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Meijun Guo
Department of Mathematics, Tianjin Polytechnic University, Tianjin, People's Republic of China

Qingjun Kong
Department of Mathematics, Tianjin Polytechnic University, Tianjin, People's Republic of China

Correspondence
Meijun Guo
Department of Mathematics, Tianjin Polytechnic University, Tianjin, People's Republic of China

A note on solvability of finite groups

Meijun Guo and Qingjun Kong

Abstract
In this paper, we obtain the solvability for a finite group based on the assumption that Sylow 2-subgroups have the semi cover-avoiding properties. Some known results are generalized.
Mathematics Subject Classification: 20D10, 20D20.

Keywords: Finite Group, Semi cover-avoiding subgroups, Solvable groups

1. Introduction

Gaschutz in [1] introduced pre-Frattini subgroups. These subgroups have cover-avoiding properties, that is, suppose that H$\leq$G, for any chief series 1=$G_0 < G_1 < \ldots < G_n$ =G, such that for every i=1,$\ldots$,n either H covers G_i/G_{i-1} or H avoids G_i/G_{i-1}. Many authors studies this property, for example, Ezquerro in [2] gave some characterization for a finite group G to be p-supersolvable and super solvable based on the assumption that all maximal subgroups of some Sylow subgroups of G have the cover-avoiding properties. Guo and Shum in [3] obtain some characterizations for a finite solvable group based on the assumption that some of its maximal subgroups or 2-maximal subgroups have the cover-avoiding properties.
Recently, Fan, Guo and Shum in [4] generalized the cover-avoiding properties of finite groups. They called semi cover-avoiding properties, that is, a subgroup H is said to be semi cover-avoiding in a group G if there is a chief series 1=$G_0 < G_1 < \ldots < G_m$ =G, such that for every i=1,$\ldots$,m either H covers G_i/G_{i-1} or H avoids G_i/G_{i-1}. They used semi cover-avoiding properties of Sylow subgroups to investigate the solvability of finite groups. They proved the following result:

Theorem A. [4] Let G be a finite group.Then G is solvable if and only if every Sylow subgroup of G is semi cover-avoiding subgroup of G.
In this paper, we will go on to study the influence of semi cover-avoiding subgroups on the structure of finite groups. In Theorem A, we can replace every Sylow subgroup of G by every Sylow 2-subgroup of G, the Theorem still holds. Our main result is the following:

Theorem B. Let G be a finite group.Then G is solvable if and only if every Sylow 2-subgroup of G is semi cover-avoiding subgroup of G.
All groups considered in this paper are finite groups. Our notation and terminology are standard. The reader may refer to ref [5].

2. Preliminary results

In this section, we give one lemma which are useful for our main results.

Lemma 2.1 [6] Let G be a group. Let H be a semi cover-avoiding subgroup of G and N be a normal subgroup of G.
(i)If H$\leq$ K$\leq$ G, then H is a semi cover-avoiding subgroup of K.
(ii)If N$\subseteq$H or gcd(|H|,|N|) = 1, where gcd(-, -) denotes the greatest common divisor, then HN/N is a semi cover-avoiding subgroup of G/N.

3. Proof of the Main Theorem

Proof of Theorem B: If G is solvable, then by Theorem A we know that every Sylow subgroup of G is semi cover-avoiding subgroup. So every Sylow 2-subgroup of G is semi cover-avoiding subgroup.

Conversely, let P be a Sylow 2-subgroup of G. If G is simple, then P covers or avoids the only chief series G/1. So G=P or $G \cap P = P = 1$. If G=P, then G is solvable. If P=1, then G is an group of odd order. By Feit-Thompson Theorem, we have that G is solvable. So we suppose that G is not simple and H cover or avoids a chief series 1=$G_0 < G_1 = N < \ldots < G_n$ =G. If $N \leq P$, then P/N is a Sylow subgroup of G/N. By Lemma 2.1 we have that P/N is a semi cover-avoiding subgroup of G/N. By induction on |G| we know that G/N is solvable. Since $N \leq P$, N is solvable. So G is solvable. If N is not contained in P, then PN$\neq$P. So P$\cap$ N=1. Since P is a Sylow 2-subgroup of G and N$\lhd$G, we have that P$\cap$ N is a Sylow 2-subgroup of N. So N is an odd group. That is, (|P|,|N|)=1. By Lemma 2.1 we have that PN/N is a Sylow 2-subgroup of G/N and is a semi cover-avoiding subgroup of G/N. So G/N satisfies the hypothesis of the theorem. By induction on |G|, we know that G/N is solvable. Since N is solvable, G is solvable. Our proof is complete now.

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