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MS Mahesh
Department of Mathematics, Franics Xavier Engineering College, Tirunelveli Tamil Nadu, India

P Namasivayam
PG and Research Department of Mathematics, the M.D.T. Hindu College, Tirunelveli, Tamil Nadu, India

Correspondence
MS Mahesh
Department of Mathematics, Franics Xavier Engineering College, Tirunelveli Tamil Nadu, India

Bounds of neighborhood connected two out degree equitable domination number

MS Mahesh and P Namasivayam

Abstract
A subset D of V(G) is said to be a neighbourhood connected two out degree equitable dominating set if D is two out degree equitable dominating set and the induced sub graph <N(D)> is connected. The minimum cardinally taken over all neighbourhood connected two out degree equitable dominating set is called neighbourhood connected two out degree equitable domination number and denoted by $\gamma_{nc2oe}(G)$. In this paper bounds neighbourhood connected two out degree equitable domination number is determined

Key words: dominating set, equitable, neighbourhood connected, two out degree
Mathematical Subject Classification: 05C69

1. Introduction

The graphs considered her are nontrivial, connected, simple finite and undirected. For a graph G=(V,E), V denoted it vertex set and E its edge set. The number of vertices and edges are denoted by p and q respectively. The open neighborhood of v is denoted by N (u) and defined as N(u)= {v/uvϵ E} [4].

The concept of domination was first studied by Ore [10] and C.Berge [3]. A set D $\subseteq V$ is said to be a dominating set of G if every vertex in V−D is adjacent to some vertex in D. The cardinality of a minimum dominating set D is called the domination number of f and is denoted by$\gamma(G)$. Anitha. A, Arumugam. S and Chellai. M [2] introduce the out degree as The out degree of v with respect to D is denoted by $od_D(u) = |\, N(v) \cap V - D|$. Ali Sahal and V. Mathad [1] introduce the concept of two out degree equitable domination in graphs. A dominating D of G is said to two out degree equitable dominating set if for any two vertices u,vϵ D such that $|od_D(u) - od_D(v)| \leq 2$. The minimum cardinality of two out degree equitable dominating set is called two out degree equitable domination number and it is donated by $\gamma_{2oe}(G)$. M.S. Mahesh and P.Namasivayam [9] introduce connected two out degree equitable domination number. A subset D of is said to be a connected two out degree equitable dominating set if D is two out degree equitable dominating set and the induced sub graph <D> is connected. The minimum cardinally taken over all connected two out degree equitable dominating set is called connected two out degree equitable domination number and denoted by $\gamma_{c2oe}(G)$ and publish some papers in [6, 7, 8]. Then M.S. Mahesh and P. Namasivayam [5] introduce neighbourhood connected two out degree equitable domination number $\gamma_{nc2oe}(G)$. In this paper the bounds of neighbourhood connected two out degree equitable domination number is discussed.

2. Neighborhood Connected Two Out Degree Equitable Domination Number

Definition 2.1

A dominating set D of a graph G is called the neighborhood connected two out degree equitable dominating set if for any two vertices u,v $\in D$ such that $|od_D(u) - od_D(v)| \leq 2$ the and the induced sub graph <N(D)> is connected. The minimum cardinality of a neighborhood connected two out degree equitable dominating set is called neighborhood connected two out degree equitable number of G and is denoted by $\gamma_{nc2oe}(G)$

Example: 2.2

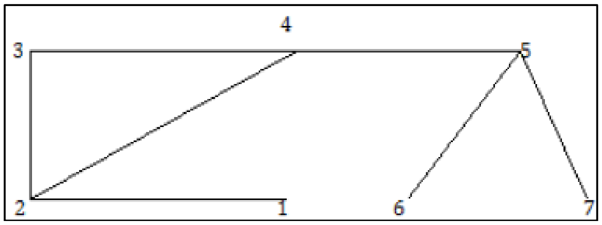

Fig 1

Let us consider a set D = {1, 3, 6, 7} and $V-D$= {2, 4, 5}
$od_D(1) = |N(1) \cap \{2,4,5\}| = 1$
$od_D(3) = |N(3) \cap \{2,4,5\}| = 2$
$od_D(6) = |N(6) \cap \{2,4,5\}| = 1$
$od_D(7) = |N(7) \cap \{2,4,5\}| = 1$
Then Clearly for any u,vϵ {1,3,67} such that $|od_D(u) - od_D(v)| \leq 2$.
D= {1, 3, 6, 7} is two out degree equitable dominating set and <N(D)> = {2,4,5} connected set then D={1,3,6,7} is neighborhood connected two out degree equitable dominating set.

3 Bounds of Neighborhood Connected Two Out Degree Equitable Domination Number
Observation 3.1
For any graph G with p vertices, 2≤ $\gamma_{nc2oe}(G) \leq p-2$

Theorem: 3.2
For any graph G: $\gamma(G) \leq \gamma_{2oe}(G) \leq \gamma_{nc2oe}(G)$

Proof:
For any graph G, any neighborhood connected two out degree equitable dominating set is a two out degree equitable dominating set and every two out degree equitable dominating set is a dominating set.

Theorem: 3.3
For any graph G: $\gamma(G) \leq \gamma_{nc}(G) \leq \gamma_{nc2oe}(G)$

Proof
For any graph G, any neighborhood connected two out degree equitable dominating set is a neighborhood connected equitable dominating set and every neighborhood connected equitable dominating set is neighborhood connected dominating set.

Theorem 3.4
Let G be a connected graph of order p, p≥ 3. Then $\gamma_{nc2oe}(G) \leq 2q-p$

Proof
Clearly by the definition of the neighborhood connected two out degree equitable dominating set we have $\gamma_{nc2oe}(G) \leq p-2$=2(p-1)-p $= 2q-p$
$\gamma_{nc2oe}(G) \leq 2q-p$

Theorem: 3.5
Let G be a graph with even order p and without isolated vertices. Then G is isomorphic to corona graph $C_p \circ K_1$ $\gamma_{nc2oe}(G) = \frac{p}{2}$

Proof:
Suppose G ≅ $C_p \circ K_1$
Let $\{v_1, v_2, v_3 - - - - - - - v_p\}$ be the vertices of C_p. Corona graph formed by adding an K_1 to each p vertices of C_p,
Let $\{v_{p1}, v_{p2}, v_{p3} - - - - - - - v_{pp}\}$ be the vertices added to each vertices of cycle C_p
Now consider D= $\{v_{p1}, v_{p2}, v_{p3} - - - - - - - v_{pp}\}$ is adjacent to remaining $\{v_1, v_2, v_3 - - - - - - - v_p\}$ and each vertices of D is adjacent to exactly one vertices in $V-D$
So clearly D is minimal dominating set and <N (D)>=$< C_p >$ is connected
Clearly D is nieghborhood two out degree equitable dominating set
Then $\gamma_{c2oe}(G) = \frac{p}{2}$

Theorem: 3.6
For any graph G $\gamma_{nc2oe}(G) \leq p-2$, the equality holds if and only if G isomorphic to $k_{1,p}$

Proof:
By theorem 3.2.1$\gamma_{nc2oe}(G) \leq p-2$
To prove the equally let us assume $\gamma_{nc2oe}(G) = p-2$
Suppose G is not isomorphic to $k_{1,p}$
Then there exists a bipartite graph with bipartition (X,Y) such that |X|=1 and |Y|=p and the vertex in X is adjacent some $p-k$ vertices k vertices are isolated.
G is divided to two sub graphs $k_{1,p-k}$ and k isolated vertices
G= $k_{1,p-k}$ ∪{k}
$\gamma_{nc2oe}(G) = \gamma_{nc2oe}(k_{1,p-k}) + k$
$\gamma_{nc2oe}(G) \leq p-k-2+k$
$\gamma_{nc2oe}(G) \leq p-2$
It is a contradiction
Hence G isomorphic to $k_{1,p}$
Suppose G isomorphic to $k_{1,p}$
By theorem 3.2.2, $\gamma_{nc2oe}(G) = p-2$

Theorem: 3.7
Let G be a graph with p ≥ 4 then $\gamma_{nc2oe}(G) \geq p-q+2$ and this bound is sharp

Proof:
Let D be a neighborhood connected two out degree equitable dominating set of G. so D is two out degree equitable dominating set and <N(D)> is connected
Then each vertex of $V-D$ is adjacent to at least one vertices in D.
Since <N(D)> is connected either $V-D$ or D contains at least two dominating edges.
Hence the number of edges $q \geq |V-D| + 2 = p - \gamma_{nc2oe} + 2$
$q \geq p - \gamma_{nc2oe} + 2$
Then $\gamma_{nc2oe}(G) \geq p-q+2$
The bound is sharp for $k_{1,p}$

Theorem: 3.8
For any graph G, $\gamma_{nc2oe}(G) \geq \frac{p}{(\Delta+1)}$
Proof:
Let D be a minimum neighborhood connected two out degree equitable dominating set and let k be the number of edges between D and $V-D$.
Since the degree of each vertex in D is almost Δ, $k \leq \Delta\gamma_{nc2oe}$

But since each vertex in $V-D$ is adjacent to at least one vertex in D, $k \geq p-\gamma_{nc2oe}$ combining these two inequalities produce

$p-\gamma_{nc2oe} \leq k \leq \Delta\gamma_{nc2oe}$

$p-\gamma_{nc2oe} \leq \Delta\gamma_{nc2oe}$

$\gamma_{nc2oe}(G) \geq \frac{p}{(\Delta+1)}$

Theorem 3.9

If T is non-trivial tree, then $\gamma_{nc2oe}(T) < p-2$ such that T is not a star.

Proof

Since T is non-trivial tree this implies that V –{u,v} is a neighborhood connected two out degree equitable dominating set of T. $\gamma_{nc2oe}(T) < p-2$

Definition: 3.10

A colouring of a graph G is an assignment of colour to the vertices of G such that no two adjacent vertices receive the same colour. The minimum number of colours required for colouring a graph G is called chromatic number and is donated by$\chi(G)$. In this section, the relation between neighborhood connected two out degree equitable domination number and chromatic number is studied..

Theorem: 3.11

For any graph G, $\gamma_{nc2oe}(G)+\chi(G) \leq 2p$ and equality holds if and only if G is isomorphic to K_2

Proof

We know that $\gamma_{nc2oe}(G) \leq p-2 \leq p$ and $\chi(G) \leq p$

Then $\gamma_{nc2oe}(G)+\chi(G) \leq p+p=2p$

Suppose $\gamma_{nc2oe}(G)+\chi(G)=2p$

This implies $\gamma_{nc2oe}(G)=p$ and $\psi(G)=p$

If $\psi(G)=p$ hence G is isomorphic to k_p

If $\gamma_{nc2oe}(G)=p=2$ hence G is isomorphic to k_2

Conversely

Suppose G is isomorphic to k_2

Then $\gamma_{nc2oe}(G)=2$ and $\chi(G)=2$

$\gamma_{nc2oe}(G)+\chi(G)=4=2p$

Theorem: 3.12

For any graph G, $\gamma_{nc2oe}(G)+\chi(G)=2p-1$ if and only if G is isomorphic to K_3.

Proof

Let us assume $\gamma_{nc2oe}(G)+\chi(G)=2p-1$

This is possible only if $\gamma_{nc2oe}(G)=p-1$ and $\psi(G)=p$

If $\gamma_{nc2oe}(G)=p-1$ then G is isomorphic to k_3

If $\psi(G)=p$ G is isomorphic to K_p.

There fore G is isomorphic to k_3

Conversely

Suppose G is isomorphic to K_3.

We know that $\gamma_{nc2oe}(G)=2=p-1$ and $\chi(G)=p$

Then $\gamma_{nc2oe}(G)+\chi(G)=p-1+3=p+2=2p-3$

Theorem: 3.13

For any graph G, $\gamma_{nc2oe}(G)+\chi(G)=2p-2$ if and only if G is isomorphic to K_4or P_3

Proof

Let us assume $\gamma_{nc2oe}(G)+\chi(G)=2p-2$

This is possible if $\gamma_{nc2oe}(G)=p$ and $\chi(G)=p-2$ or $\gamma_{nc2oe}(G)=p-1$ and $\chi(G)=p-1$ or $\gamma_{nc2oe}(G)=p-2$ and $\chi(G)=p$

Suppose $\gamma_{nc2oe}(G)=p$ and$\chi(G)=p-2$, since $\gamma_{nc2oe}(G)=p$ which gives G is isomorphic to K_2 and hence $\chi(G)=2 \neq p-2$ so the condition (i) not holds.

Suppose $\gamma_{nc2oe}(G)=p-1$and $\chi(G)=p-1$,

since $\chi(G)=p-1$,G contains a complete sub graph K on $p-1$ vertices

Let V(K)=$\{v_1, v_2, v_3 ------v_{p-1}\}$ and V(G) – $V(K)$=$\{v_p\}$

Then v_p is adjacent to v_i for some $v_i \epsilon V(K)$ and $|od_D(u)-od_D(u)| \leq 2$

If deg $(v_p)=1$ and p≥ 4 then $\{v_i, v_j, v_k\}$ $i \neq j$ is a neighborhood connected two out degree equitable dominating set

Hence $p=4$ and $G=k_4$

Thus G is isomorphic to $k_3(1,0,0)$

If deg$(v_p)=1, p=3$

Thus G is isomorphic p_3

If deg (v_j)>1 then $\gamma_{nc2oe}(G)=2$ $then$ p=3, which gives G is isomorphic to k_3, but it is not possible because $\chi(G)=p-1$

Suppose $\gamma_{nc2oe}(G)=p-2$ and $\psi(G)=p$

Since $\chi(G)=p$, isomorphic to K_p. but $\gamma_{nc2oe}(K_p)$=p-2. Therefore p=4

Then G is isomorphic to K_4

Conversely

Suppose G is isomorphic to K_4.

We know that $\gamma_{nc2oe}(G)=2$ and $\psi(G)=4$

Then $\gamma_{nc2oe}(G)+\chi(G)=2+4=6=2(4)-2$

$=2p-2$

Theorem 3.14

Let G be any graph order p, Then $\gamma_{nc2oe}(G)+\chi(G)=p+2$ if and only if G=K_p

Proof

Suppose $\gamma_{nc2oe}(G)+\chi(G)=p+2$

It is possible if $\gamma_{nc2oe}(G)=2$ and $\chi(G)=p$

Since $\gamma_{nc2oe}(G)=2$ and $\chi(G)=p$

Then G is isomorphic to K_p

Conversely

G is isomorphic to K_p

Then $\gamma_{nc2oe}(G)=2$ and $\chi(G)=p$

Then $\gamma_{nc2oe}(G)+\chi(G)=p+2$

Corollary: 3.15

Let T be a tree of order p, Then $\gamma_{nc2oe}(G)+\chi(G)=p$ if and only if G is isomorphic to P_4,P_5, $K_{1,3}$ or C_5

Corollary: 3.16

Let G be any graph of order p, Then $\gamma_{nc2oe}(G)+\chi(G)=2p-3$ if and only if G is isomorphic to K_5or P_5,P_6

Corollary: 3.17

Let G be any graph of order p Then $\gamma_{nc2oe}(G)+\chi(G)=p-1$ if and only if G is isomorphic to P_6

4. Conclusions

In this paper some upper bound of non split two out degree equitable domination number is discussed. Further we like to compare the non split two out degree equitable domination number with other domination number parameter. Finally we like to study the application of non split two out degree equitable domination number in real life/

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