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Almost periodic points and minimal sets in topological spaces

Dr. Satya Prakash Gupta

Abstract
This Paper we characterize in this paper, we need the following terminology and concepts. Let N= $\{0,1,2,\ldots,\}$ be the set of natural numbers and let Z = $\{0,\pm 2\ldots,\}$ be the set of integers. For a set A, |A| denotes the cardinality of the set A. If f :X → X is map (=continuous function) of a topological space X, then f0=Id $(n \geq 1)$ denotes the composition with itself n times. The orbit of a point x∈ X under f, denoted by 0+(x, f), is the set {fn(x)|n∈ N}. Also if f: X → X is a homeomorphism, then we put f-n=(f-1)n(n ≥ 1), where f-1 is the inverse of f. The two-sided orbit of a point x∈ X under f, denoted by O + (x, f), is the set {fn(x)|n∈ Z}. A point x∈ X is called a periodic point of f if there exists a positive number N ∈ N such that fN(x)=x. A point x∈ X is called an almost periodic point of f provided that for any neighborhood U of x in X, there exists N ∈ N such that {fn + i(x)|i=0,1,2,.........N} ∩ U ≠ for all N ∈ N. We denote the set of all almost periodic points of f by AP(f). A subject W of X is invariant of f if W ≠ and f(W) ⊆ W. A subset W of X is a minimal set of f if W is a closed invariant set off and W does not contain any proper closed invariant set off. A map f: X → X is minimal if X is a minimal set off. It is well known that if f: S1 → S1 is an irrational rotation of the unit circle SI, then f is minimal.

Keywords: Periodic points, minimal, topological

Introduction

A topological space X is a T1-space if for any distinct point x and y in X, there exist open sets U and V of X such that x∈ X, y∈ V, y U and xV.A topological space X is a Housdorif space if for any distinct points x and y in X, there exist disjoint open sets U and V of X such that x∈ X U and y∈ V. A topological space X is regular space if for any closed subset W of X, any point x∈ X − W, there exist disjoint open sets U and V such that x∈ U and W ⊆ V. A topological space x is an !-regular space if for any closed subset W of X, any point x∈ X − W and any countable subset A of W, exist disjoint open sets U and V such that x∈ U and A ⊆ V [1, 2, 3].
Now we assert and prove the main theorem on compact minimal set in topological dynamics.

Theorem:-

Let E be a compact minimal set in X. The set E is almost periodic minimal if and only if the flow π is equicontinuous on E.

Proof:-

Let us assume E be almost periodic minimal set and let E=H(x), where x is and almost periodic point. We claim that the flow π is equicontinuous on E.
We first show that π is equicontinuous on γ(x). Let an index $a \in$ A be preassigned and select b ∈ A, so by the uniformity condition if follows that $X_i \in V_b(x_{i+1}), i = 1,2,3.$

$$\Rightarrow x_i \in V_a(x_4) \ldots \ldots \ldots \ldots \ldots \ldots (1)$$

Let L.0be an inclusion interval for the relatively dense set E $(b, \pi x)$.
Since the continuity of π restricted to the compact set Ex [0,L] is uniform, for every b ∈ A there exists and index c such that for $y, z \in, y \in V_c(z)$ implies that,

$$\pi(y,\mathrm{t}) \in \mathrm{V_b}\big(\pi(\mathrm{z},\mathrm{t})\big) \text{ for all t } \in \mathrm{R} \ldots \ldots \ldots .. (2)$$

In order to show that π is equicontinuus on $\gamma(x)$ for any tow points $\pi(x,t_1)$ and $\pi\pi(x,t_2)$ of $y(x), \pi(x,t_1+t) \in V_a(\pi(x,t_{2+}))$ whenever $\pi(x,t_1) \in V_c(\pi(x,t_2))$ for all $t \in R$.
Let t ∈ R be fixed. Then there exists a T in E(b,πx) such that $0 \leq \tau + t \leq L$. From the definition of b-displacement of πx, it follows that, $\pi(x,\tau+t+t_1) \in V_b\big(\pi(x,t_1+t)\big)$and $\pi(x,\tau+t_1+t) \in V_b(\pi(x,t_2+t))$
Applying the condition (2) of uniform continuity one has $\pi(x,t_1+t) \in V_b(\pi(x,t_2+t)$
Hence finally, by uniformity condition (1) we have $\pi(x,t_1+t) \in V_a(\pi(x,t_2+t))$ i.e., the equicontinuity of π on $\gamma(x)$ follows.
In order to show that π is equicontinuos on E, let a ∈ A be any index. Then there b ∈ A in consistent with the uniformity condition (i). By using the equicontinuity of π on $y(x)$ we can choose C∈A such that $\pi(x,t_1+t)) \in V_b(\pi(x,t_2 t))$ whenever $\pi(x,t_1) \in V_c(\pi(x,t_2))$
Now let us choose d so that $x_i \in V_d(x_i+1), i = 1,2,3$

$$\Rightarrow x_i \in V_c(x_4) \ldots\ldots\ldots\ldots\ldots..(3)$$

We will now show that if y, z are points in E=H (x) with $y \in V_d(z)$, then $\pi^t(y) \in V_a\big(\pi^t(z)\big), i.e., \pi(y,t) \in V_a\big(\pi(z,t)\big)$.
Since $y,z \in E = H(x)$, there are nets $\{\pi(x,t_n)\}\, and\, \pi(x,S_n)\}$ in $y(x)$ with $\pi(x,t_n) \to y\, and\, \pi(x,s_n) \to z$ without the loss of generality, $\pi(x,t_n) \in V_d(y) and\, \pi(x,s_n) \in V_d(z)$.
Since $y \in V_d(z) \Rightarrow \pi(x,s_n+t) \in V_b(\pi(x,t_n))$ for all n all $t \in R$. Thus we have two convergent nets $\{\pi(x,S_n+t)\}\, and\, \{\pi(x,t_n+t)\}$ in the compact hull H(x)=E, so they are uniformly convergent.
Now for the index b ∈ A, there exists mo such that $\pi(x,S_n+t) \in V_b(z,t) and\, \pi(x,t_n+t) \in V_b(y,t)$ for all $n \geq m_0$ and $t \in R$. Finally, for the uniformity condition (1) where $x_1 = \pi(y,t) x_2 = \pi(x,t_n+t), x_3 = \pi(x,S_n+t)$ and $x_4 = \pi$ (z,t), it follows that $\pi(y,t) \in V_a(\pi(z,t)$.
Conversely, let us assume that the flow π is equicontinuous on E. We show that E is almost periodic minimal. Since E is compact minimal, then by Birkhoff recurrence theorem every motion in E is recurrent. Therefore, for any $x \in E$ and any index by the set $E(b,\pi x) = \{\tau: \pi(x,\tau) \in V_b(x)\}$ is relatively dense in R. Let the index a be preassigned, then the resists $b \in A$ for the equicontinuity of π. Thus we have. $E(b, \pi(x,t) \in V_b(x)\}c$.
$\{t: \pi(x,t+\tau) \in V_b\big(\pi(x,t)\big), \text{for all } t \in R\}$ Assuring the almost periodicity of πx. Therefore, E is almost periodic minimal.

References

1. Birkhoff GD. Dynamical Systems, revised editions, Amer. Math. Soc. Colloq. Publ., Amer Math. Soc., Providence, RI 1966, IX.
2. Gottschalk WH. Orbit-closure decompositions and almost periodic properties, Bull. Amer. Math. Soc 1944;50:915-919.
3. Gottschalk WH. Almost periodic points with respect to transformation semi-groups, Ann. Math 1946;47:762-766.