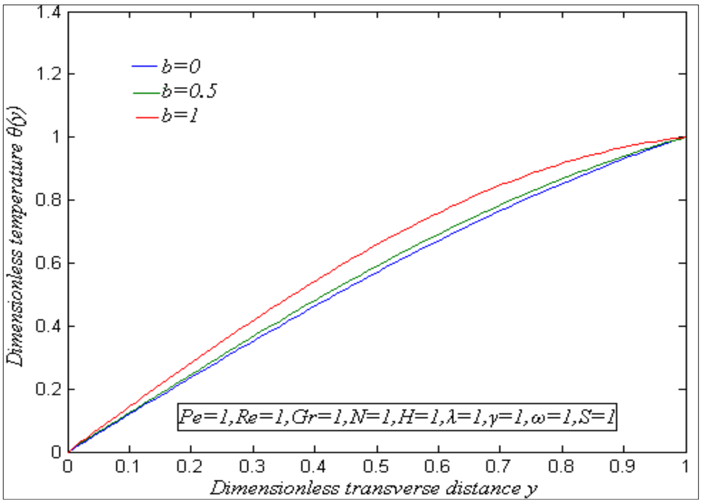

Fig 2b

Fig. 2: Dimensionless temperature $\theta(y)$ versus dimensionless transverse distance y. The curves are plotted using the eqn. (14) for various values of the dimensionless heat generating source parameter b and in some fixed values of the other dimensionless parameters.

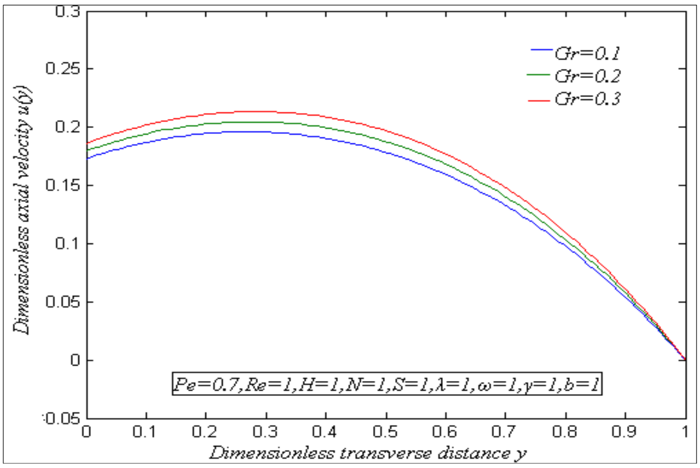

Fig 3(a)

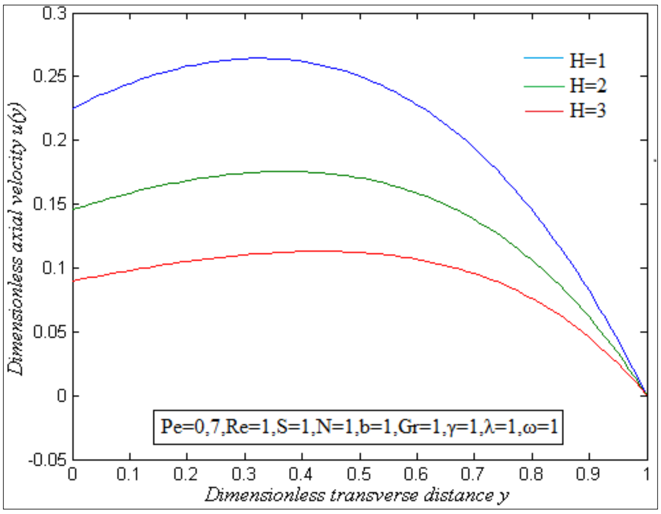

Fig 3(b)

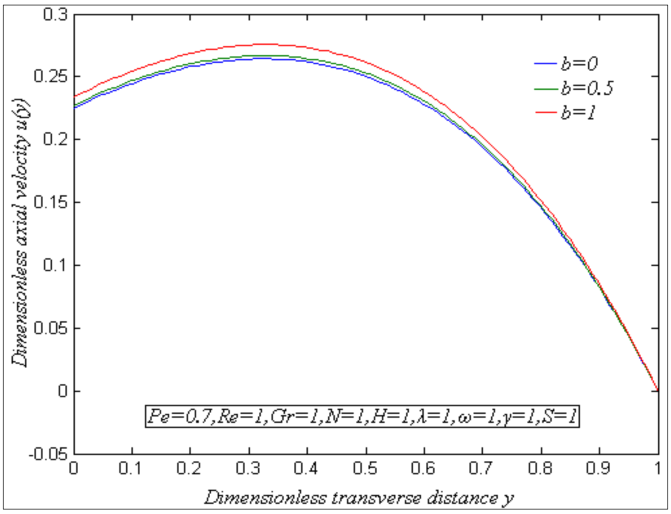

Fig 3(d)

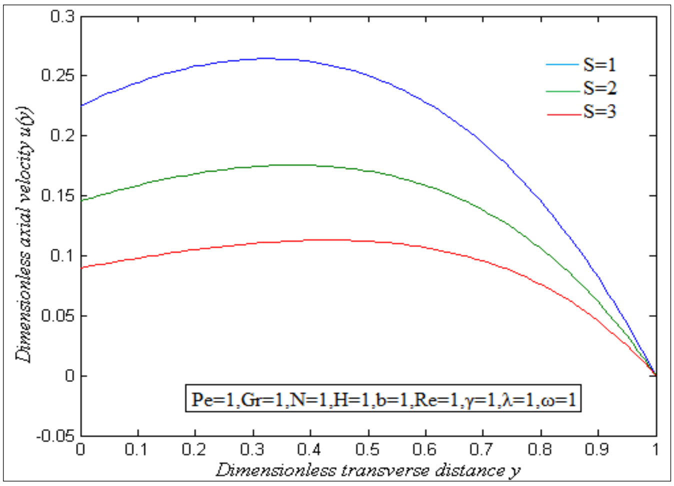

Fig 3(c)

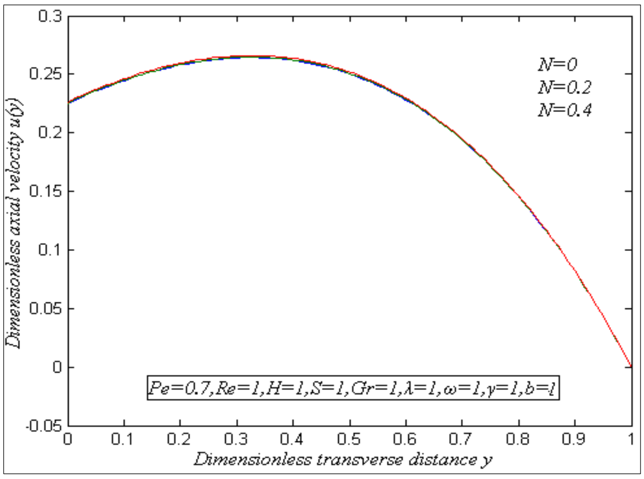

Fig 3(e)

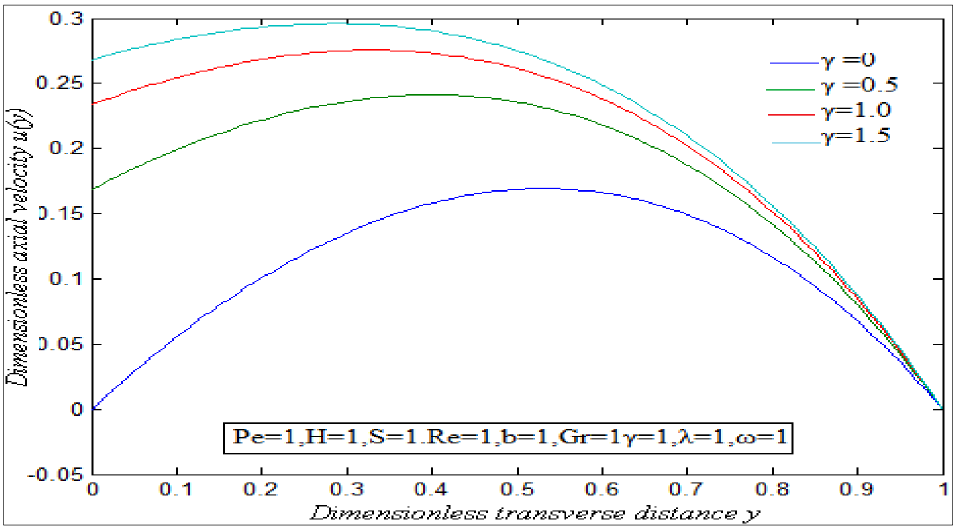

Fig (f)

Fig 3: Dimensionless axial velocity $u(y)$ versus dimensionless transverse distance y. The curves are plotted using eqn. (13) for various values of the dimensionless parameters H, N, S, Gr, b and in some fixed values of the other dimensionless parameters, when

(a) $Pe = 0.7, H = 1, S = 1, N = 1, \mathrm{Re} = 1, b = 1, \lambda = 1, \omega = 1, \gamma = 1$

(b) $Pe = 0.7, Gr = 1, S = 1, N = 1, \mathrm{Re} = 1, b = 1, \lambda = 1, \omega = 1, \gamma = 1$

(c) $Pe = 0.7, Gr = 1, H = 1, N = 1, \mathrm{Re} = 1, b = 1, \lambda = 1, \omega = 1, \gamma = 1$

(d) $Pe = 0.7, Gr = 1, H = 1, N = 1, \mathrm{Re} = 1, S = 1, \lambda = 1, \omega = 1, \gamma = 1$

(e) $Pe = 0.7, Gr = 1, H = 1, b = 1, \mathrm{Re} = 1, S = 1, \lambda = 1, \omega = 1, \gamma = 1$.

5. Conclusion

The analytical expressions of the dimensionless axial velocities and dimensionless temperatures for the steady MHD fluid flow problem are derived mathematically and graphically. We conclude from the plotted velocity profile as fluid velocity increases while increasing Grashoff number Gr, Radiation number N, heat generating source parameter b and the non-dimensional slip parameter γ and it decreases with increasing of the Hartmann number H and also Porous medium shape factor S.

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Appendix-A: Nomenclature

Symbol	Meaning
C_p	Specific heat at constant pressure
Da	Darcy number
g	Gravitational force
Gr	Grashoff number
H	Hartmann number
K	The porous medium permeability
N	Radiation parameter
P	Pressure
Pe	Peclet number
q	Radioactive heat flux
Re	Flow Reynolds number
S	Porous medium shape factor
b	Heat generating source parameter
t	Non-dimensional time variable
T	Fluid temperature
T_0	Temperature at y=0
T_w	Temperature at y=a
u	The axial velocity
x	The axial distance
y	Transverse distance
Q	Dimensional heat generation or absorption coefficient
β	Volumetric coefficient of thermal expansion
λ	A constant
γ	The non-dimensional slip parameter
ω	Frequency of the oscillation
ν	Kinematic viscosity coefficient
ρ	The fluid density
μ	Coefficient of viscosity
θ	Non-dimensional temperature

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