

International Journal of Statistics and Applied Mathematics



ISSN: 2456-1452
Maths 2017; 2(2): 26-30
© 2017 Stats & Maths
www.mathsjournal.com
Received: 16-01-2017
Accepted: 17-02-2017

Ze Zhong Yang
Ph.D., The School of
Mathematics and Statistics,
Shandong Normal University,
Jinan, China

Tong Wang
M.A., The School of Mathematics
and Statistics, Shandong Normal
University, Jinan, China

Study on the data in *Bi Li Si Xian Xin Biao*

Ze Zhong Yang and Tong Wang

Abstract

The *Bi Li Si Xian Xin Biao* was an important book compiled by Xue Fengzuo, a famous calendarian and a mathematician living in the end of Ming dynasty and early Qing dynasty. This book introduced the western trigonometric function logarithm, and received widespread attention and acclaim at that time and the later. However, after analyzing the data in this book carefully, it could be found that about 80% of the data actually were of relatively large deviation, and some of them were even completely wrong. The reason for this might be that Xue Fengzuo misunderstood the nature of the trigonometric function, and used the incorrect calculation method. This showed that, when compiling the book, Xue Fengzuo did not conduct strict logarithmic calculation, instead just did some simple arithmetic operation.

Keywords: *Bi Li Si Xian Xin Biao*, Trigonometric functions, Logarithm, Xue Fengzuo

1. Introduction

Bi Li Si Xian Xin Biao (also known as *Zhong Fa Si Xian*) is an important book compiled by Xue Fengzuo (1599-1680), a famous calendarian living in the late Ming dynasty and early Qing dynasty in China. This book introduced the western trigonometric function logarithm through giving all values of the sine, cosine, tangent and cotangent within 90 degrees at the interval of 1 minute (centesimal system) firstly. Even though these knowledge came from his teacher, a Poland missionary named Jean Nicolas Smogolenski (1610-1656), his book (*Bi Li Si Xian Xin Biao*) received widely attention and acclaim in history. Since it was the first time that these knowledge were introduced into China, and the works of Xue Fengzuo were not only careful but also complete (Yuan, Z.T., 1991)^[1]. However, some of the issues on the book have not been able to get a better answer now, such as, were the data in book correct? How did they be calculated? were they translated from the western mathematics book directly? and so on. The study on these issues are meaningful, because many works of Xue Fengzuo were conducted based on these data.

Ruan Yuan ever said, "Because it was not convenient to calculate with 60 degrees, Xue Fengzuo' *Zhong Fa Si Xian* switched to old methods---with a degree of 100. What values he listed only were the values of sine, cosine, tangent and cotangent, so the name *Si Xian* came into being. This influenced many books, such as *Tai Yang Tai Yin Zhu Xing Fa Yuan*, *Mu Huo Tu San Xing Jing Xing Fa Yuan*, *Jiao Shi Fa Yuan*, *Li Nian Jia Zi* and *Qiu Sui Shi*." (Ruan, Y., 2009)^[2]

“(薛凤祚) 中法四线者，以西法六十分为度不便于算，改从古法，以百分为度表，所列止正弦、余弦、正切、余切，故曰四线。其推步诸书，曰《太阳太阴诸行法原》，曰《木火土三星经行法原》，曰《交食法原》，曰《历年甲子》，曰《求岁实》……皆从此起算。” At that time, Mei Wending also said, “Xue Yifu compiled *Bi Li Si Xian Xin Biao* by using *Si Xian*...Mu (that was Jean Nicolas Smogolenski) wrote *Tian Bu Zhen Yuan*, while Xue wrote *Tian Xue Hui Tong*. If you do not know this, you should not read these two books.” (Mei, W.D., 1935)^[3]

“(薛仪甫又有四线新比例，用四线……穆有天步真原，薛有天学会通，并依此立算，不知此则二书不可以读。”)

Correspondence
Ze Zhong Yang
Ph.D., The School of
Mathematics and Statistics,
Shandong Normal University,
Jinan, China

However, these works have been controverted for a long time (Zhao, E.X., 1986) [4]. There are different opinions on the scientificity and rationality of these works (Liu, X.X., 2011) [5]. As a result, studying the *Bi Li Si Xian Xin Biao* is beneficial for people to understand Xue Fengzuo' research further more, especially the work in astronomical calendar, and give a more precise statement, etc. In order to answer the above questions, this paper will analyze the data in Xue Fengzuo' *Bi Li Si Xian Xin Biao*.

2. The analysis of the data in *Bi Li Si Xian Xin Biao*

Bi Li Si Xian Xin Biao has only one volume, and it is mainly composed of some data. These data can be divided into four categories ---sine function logarithm values, cosine function logarithm values, tangent function logarithm values, and cotangent function logarithm values. Each category includes 4500 data, so there are a total of 18000 data. These data are all positive integers, no negative numbers and decimals. In addition, the data are all seven digits, only the cotangent function logarithmic values are eight digits. The data are shown in table 1.

Table 1

	Cotangent	Tangent	Cosine	Sine	θ (degree)
.....
89 (minutes)	12722903	7277095	9999998	7277095	11 (minutes)
88	12687185	7312313	9999998	7312313	12
87	12652466	7347531	9999998	7347531	13
86	12617248	7382749	9999998	7382749	14
85	12582030	7417969	9999998	7417968	15
84	12557042	7442957	9999998	7442955	16
83	12532054	7467945	9999998	7467943	17
82	12507066	7492931	9999998	7492930	18
81	12482078	7517919	9999997	7517918	19
80	12457091	7542909	9999997	7542906	20
.....
89 (degree)	Tangent	Cotangent	Sine	Cosine	

Starting from the angle 0°5', 0°10', 0°15', 0°20', 98°90' that are easy to transform into sixtieth system, it can be found that Xue Fengzuo is likely to apply the following four formulas to calculate function logarithm values. The reason is that the values in the book are almost the same as the results calculated by the following formulas (there is only a slight difference in the last digit of some values).

- sine function logarithm values = Round (10⁶ log1010 sin a, 0)
- cosine function logarithm values = Round (10⁶ log1010 cos a, 0)
- tangent function logarithm values= Round (10⁶ log1010 tan a, 0)
- cotangent function logarithm values = Round (10⁶ log1010 cot a, 0)

For example, using 0°20' to verify, the results calculated by the above four formulas are as follows:

Round(10⁶ log10¹⁰ sin 0°20', 0) =7542906

Round(10⁶ log 10¹⁰ cos 0°20', 0) =9999997

Round(10⁶ log 10¹⁰ tan 0°20', 0) =7542909

Round(10⁶ log 10¹⁰ cot 0°20', 0) =12457091

The results are exactly the same as the results given in *Bi Li Si Xian Xin Biao*. Did Xue Fengzuo really use the above formulas or a similar approach?

To verify with the other angles, we find that the calculation results are quite different from those given in *Bi Li Si Xian Xin Biao*. For example, use the above formulas to calculate the trigonometric function logarithm values of the angles between 0°10' and 0°20' (as shown in table 2), and then compare with the results in table 1, it can be found that they are significantly different.

Table 2

	Cotangent	Tangent	Cosine	Sine	θ (degree)
.....
89 (minutes)	12716729	7283271	9999999	7283270	11 (minutes)
88	12678941	7321059	9999999	7321058	12
87	12644179	7355821	9999999	7355820	13
86	12611994	7388006	9999999	7388005	14
85	12582030	7417970	9999999	7417968	15
84	12554002	7445998	9999998	7445997	16
83	12527672	7472328	9999998	7472326	17
82	12502849	7497151	9999998	7497149	18
81	12479367	7520633	9999998	7520630	19
80	12457091	7542909	9999997	7542906	20
.....
89 (degree)	Tangent	Cotangent	Sine	Cosine	

What we get after analyzing the differences between the calculation results and the numbers given in *Bi Li Si Xian Xin Biao* carefully is that most of the differences are found in the last five digits of seven digits. What's more, in a part of

results, the last six digits are all different, such as the sine function logarithm values and tangent function logarithm values of angles between 0°1' to 0°4'. The two different kinds of calculation results are shown in table 3.

Table 3

	Cotangent	Tangent	Cosine	Sine	0 (degree)
Calculation result given in <i>Bi Li Si Xian Xin Biao</i>					
99	13421444	6343346	10000000	6343346	1 (minute)
98	13330871	6492721	10000000	6492721	2
97	13240298	6642096	10000000	6642096	3
96	13149725	6791472	10000000	6791472	4
Calculation result by modern computer technology					
99	13758123	6241877	10000000	6241877	1 (minute)
98	13457093	6542907	10000000	6542907	2
97	13281001	6718999	10000000	6718999	3
96	13156063	6843937	10000000	6843937	4
89 (degree)	Tangent	Cotangent	Sine	Cosine	

Thus, it can be sure that Xue Fengzuo didn't use the above formulas or a similar way. If he use the above formulas, there will never be such a big error, also there will not be so many errors. Moreover, there won't appear a strange phenomenon that the trigonometric function logarithm values of special angles (the angles that are positive integer multiple of 5) are correct but those of other angles are wrong.

The above results were calculated with the help of the modern computers. Modern computing technology has a strong computing function, the precision and accuracy of calculation is of no doubt. From this we can know that the trigonometric function logarithm values of angles within 90 degrees at the interval of 1 minute (centesimal system) calculated by above formulas are certainly not wrong. However, the differences between the results calculated by the above formulas with the help of modern computer technology and the data in *Bi Li Si Xian Xin Biao* are so big and so many--- the data that are different almost accounts for 80% of all the data in *Bi Li Si Xian Xin Biao*. The conclusion only can be that the errors of the data in *Bi Li Si Xian Xin Biao* are so large that the accuracy and scientificity of the authors' calculation are not believable.

3. The analysis about the causes of numerical error in *Bi Li Si Xian Xin Biao*

Why there are so many wrong data in *Bi Li Si Xian Xin Biao* ? Figuring out this problem is undoubtedly very important to have a comprehensive understanding of Xue Fengzuo' work.

From the the above analysis, it can be known that the data in Xue Fengzuo' *Bi Li Si Xian Xin Biao* are not calculated by the above formulas or its equivalent method. So the question is how he calculated it. In order to figure out this problem, the author looked through the Xue Fengzuo' related books in *Li Xue Hui Tong* carefully, and found an instruction about the method of making the trigonometric function table in the first book *Sine of Li Xue Hui Tong*. More, using the method described in the book, we can completely calculate all the 9000 sine function values of the angles that interval of 1 minute (centesimal system). The formula adopted to calculate the sine function values of very small angles in this book is: $\alpha : \sin \alpha = \beta : \sin \beta$. For example, calculate the value of $\sin 10'$ from the known value of $\sin 15'$. The concrete

computation method is as follows: $\sin 10' = \frac{\sin 15'}{15'} \times 10'$. Take

another instance. Calculate the value of $\sin 8'$ from the known value of $\sin 7'$. The concrete computation method is

as follows: $\sin 7' = \frac{\sin 8'}{8'} \times 7'$ (Yang, Z.Z., 2011)^[6].

So whether the author used the above method? The four kinds of trigonometric function values can be worked out in this

way obviously. The process is: calculate the sine value of angle, then use relevant formulas to calculate the cosine value of angle, and then find out the tangent and cotangent value. The author carried out a verification, and found that the calculation results by this method were also different from that in *Bi Li Si Xian Xin Biao*. It can be illustrated by the examples of the logarithm of $\sin 4'$, $\sin 3'$, $\sin 2'$ and $\sin 1'$. The corresponding calculation results are 6241877, 6542907, 6718999, 6843937. These results were as same as that calculated by the above formula--- $\text{Round}(10^6 \log 10^{10} \sin \alpha, 0)$. From this we can see that Xue Fengzuo did not apply the method in the *Sine*.

So are these data directly from the western mathematical books? In order to figure out this problem, the author reviewed many kinds of trigonometric function logarithm tables published in western society at that time --including that had been introduced into China and had not been introduced into our country, but none was been found to be identical. John Newton, a westerner, published the book *Trigonometria Britanica* in 1658. He also put forward a trigonometric function logarithm table within 90 degrees at the interval of 1 minute (centesimal system), but its values were 9 digits. The results were exactly the same as the results calculated by modern computing tool (as shown in table 1), so what John Newton had calculated were correct and it had not any similar errors appeared in Xue Fengzuo' *Bi Li Si Xian Xin Biao*(John, N.,1658)^[7].

So whether these results were from the old *Bi Li Si Xian Biao* written by Jean Nicolas Smogolenski-Xue Fengzuo's teacher? Because Jean Nicolas Smogolenski' *Bi Li Si Xian Biao* was missing, we are unable to do any direct examination now (Li, Y. & Qian, B.C., 1998)^[8]. According to the contents of *Zhong Fa Si Xian Yin* written by Xue Fengzuo, it can be concluded that the above conjecture is impossible too. To sum up, the results should be calculated by Xue Fengzuo himself. In *Zhong Fa Si Xian Yin*, Xue Fengzuo said, "There is a thought that to unify the advanced western mathematical methods with the traditional Chinese mathematics research habits, but worried that Chinese traditional approach is too unrestrained. According to the old methods, the sexagesimal system and the centesimal system are not compatible so we have to change one to unify. After that, the research method of all the books including *Ba Xian* is the centesimal system instead of the sexagesimal system and therefore the unification is

achieved."("今有较正会通之役, 复患中法太脱略, 而旧法又以六成十不能相入, 乃取而通之, 自诸书以及八线皆取其六数通以十数。") In addition, beneath the title of *Bi Li Si Xian Xin Biao*, it was marked with " written by Xue Fengzuo, revised by Liu gongjian

"(“海岱薛凤祚纂著，颍川后学刘拱键较”)。From this, it can be seen that this book should be the work of Xue Fengzuo, rather than using the values of Jean Nicolas Smogolenski. Then how did Xue Fengzuo calculate the results in the book? The author collated the data. Firstly, sort all the angles from small to large. After that, put every 5 angles into one group to obtain several small units. When comparing the data, the

author found a phenomenon. In every unit contained 5 trigonometric function logarithm values, the result of a number minus the number in front of it was basically the same. For example, the difference of ten sine function logarithms and tangent function logarithms between $0^{\circ}10'$ and $0^{\circ}20'$ were 35218, 24987, 35218 and 24988 respectively. The specific situation was shown in table 4.

Table 4

θ (degree)	Sine	The difference of adjacent two sine function logarithm	Tangent	The difference of adjacent two tangent function logarithm
.....
10 (minutes)	7241877	7241878
11 (minutes)	7277095	35218	7277095	35217
12	7312313	35218	7312313	35218
13	7347531	35218	7347531	35218
14	7382749	35218	7382749	35218
15	7417968	35219	7417969	35220
16	7442955	24987	7442957	24988
17	7467943	24988	7467945	24988
18	7492930	24987	7492931	24988
19	7517918	24988	7517919	24988
20	7542906	24988	7542909	24990
.....

This rule was throughout the whole indicator. As a result, the author speculated that Xue Fengzuo was likely to calculate by the following ways:

- (1) calculated the trigonometric function logarithm of two adjacent angle that was positive integer multiples of 5, then got the difference;
- (2) divided the difference into five equal parts;
- (3) based on the trigonometric function logarithm of the smaller angle that was positive integer multiples of 5, added the number just calculated in last step successively---if there was decimal, the digit after decimal point should be leaved or into 1 appropriately, then the four trigonometric function logarithm value in the middle can be got.

Take the calculation of the sine function logarithm of $0^{\circ}11'$, $0^{\circ}12'$, $0^{\circ}13'$, $0^{\circ}14'$ for example:

- (1) first, calculate the difference of sine function logarithm of $0^{\circ}10'$ and $0^{\circ}15'$: $7417968-7241877 = 176091$;
- (2) divide the above difference into five equal parts: $176091 \div 5 = 35218.2$
- (3) based on the sine function logarithm of $0^{\circ}10'$, add the number calculated in last step successively, then the required four logarithm can be got.

$$7241877+35218.2 \approx 7277095 \quad 7277095+35218.2 \approx 7312313$$

$$7312313+35218.2 \approx 7347531 \quad 7347531+35218.2 \approx 7382749$$

Again for instance, calculate the sine trigonometric function logarithm of $0^{\circ}16'$, $0^{\circ}17'$, $0^{\circ}18'$, $0^{\circ}19'$:

- (1) first, calculate the difference of sine function logarithm of $0^{\circ}15'$ and $0^{\circ}20'$: $7542906-7417968=124938$
- (2) divide the above difference into five equal parts: $124938 \div 5=24987.6$
- (3) based on the sine function logarithm of $0^{\circ}15'$, add the number calculated in last step successively, then the required four logarithm can be got.

$$7417968+24987.6 \approx 7442955 \quad 7442955+24987.6 \approx 7467943$$

$$7467943+24987.6 \approx 7492930 \quad 7492930+24987.6 \approx 7517918$$

After verification, the author found that all the trigonometric function logarithm can be calculated by this method, and the

results were almost as same as the data in Xue Fengzuo' *Bi Li Si Xian Xin Biao*.

Thus, here Xue Fengzuo should evaluate trigonometric function logarithm by the method that calculate the numbers in the middle of arithmetic progression based on the first and the last number.

This method was obviously easy to operate and extremely fast to calculate, but it was also obviously incorrect. This question was very easy to proof. Xue Fengzuo regarded triangle function logarithm of consecutive angle as the arithmetic progression, and calculated the values by using the properties of arithmetic progression. Then, if Xue Fengzuo' method was right, we can deduce that the triangle function value of continuous angle should be geometric progression. For example, $0^{\circ}11'$, $0^{\circ}12'$, $0^{\circ}13'$, $0^{\circ}14'$ and $0^{\circ}15'$ are five continuous angles, their sine function value should comply with the following relations: $\sin 0^{\circ}11' : \sin 0^{\circ}12' = \sin 0^{\circ}12' : \sin 0^{\circ}13'$, $\sin 0^{\circ}12' : \sin 0^{\circ}13' = \sin 0^{\circ}14' : \sin 0^{\circ}15'$. That is obviously not right. Therefore, the Xue Fengzuo' method that used to calculate the trigonometric function logarithm of the angles that was not positive integer multiples of 5 was not correct.

Thus, the author considers that Xue Fengzuo may more likely to take the following ways to give the data in *Bi Li Si Xian Xin Biao*: firstly, reference the trigonometric function logarithm of the angles that was positive integer multiples of 5 in other scholar' *Bi Li Si Xian Biao*. It should be noted that the angles that was positive integer multiples of 5 in Xue Fengzuo' book were positive integer multiples of 3 in sexagesimal, and the trigonometric function logarithm table had already been introduced into our country by western missionaries at that time, so the first step is very easy to implement. What's more, among the incoming table, some trigonometric function logarithms are precise to ten decimal places (Wu, W.J., 2000) [9]. Secondly, use the method that calculate the intermediate numbers in arithmetic progression to evaluate trigonometric function logarithms of four angles between two successive angles that was positive integer multiples of 5. Pay attention to rounding decimals in this process.

4. Conclusions

Bi Li Si Xian Xin Biao is an important book for Xue Fengzuo to complete *Li Xue Hui Tong*. It is popular with people for its calculation and instruction of trigonometric function logarithm. In *Zhong Fa Si Xian Yin*, Xue Fengzuo just broadly gave a general statement about the writing process of the book, he did not explain the specific description of the contents of book. Through in-depth analysis of the data in this book, it can be found that about 80% of the data are of relatively large deviation, and some even can be said completely wrong. Xue Fengzuo neither made use of reasonable formulas nor utilized the method for calculating function value introduced in *Sine*, but used an unreasonable way instead to calculate intermediate numbers in arithmetic progression. Why did Xue Fengzuo take in such a way? It is either because this method is much more easier, or he misunderstands the nature of the trigonometric function and mistakenly treats trigonometric function value of continuous angle as a geometric sequence, and the latter is more likely to be the reason. Thus, when Xue Fengzuo compiled the *Bi Li Si Xian Xin Biao*, he probably did not use the *Bi Li Shu Biao* (logarithmic tables) written by Smogolenski and himself. Neither calculated with logarithm nor referred to the previous sine calculation method, he just did some simple elementary arithmetic of addition, subtraction, multiplication, and division based on some existing data. Xue Fengzuo didn't do a overall consideration linked with the previous work, this is really a pity in his research work.

References

1. Yuan ZT. Xue Fengzuo. Famous Calendarian in Qing Dynasty. History Teaching University Edition, 1991, (06).
2. Ruan Y. Biography of the Academic Activities of Chinese Scholars in Past Dynasties. Guangling Publishing House, 2009, 405.
3. Mei WD. Calendar Records of Wu An. The Commercial Press, 1935, 29.
4. Zhao EX. The Draft of History of Qing Dynasty. Shanghai Classics Publishing House. 986:1596-1607.
5. Liu XX. Radical Changes of the Big Dipper and Stars' Positions Resulted from Errors of Root Number. Pioneers of Chinese and Western culture. Qilu Press. 2011, 311-328.
6. Yang ZZ. The Study on Xue Fengzuo's book *Sine*. Shandong Social Science. 2011, (06).
7. John NT. Rigonometria Britanica. R. & W. Leybourn. 1658, 298-415.
8. Li Y, Qian BC. Collected Works in History of Science by Li Yan, Qian Baocong. Liaoning Education Press, 1998; 7:240.
9. Wu WJ. History of Mathematics in China. Beijing Normal University Press. 2000; 7:102.