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A scientific model of finite element technique to study spiral heat regulation in human limbs

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Abstract
We consider this paper as the one-dimensional bio-heat exchange condition with the perfusion of blood, reliant on temperature which oversees physiological heat dispersion issues relating to limbs and the organic properties are supposed to be distinctive along the radial direction. By methods for an extensive technique with metaphorical estimation, we acquire the correct analytical answer for inspect the heat circulation in the tissues. The dermal region is comprised of three layers, to be specific epidermis, dermis, and subcutaneous tissues. The model consolidates critical varieties of physical and physiological parameters like blood mass stream rate, rate of metabolic heat generation, and thermal conductivity in each layer. Numerical outcomes have been acquired for different instances of functional interest.

Keywords: Rate of metabolism, blood mass stream rate, warm conductivity, warm era, limited component method, Pennes Bio - Heat Model

Introduction

The heat transmit methodology through human dermal locales is an extremely unpredictable phenomena because of various reasons. At lower environmental temperatures, the one dimensional models for heat distribution in limbs have wide degree as the center temperature of the human appendages fluctuates broadly at lower air temperatures. For falling climatic temperature, the body center therapists quickly and isotherm shells in the appendage change their separate position progressively. This might be because of the way that the blood vessel blood has chilled off while streaming towards the furthest points. Likewise the two inverse side of internal center of a human appendage might be at various temperature. This might be on the grounds that one side of the appendage contains real veins (corridors) with blood originating from fundamental trunk at body center temperature and the external side of the appendage contains veins with blood coming back from the furthest points at lower temperature.

Perl [12] took collectively the forms of differential equations and Fick's perfusion principle with heat conduction and matter diffusion equations and metabolic term to get equation. He used equation to solve its simple cases by taking all parameters as constant throughout the region. Perl and Hirsch [7] used this equation to test the transient response for measuring local tissue blood flow on dog and rabbit kidney. Trezek and Cooper [1] computed thermal conductivity of tissue by taking all parameters as constant. Cooper and Trezek [2] obtain solution of equation in SST region by taking all parameters as constant. Patterson [6] made experimental attempts, to determine temperature profiles in skin and subcutaneous region.

This paper employs a variational finite element approach to study the temperature distribution in a normal cross-sectional region of a limb. Due to unsymmetric situations of large blood vessels passing through the core of the limb the inter-face has angular variation. The peripheral part of limb is directly exposed to atmosphere. Different types of variations of parameters have been considered for different natural subregions such as stratum corneum, stratum germinativum, dermis and underlying tissue (Montagana, Jarrett and Gray) [5, 4, 3]. Finite element formulation provides necessary flexibility in taking care of different behavior of distinctly different subregions.

2. Material and Methods

The heat flow in SST region is given by the following partial differential equation

$$div\ (K\ grad\ T) + m_b\ C_b\ (T_b - T) + S = \rho \bar{c} \frac{\partial T}{\partial t} \quad (2.1)$$

Where T and Tb are Temperature and Body center temperature, S is Rate of metabolic warmth generation, mbis Blood mass stream rate in the tissue, is thickness of the tissue. Cb=Specific warmth of blood, K is warm conductivity of the tissue. Above condition has been adjusted and extensively used by Saxena [11], Saxena and Arya [8], Saxena and Bindra [9, 10] in the warm investigation of human skin and subcutaneous tissue.

In this, skin and fundamental tissues of round and hollow locales, for example, appendages of a human are partitioned into four annular layers. They can be considered as multi layered districts. Each layer has diverse physical and physiological properties. The external body surface is presented to the earth and warmth misfortune from the body surface happens because of conduction, convection, radiation and vanishing. Here we utilize for a human appendage with round symmetry. The properties and temperature conveyance are thought to be uniform along and Z headings. In this manner the condition diminishes in one dimensional precarious state case for each layer to the accompanying round and hollow from.

$$\frac{1d}{r\,dr}\left(k^{(i)} r \frac{dT_j}{dr}\right) + M_i\ (T_b - T_i) + S_i = \rho \overline{C} \frac{\partial T}{\partial t}, \qquad i = 1\ to\ 4$$

Where $K^{(i)}$,M_i,S_i and T_i denote the values of K, M, S and T in i the sub-region.

Boundary and Interface Conditions:

Inview of continuity of temperature and temperature gradient in various sublayers, the following boundary and interface conditions can be formulated

(i) $T^{(1)}=T^{(2)}$ at $r= a_1$

(ii) $K_1 \frac{dT^{(1)}}{dr} = K_2 \frac{dT^{(2)}}{dr}$ at $r = a_1$

(iii) $T^{(2)} = T^{(3)}$ at $r= a_2$

(iv) $K_2 \frac{dT^{(2)}}{dr} = K_3 \frac{dT^{(3)}}{dr}$ at $r = a_2$

(v) $T^{(3)}=T^{(4)}$ at $r = a_3$

(vi) $K_3 \frac{dT^{(3)}}{dr} = K_4 \frac{dT^{(4)}}{dr}$ at $r = a_3$

It is assumed that at the outermost layer ($r = a_0$) the heat is lost to the environment by conduction, convection, radiation and evaporation. Therefore at this layer we take

$$K_1 \frac{dT}{dr} = -h\ (T - T_A) + LE$$

Where h is coefficient of convection,L is dormant warmth of dissipation and E is the Rate of sweat vanishing TA is the barometrical temperature.At the deepest layer the temperature will be same as that of the body center. Thus the limit conditions will be$T4 =Tb$ at$r= a4$

3. Solution of the Problem

The variational form is defined in the region as I1, I2, I3 and I4 respectively for stratumcorneum, stratum germinativum, dermis and subdermal parts. Assigning the values to T as Ti (i = 0, 1, 2, 3, 4) called as nodal values. The distance between the nodal values is given by ai (I = 0, 1, 2, 3, 4) from the outermost layer of epidermis to innermost layer of subdermallayer. Let$T^{(i)}$ (*i* = 1, 2, 3, 4) denote the linear values of T (r) for $a_i < r < a_{i-1}$

Now, applying the shape function to approximate the solution of the problem.

$$T^{(i)} \approx A_i + B_i\, r \quad \text{for} \quad a_i < r < a_{i-1}$$

Where,

$$A_i = \frac{T_i - T_{i-1}}{a_i - a_{i-1}}, \qquad B_i = \frac{a_{i-1}\ T_i - a_i T_{i-1}}{a_{i-1} - a_i} \quad for \quad i = 2, 3, 4$$

Thus the equation in one dimensional unsteady state case for each layer to the following cylindrical form

$$\frac{1}{r}\frac{d}{dr}\left(Kr\frac{dT}{dr}\right) + M\ (T_b - T) + S = \rho \overline{C} \frac{\partial T}{\partial t},$$

After camparing with Euler's Lagrange's equation, we get the varitional forms

$$I = \frac{1}{2}\int_{a_i}^{a_{i-1}} \left[K_i r \left(\frac{\partial T^i}{\partial t}\right)^2 + r m_b C_b\ (T^i - T_b)^2 - 2s^{(i)}\, rT^{(i)} + \rho \bar{c} r \frac{\partial T(i)^2}{\partial T} \right] dr + \frac{1}{2} h\ (T^{(c)}{}_{(1)} - T_a)^2 + LET_{(i)}$$

We have,

$I=I_1 + I_2 + I_3 + I_4$

Clearly,

$I = f(T^0, T^1, T^2, T^3, T^4)$

Now minimizing I with respect to parameters T^0, T^1, T^2, T^3 ,therefore

$$\frac{\partial I}{\partial T_0} + \frac{\partial I}{\partial T_1} + \frac{\partial I}{\partial T_2} + \frac{\partial I}{\partial T_3} = 0$$

Finally taking La-place transform, we get four non homogenous simultaneous equations as

$x^1 T^0 + y^1 T^1 = n^1$

$x^2\ T^0 + y^2\ T^1 + z^2\ T^2 = n^2 = n^2$

$y^3 T^1 + z^3 T^2 + w^3 T^3 = n^3$

$z^4\ T^2 + w^4\ T^4 = n^4$

solving for $\overline{T_0}, \overline{T_1}, \overline{T_2}, \overline{T_3}$ by using matrix method, we get the values of $\overline{T_0}, \overline{T_1}, \overline{T_2}, \overline{T_3}$ in the form of polynomials

$$\begin{bmatrix} x_1 & y_1 & 0 & 0 & \eta_1 \\ x_2 & y_2 & z_2 & 0 & \eta_2 \\ 0 & y_3 & z_3 & \omega_3 & \eta_3 \\ 0 & 0 & z_4 & \omega_4 & \eta_4 \end{bmatrix}$$

$$\overline{T_1} = \frac{X_i(p)}{Y_1(p)}$$

Where x^i (p) is a polynomial of degree less than n that of Y ^{j} (P)i.e.,(n-1)

The value of nodal temperature $T^0 T^1 T^2 T^3$,can be obtained by taking the inverse La-place transform

$$T^1 = \sum_{n=1}^{N} \frac{X_1(p_N)}{y_1(p_N)} \mathbf{e}^{pnt}$$

Therefore, we can see that nodal values are dependent on time.

4. Tables

The numerical result have been obtained with the help of following values –

Thermal Conductivity (cal/cm min ºC)	Heat Transfer Coefficient h (cal/cm² min ºC)	Specific Heat of Tissues c (cal/gmºC)
K_1=0.060, K_2=0.045, K_3=0.030	0.009	0.830
Blood Density of Tissuesρ (gm/cm³)	Latent HeatL (cal/gm)	Body Core Temperature T_b (ºC)
1.090	579.0	37

The numerical result have been computed for three case of atmospheric temperatures T = 15 °C, 23°C and 33°C. The following sets of numerical values have been taken and graphical representation obtained for temperature distribution in SST region. The three different set of values for thickness of layers in SST region are taken for a_0, a_1, a_2, a_3 and a_4.

Thickness of Skin	a_0	a_1	a_2	a_3	a_4
Set I	8	7.5	7	6	5.5
Set II	7.5	7	6	5	4.5
Set III	7	6	6.5	5	5.5

5. Conclusion

Distinctive T(i) (i = 0, 1, 2, 3) versus time t for various thickness of skin and diverse estimations of climatic temperatures have been ascertained On looking at the nodal temperature on external surface and in each sub district, it is watched that these nodal temperatures differ extensively with the change in environmental temperatures and rate of sweat dissipation.

Clearly sharpness of slope is more set apart in the epidermal sub-layer which is specifically presented to condition. In addition blood compliment in the dermal and sub dermal sub layers.

6. Reference

1. Cooper TE, Trezek GJ. A Probe technique for determining the thermal conductivity of tissues, J Heat transfe. 1972a, 133-140. ASME 94.
2. Cooper JE, Trezek GT. on the freezing of tissues, J Heat transfer.1972b, 251-253. ASME, 94.
3. GRAY H. Text Book of Gray's Anatomy, Longmans Press, 1973.
4. JARRETr A. The Physiology and Pathophysiology of Skin, Academic Press, 1973.
5. MONTAGNA W. The Structure and Function of Skin, Academic Press, 1962.
6. Patterson AM. Measurement of temperature profiles in human Skin, S. Afr J Sci. 1976; 72:78-79.
7. Perl W, Hirsch RL. Local blood flow in kidney tissue by heat clearance method. J Theoret Biol. 1966; 10(2)251-280.
8. SAXENA VP, ARYA D. Exact Solution of the Temperature Distribution Problem in Epidermis and Dermis Regions ofHuman Body, Proc. VNM, Medical and Biological Engineering, Sweden, 1981, 364-366.
9. SAXENA VP, BINDRA JS. Steady State Temperature Distribution in Dermal Regions of Human Body with Variable Blood Flow, Perspiration and Self-Controlled Metabolic Heat Generation, J Pure Appl Math. 1984; 15(1):31-42.
10. SAXENA VP, BINDRA JS. Quadratic Shape Functions in Variational Finite Element Approach to Heat Distribution in Cutaneous and Subcutaneous Tissues, Ind. J. pure Appl Math L. 1987; 8(9):846-855.
11. SAXENA VP. Temperature Distribution in Human Skin and Subdermal Tissues, 1, Theo. Biol, 1983; 102:277-286.
12. Perl W. Heat and matter distribution in body tissue and determination of tissue blood flow by local clearance methods. Journal of theoretical biology. 1962; 2(3):201-235.