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Peristaltic flow of a couple stress fluid in an inclined channel under the effect of magnetic field with slip condition

Dr. NG Sridhar

Abstract
The present paper investigates the peristaltic motion of a couple stress fluid in a two dimensional inclined channel with the effect of magnetic field using slip condition. The effects of various physical parameters on velocity, pressure gradient and friction force have been discussed & computed numerically. The effects of various key parameters are discussed with the help of graphs.

Keywords: Peristaltic transport, Couple stress fluid, Magnetic field and inclined channel

1. Introduction

Peristalsis is known to be one of the main mechanisms of transport for many physiological fluids, which is achieved by the passage of progressive waves of area contraction and expansion over flexible walls of a tube containing fluid. Various studies on peristaltic transport, experimental as well as theoretical, have been carried out by many researchers to explain peristaltic pumping in physiological systems. The study of couple stress fluid is very useful in understanding various physical problems because it possesses the mechanism to describe rheological complex fluids such as liquid crystals and human blood. By couple stress fluid, we mean a fluid whose particles sizes are taken into account, a special case of non-Newtonian fluids.

Srivastava *et al*., [1] peristaltic transport of a physiological fluid: part I flow in non- uniform geometry. Latham [2] investigated the fluid mechanics of peristaltic pump and science. Mekhemier [3] studied non-linear peristaltic transport a porous medium in an inclined planar channel. Srivastava and Srivastava [4] studied peristaltic transport of a non-newtonian fluid: applications to the vas deferens and small intestine. El-dabe and El-Mohandis [5] have studied magneto hydrodynamic flow of second order fluid through a porous medium on an inclined porous plane. Rathod and Asha [6] effects of magnetic field and an endoscope on peristaltic motion. Rathod and Mahadev [7] studied effect of magnetic field on ureteral peristalsis in cylindrical tube. Rathod and Pallavi [8] studied the influence of wall properties on MHD peristaltic transport of dusty fluid. Rathod and Pallavi [9] studied the effect of slip condition and heat transfer on MHD peristaltic transport through a porous medium with complaint wall. Rathod and Mahadev [10] studied slip effects and heat transfer on MHD peristaltic flow of Jeffrey fluid in an inclined channel. Rathod and Laxmi [11] investigated effects of heat transfer on the peristaltic MHD flow of a Bingham fluid through a porous medium in an inclined channel. Rathod and Laxmi [12] studied effects of heat transfer on the peristaltic MHD flow of a Bingham fluid through a porous medium in a channel. Jayarami Reddy *et al*., [13] studied peristaltic flow of a Williamson fluid in an inclined planar channel under the effect of a magnetic field. Ramana Kumari and Radhakrishnamacharya [14] studied effect of slip on peristaltic transport in an inclined channel with wall effects. Reddappa *et al*., [15] studied the peristaltic transport of a Jeffrey fluid in an inclined planar channel with variable viscosity under the effect of a magnetic field. Sridhar [16] studied effect of thickness of the porous material using porous media on the peristaltic pumping of couple stress fluid through non - erodible porous lining tube. Rathod and Sridhar [17] studied peristaltic transport of couple stress fluid in uniform and non-uniform annulus through porous medium.

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Rathod *et al*., [18] studied peristaltic pumping of couple stress fluid through non - erodible porous lining tube wall with thickness of porous material. Rathod and Sridhar [19] studied effect of magnetic field on peristaltic transport of a couple stress fluid in a channel. Rathod and Sridhar [20] studied peristaltic flow of a couple stress fluid in an inclined channel. Rathod *et al*., [21] studied peristaltic flow of a couple stress fluid in an inclined channel under the effect of magnetic field. Rathod *et al*., [22] studied peristaltic transport of a conducting couple stress fluid through a porous medium in a channel. Sridhar [23] studied effects of magnetic field with variable viscosity and an endoscope on peristaltic transport of couple stress fluids through a porous medium.

The present research aim is to investigate the interaction of peristalsis for the flow of a couple stress fluid in a two dimensional inclined channel with the effect of magnetic field using slip condition. The computational analysis has been carried out for drawing velocity profiles, pressure gradient and frictional force.

2. Formulation of the problem

We consider a peristaltic flow of a Couple stress fluids through two-dimensional channel of width 2a and inclined at an angle α to the horizontal symmetric with respect to its axis. The walls of the channel are assumed to be flexible.

The wall deformation is

$$H(x,t)=a+bCos(\frac{2\pi}{\lambda}(X-ct)) \tag{1}$$

Where 'b' is the amplitude of the peristaltic wave, 'c' is the wave velocity, 'λ' is the wave length, t is the time and X is the direction of wave propagation.

The governing equations are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2}$$

$$\rho(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y})=-\frac{\partial p}{\partial x}+\mu\nabla^2u-\eta^*\nabla^4u+\rho g\sin\alpha-\sigma B_o^2u \tag{3}$$

$$\rho(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y})=-\frac{\partial p}{\partial y}+\mu\nabla^2v-\eta^*\nabla^4v-\rho g\cos\alpha-\sigma B_o^2v \tag{4}$$

Where, $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$

u and v are velocity components, 'p' is the fluid pressure, 'ρ' is the density of the fluid, 'μ' is the coefficient of viscosity, 'η^*' is the coefficient of couple stress, 'g' is the gravity due to acceleration, 'α' angle of inclination, 'σ' is electric conductivity and '$B_\circ$' is applied magnetic field.

Introducing a wave frame (x, y) moving with velocity c away from the fixed frame (X, Y) by the transformation

$$x=X-ct, y=Y, u=U, v=V, p=P(X,t) \tag{5}$$

We introduce the non-dimensional variables:

$$x^*=\frac{x}{\lambda},\ y^*=\frac{y}{a},u^*=\frac{u}{c},\ v^*=\frac{v}{c\delta},\ t^*=\frac{tc}{\lambda},p^*=\frac{pa^2}{\mu c\lambda},G=\frac{\rho ga^2}{\mu c},M=B_\circ\sqrt{\frac{\sigma}{\mu a^2}},\phi=\frac{b}{a} \tag{6}$$

Equation of motion and boundary conditions in dimensionless form becomes

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{7}$$

$$\mathrm{Re}\,\delta(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y})=-\frac{\partial p}{\partial x}+(\delta^2\frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial y^2})-\frac{1}{\gamma^2}(\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2})(\delta^2\frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial y^2})$$
$$-M^2u+G\sin\alpha \tag{8}$$

$$\mathrm{Re}\,\delta^3(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y})=-\frac{\partial p}{\partial y}+\delta^2(\delta^2\frac{\partial^2v}{\partial x^2}+\frac{\partial^2v}{\partial y^2})-\frac{1}{\gamma^2}\delta^2(\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2})(\delta^2\frac{\partial^2v}{\partial x^2}+\frac{\partial^2v}{\partial y^2})\quad -M^2\delta^2v-\rho g\delta\cos\alpha \tag{9}$$

Where, $\gamma^2=\frac{\eta^*}{\mu a^2}$ couple-stress parameter and $M^2=B_o^2\frac{\sigma}{\mu a^2}$ Hartmann number.

The dimensionless boundary conditions are:

$$\frac{\partial u}{\partial y}=0; \quad \frac{\partial^2 u}{\partial y^2}=0 \qquad at \quad y=0$$

$$u=-k_n\frac{\partial u}{\partial y};\frac{\partial^2 u}{\partial y^2} \quad finite \quad at \quad y=\pm h=1+\phi Cos[2\pi x] \tag{10}$$

where, k_n is Knudsen number (slip parameter)

Using long wavelength approximation and neglecting the wave number δ, one can reduce governing equations:

$$\frac{\partial p}{\partial y}=0 \tag{11}$$

$$\frac{\partial p}{\partial x}=\frac{\partial^2 u}{\partial y^2}-\frac{1}{\gamma^2}\frac{\partial^4 u}{\partial y^4}-M^2u+G\sin\alpha \tag{12}$$

Solving the Eq.(12) with the boundary conditions (10), we get

$$u=\frac{\partial p}{\partial x}[D(1-\frac{D}{\gamma^2})-\frac{1}{M^2}]-G.D\sin\alpha \tag{13}$$

Where $D=\frac{y^2}{2}-\frac{h^2}{2}-\beta y$

The volumetric flow rate in the wave frame is defined by

$$q=\int_0^h udy=\frac{\partial p}{\partial x}[A-\frac{1}{\gamma^2}(\frac{2h^5}{15}+\frac{h^4\beta}{4}+\frac{h^3\beta^2}{3})-\frac{h}{M^2}]-GA\sin\alpha \tag{14}$$

Where, $A=\frac{h^3}{6}-\frac{h^2\beta}{2}-\frac{h^3}{2}$

The expression for pressure gradient from Eq.(14) is given by

$$\frac{\partial p}{\partial x}=\frac{q+GA\sin\alpha}{A-\frac{1}{\gamma^2}(\frac{2h^5}{15}+\frac{h^4\beta}{4}+\frac{h^3\beta^2}{3})-\frac{h}{M^2}} \tag{15}$$

The instantaneous flux Q (x, t) in the laboratory frame is

$$Q(x,t)=\int_0^h (u+1)dy=q+h \tag{16}$$

The average flux over one period of peristaltic wane is $\overline{Q}$

$$\overline{Q}=\frac{1}{T}\int_0^T Qdt=q+1 \tag{17}$$

From equations (15) and (17), the pressure gradient is obtained as

$$\frac{\partial p}{\partial x}=\frac{(\overline{Q}-1)+GA\sin\alpha}{A-\frac{1}{\gamma^2}(\frac{2h^5}{15}+\frac{h^4\beta}{4}+\frac{h^3\beta^2}{3})-\frac{h}{M^2}} \tag{18}$$

The pressure rise (drop) over one cycle of the wave can be obtained as

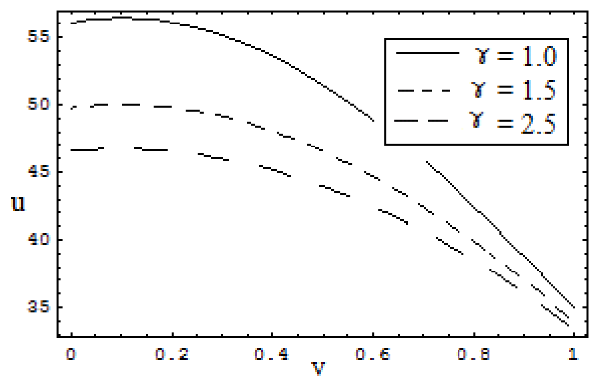
$$\Delta P=\int_0^1 (\frac{dp}{dx})dx \tag{19}$$

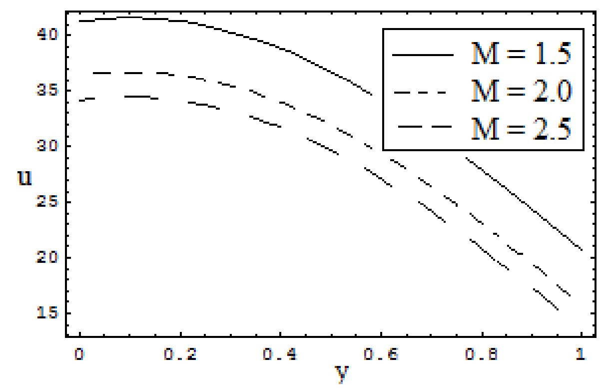
The dimensionless frictional force F at the wall across one wavelength is given by

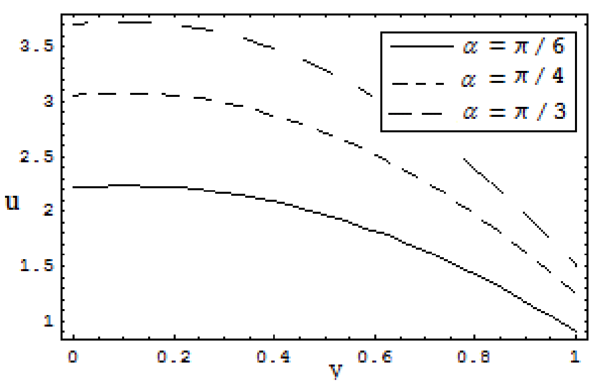
$$F = \int_0^1 h(-\frac{dp}{dx})dx \tag{20}$$

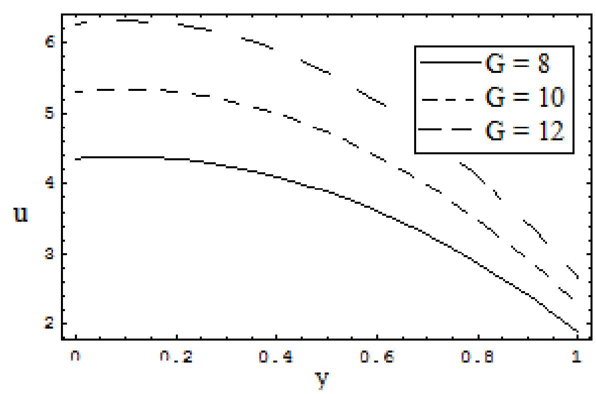
3. Results and discussions

In this section we have presented the graphical results of the solutions axial velocity u, pressure rise ΔP , friction force F for the different values of couple stress (γ), magnetic field (M), angle of inclination (α), gravitational parameter (G) and slip parameter (β). The axial velocity is shown in Figs. (1 to 5). The Variation of u with γ , we find that u depreciates with increase in γ (Fig. 1). The Variation of u with magnetic field M shows that for u decreases with increasing in M (fig. 2).The Variation of u with angle of inclination α shows that for u increases with increasing in α (Fig 3). The Variation of u with gravitational parameter G shows that for u increases with increasing in G (Fig 4). The Variation of u with slip parameter β shows that for u increases with increasing in G (Fig 5).









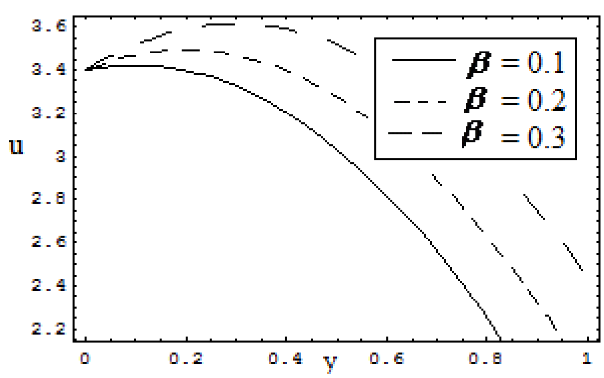
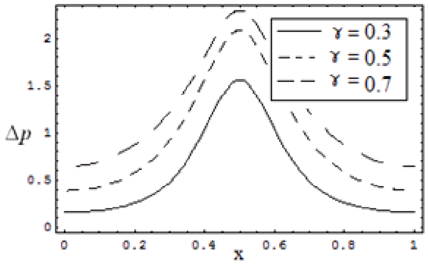
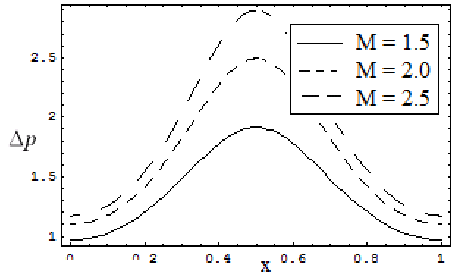
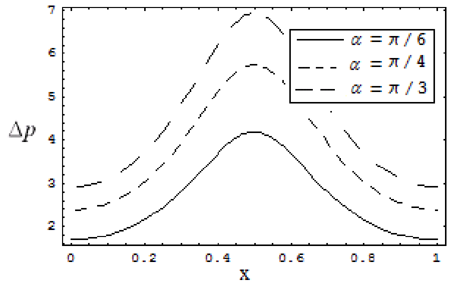


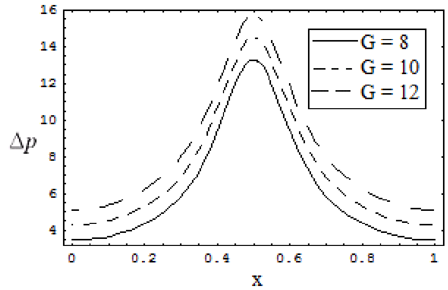
Fig. 1: Effect of γ on u, when ϕ =0.2, x=0.1 & p=-25, G=6, M=1, β = .1 & $\alpha = \pi/4$.
Fig. 2: Effect of **M** on u, when γ =1, ϕ =0.2, x=0.1, p=-25, G=4, $\alpha = \pi/4$, ϕ=0.2 & β=.1
Fig. 3: Effect of α on u, when γ =3, ϕ =0.2, x=0.1, p=-.25, M=5, β = .1 & G=6.
Fig. 4: Effect of G on u, when γ =1, ϕ=.2, x=0.1, p=-.25, M=1, β = .1 & $\alpha = \pi/4$.
Fig. 5: Effect of β on u, when γ =1, x=0.1, p=-.25, G=6, ϕ=0.2, M=1 & $\alpha = \pi/4$.

The variation of pressure rise ΔP is shown in Figs (6 to 10) for different values of γ , M, α , G & β . We find that ΔP increases with increasing in γ (Fig. 6). The Variation of ΔP with M shows that for ΔP increases with increasing in M (Fig 7).The Variation of ΔP with angle of inclination α shows that for ΔP increases with increasing in α (Fig 8). The Variation of ΔP with gravitational parameter G shows that for ΔP increases with increasing in G (Fig 9). The Variation of ΔP with slip parameter β shows that for ΔP depreciates with increase in β (Fig 10).









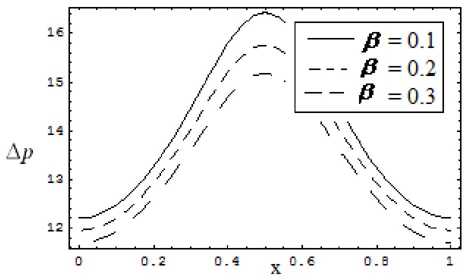
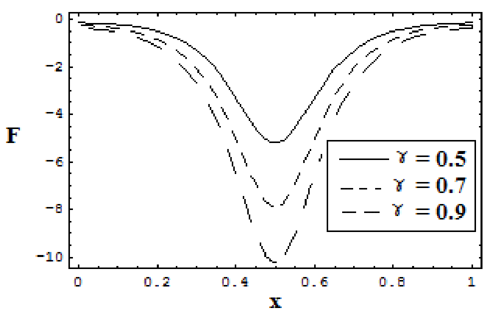
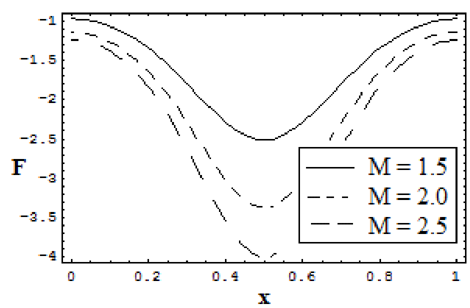
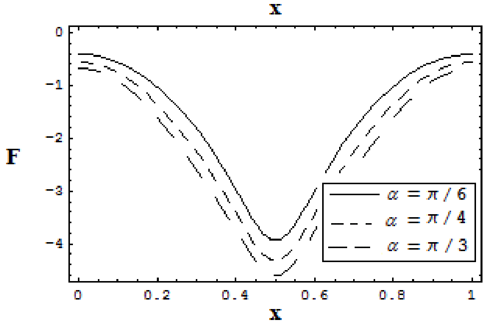


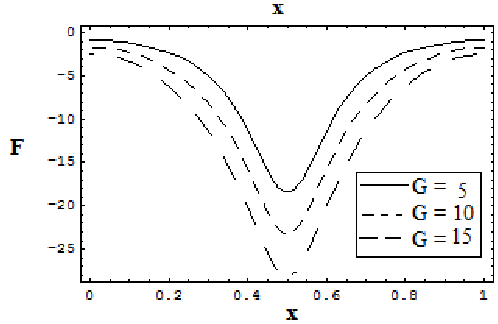
Fig 6: Effect of γ on Δp, when $\phi = 0.6$, $G = 4$, $\alpha = \pi/4$, $Q = 0$, $\beta = 1$ & $M = 1$.
Fig 7: Effect of M on Δp, when $\gamma = .7$, $\phi = 0.2$, $G = 2$, $\alpha = \pi/4$, $\beta = .1$ & $Q = 0$.
Fig 8: Effect of α on Δp, when $\gamma = 1.5$, $\phi = 0.6$, $M = 1$, $Q = 0$, $G = 20$ & $\beta = 10$
Fig 9: Effect of G on Δp, when $\gamma = 1.5$, $\phi = 0.6$, $M = 10$, $Q = 0$, $\beta = 1$ & $\alpha = \pi/4$.
Fig 10: Effect of β on u, when $\gamma = 1.5$, $G = 20$, $\phi = 0.2$, $Q = 0$, $M = 10$ & $\alpha = \pi/4$.

The variation of friction force F is shown in Figs. (11 to 15) for a different values of γ, M, α, G & β. Here, it is observed that the effect of all the parameters on friction force are opposite behavior as to the effects on pressure with time average mean flow rate is observed.









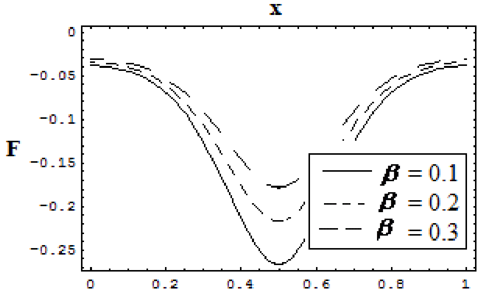


Fig. 11: Effect of γ on F, when $G = 4$, $\phi = 0.6$, $\alpha = \pi/4$, $Q = 0$, $\beta = 1$ & $M = 1$.
Fig. 12: Effect of M on F, when $\gamma = 1$, $\phi = 0.2$, $\alpha = \pi/4$, $Q = 0$, $\beta = .1$ & $G = 2$.
Fig. 13: Effect of α on F, when $\gamma = 1$, $\phi = 0.6$, $G = 6$, $Q = 0$, $\beta = 1$ & $M = 1$.
Fig. 14: Effect of G on F, when $\gamma = 1.5$, $\phi = .6$, $\alpha = \pi/4$, $Q = 0$, $\beta = 1$ & $M = 10$.
Fig. 15: Effect of β on u, when $\gamma = .1$, $G = 2$, $\phi = 0.2$, $Q = 0$, $M = 5$ & $\alpha = \pi/4$.

4. Conclusion

In this paper we presented a theoretical approach to study the peristaltic flow of a couple stress fluid in an inclined channel with the effect of a magnetic field using slip condition. The governing Equations of motion are solved analytically. Furthermore, the effect of various values of parameters on Velocity, Pressure rise and Friction force have been computed numerically and explained graphically.

We conclude the following observations:

1. The velocity u increases with increasing in gravitational parameter G, angle of inclination α, slip parameter β but, decreases with increasing in couple stress parameter γ & magnetic field M.
2. The pressure ΔP increases with increasing in gravitational parameter G, angle of inclination α, couple stress parameter γ & magnetic field M but, decreases with increasing in slip parameter β.
3. The friction force F decreases with increasing in gravitational parameter G, angle of inclination α, couple stress parameter γ & magnetic field M but, increases with increasing in slip parameter β.

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