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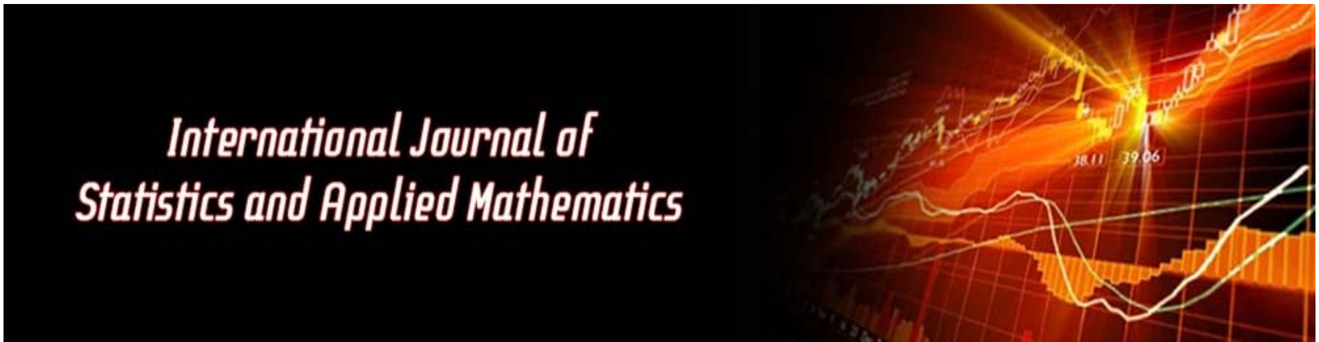

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A note on semi cover-avoiding subgroups of finite groups

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Abstract
In this paper, we obtain the supersolvability for a finite group based on the assumption that minimal subgroups and cyclic subgroups of order 4 have the semi cover-avoiding properties. Some known results are generalized.
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Keywords: Finite Group, Semi cover-avoiding subgroups, Supersolvable groups

1. Introduction
In 1962, Gaschutz [1] introduced pre-Frattini subgroups. These subgroups have cover-avoiding properties, that is, suppose that H$\leq$G, for any chief series 1=$G_0 < G_1 < \ldots < G_n$ =G, such that for every i=1,$\ldots$,n either H covers G_i/G_{i-1} or H avoids G_i/G_{i-1}. Many authors studies this property, for example, In 1993, Ezquerro [2] gave some characterization for a finite group G to be p-supersolvable and supersolvable based on the assumption that all maximal subgroups of some Sylow subgroups of G have the cover-avoiding properties. Guo and Shum in [3] obtain some characterizations for a finite solvable group based on the assumption that some of its maximal subgroups or 2-maximal subgroups have the cover-avoiding properties.
Recently, Fan, Guo and Shum [4] generalized the cover-avoiding properties of finite groups. They called semi cover-avoiding properties, that is, a subgroup H is said to be semi cover-avoiding in a group G if there is a chief series 1=$G_0 < G_1 < \ldots < G_m$ =G, such that for every i=1,$\ldots$,m either H covers G_i/G_{i-1} or H avoids G_i/G_{i-1}. They used semi cover-avoiding properties of Sylow and maximal subgroups to investigate the solvability of finite groups. We [9] obtain the supersolvability for a finite group based on the assumption that minimal subgroups and cyclic subgroups of order 4 have the semi cover-avoiding properties. We proved the following result:

Theorem A. [9] Let G be a finite group. N$\lhd$G and G/N is supersolvable. If every minimal subgroup and every cyclic subgroup of order 4 of N are semi cover-avoiding subgroups of G, then G is supersolvable.
In this paper, we will go on to study the influence of semi cover-avoiding subgroups on the structure of finite groups. In Theorem A, if G is a finite solvable group, we can replace N by F(N). Our main result is the following:

Theorem B. Let G be a finite solvable group. N$\lhd$G and G/N is supersolvable. If every minimal subgroup and every cyclic subgroup of order 4 of F(N) are semi cover-avoiding subgroups of G, then G is supersolvable.
All groups considered in this paper are finite groups. Our notation and terminology are standard. The reader may refer to ref [5].

2. Preliminary results

In this section, we give one lemma which are useful for our main results.

Lemma 2.1 [9] Let G be a group. Let H be a semi cover-avoiding subgroup of G and N be a normal subgroup of G.

(i)If H $\leq$ K $\leq$ G, then H is a semi cover-avoiding subgroup of K.

(ii)If N $\subseteq$ H or gcd (|H|,|N|) = 1, where gcd(-, -) denotes the greatest common divisor, then HN/N is a semi cover-avoiding subgroup of G/N.

3. Proof of the Main Theorem

Proof of Theorem B. Assume that the theorem is false and let G be a counterexample of minimal order. Then

(i) Φ (G)=1.

In fact, since G is solvable,by [6,III,4.2,c], we have that F(G)/ Φ (G)>1. If Φ (G)>1, we can suppose that K is a minimal normal subgroup of G such that K $\leq \Phi$ (G), then F(G/K)=F(G)/K. By lemma 2.3, we know that G/K satisfies the hypothesis of the theorem, so G/K is supersolvable. Thus G is supersolvable. a contradiction, so Φ (G)=1.

(ii)F(G)= $N_1 \times N_2 \times \cdots \times N_s \times M_1 \times M_2 \times \cdots \times M_t$, where N_j (j=1,2,…,s) and M_j (j=1,2,…,t) are minimal normal subgroups of G, F(N)= $N_1 \times N_2 \times \cdots \times N_s$ and $M_j \cap N$ =1(j=1,2…,t).

In fact, by [7, Lemma 2.3 of Appendixes] we know that F (G) is the direct product of minimal normal subgroups of G. By the Lemma 2.3 in [8], we know F(N)= $N_1 \times N_2 \times \cdots \times N_s$, where N_j (j=1,2…,s) are minimal normal subgroups of G. Noticing that F(N) $\leq$ F(G), we know (ii) is true.

(iii)Every N_j (j=1,2,…,s) and M_j (j=1,2,…,t) are cyclic groups of prime order.

In fact, by Theorem 3.2 in [4], we know that F(N) is supersolvable, so every N_j (j=1,2,…,s) is a cyclic group of prime order. Since M_jN/N is a minimal normal subgroup of G/N, we have that M_jN/N is a cyclic group of prime order by the supersolvability of G/N. Hence $M_j \cong M_j / M_j \cap N \cong M_jN/N$ is a cyclic group of prime order.

(iv) G is supersolvable.

Let M be a maximal subgroup of G. If F (G) is not contained in M, then there exists some N_j or M_j such that N_j is not contained in M or M_j is not contained in M. No matter what case it is, we have that [F(G): M $\cap$ F(G)] is a prime. By [8,p12 Theorem 3.3], G is supersolvable. Our proof is complete now.

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