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On rgwα-Homeomorphism in topological spaces

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Abstract
A bijection f: (X, τ) → (Y, σ) is called regular generalized weakly α- homeomorphism if f and f^{-1} are rgwα-continuous. Also we introduce the new class of maps, namely rgwα*-homeomorphisms which form a subclass of rgwα-homeomorphisms. The class of rgwα- homeomorphism and rgwα*-homeomorphism maps are closed under composition of maps. We prove that the set of all rgwα-homeomorphism and rgwα*-homeomorphisms forms a group under the operation composition of maps.

Keywords: rgwα-closed set, rgwα-open set, rgwα-continuous mapping, rgwα-homeomorphism, rgwα*-homeomorphism

1. Introduction
The homeomorphism in topological space plays a significant role in the study of topology. First we provide basic requirement for better understanding of rgwα -Homeomorphism in topological space. In this paper, we define new class of homeomorphism called rgwα-Homeomorphism and study the properties of this map and also compare this homeomorphism with well-known homeomorphisms w-homeomorphisms, g-homeomorphisms, rgα-homeomorphisms and rwg-homeomorphism etc. Also we introduce new class of maps rgwα*-homeomorphisms which form a subclass of rgwα-homeomorphisms. The class of rgwα-homeomorphism and rgwα*- homeomorphism maps are closed under composition of maps.We prove that the set of all rgwα- homeomorphism and rgwα*- homeomorphism maps forms a group under the operation composition of maps.

2. Preliminaries
Let us recall the following definition which we shall require later.

Definition 2.1: A subset A of a space (X, τ) is called
i. Regular open [18] if A= int(cl(A)) and regular closed if A= cl(int(A)).
ii. Pre-open [16] if A ⊂ int(cl(A)) and pre-closed if cl(int(A)) ⊂ A.
iii. Semi-open [9] if A ⊂ cl(int(A)) and semiclosed if int(cl(A)) ⊂ A.
iv. α-open [13] if A ⊂ int(cl(int(A))) and α-closed if cl(int(cl(A))) ⊂ A.
v. Semi-pre-open [10, 24] if A ⊂ cl(int(cl(A))) and a semi-preclosed if int(cl(int(A))) ⊂ A.
vi. Regular α-open [4] if there is a regular open set U such that U ⊂ A ⊂ αcl(U).
vii. Regular semi open [8] if there is a regular open set U such that U ⊂ A ⊂ cl(U).

Definition 2.2: A subset A of a space (X, τ) is called
i. Generalized closed set (briefly, g-closed) [22] if cl(A) ⊂ U whenever A ⊂ U and U is open in X.
ii. Generalized α-closed set (briefly, gα-closed) [3] if αcl(A) ⊂ U whenever A ⊂ U and U is α-open in X.
iii. α-generalized closed set (briefly, αg-closed) [2] if αcl(A) ⊂ U whenever A ⊂ U and U is open in X.
iv. Regular generalized closed set (briefly, rg-closed) [14] if cl(A) ⊂ U whenever A ⊂ U and U is regular open in X.

v. Generalized pre -regular closed set (briefly, gpr-closed) [12] if pcl(A) ⊂ U whenever A ⊂ U and U is regular open inX.
vi. Weakly generalized closed set (briefly, ωg-closed) [9, 14] if clint(A) ⊂ U whenever A ⊂ U and U is open in X.
vii. Weakly closed sets (briefly, ω-closed) [7, 11] if cl(A) ⊂ U whenever A ⊂ U and U is semi open in X.
viii. Regular weakly generalized closed set (briefly, rωg-closed) [1] if cl(int(A)) ⊂ U whenever A ⊂ U and U is regular open in X.
ix. Regular generalized α-closed set (briefly, rgα-closed) [4] if αcl(A) ⊂ U whenever A ⊂ U and U is regular α- open in X.
x. Regular ω-closed (briefly, rω-closed) [11] if cl(A) ⊂ U whenever A ⊂ U and U is regular semi open in X.
xi. Regular generalized weakly α-closed set (briefly rgwα-closed) [19] if rαcl (A) ⊆U whenever A⊆U & U is weak α-open set in X.

Definitions 2.3: A map f : (X, τ) → (Y, σ) is said to be
i. continuous [15] if $f^{-1}(V)$ is closed in X, for every closed set V in Y.
ii. ω-continuous [1] if $f^{-1}(V)$ is ω-closed in X, for every closed set V in Y.
iii. α-continuous [11] if $f^{-1}(V)$ is α-closed in X, for every closed set V in Y.
iv. gα-continuous [3] if $f^{-1}(V)$ is gα-closed in X, for every closed set V in Y.
v. rgα-continuous [4] if $f^{-1}(V)$ is rgα-closed in X, for every closed set V in Y.
vi. swg-continuous [11] if $f^{-1}(V)$ is swg-closed in X, for every closed set V in Y.
vii. rwg-continuous [11] if $f^{-1}(V)$ is rwg-closed in X, for every closed set V in Y.
viii. rgwα-continuous [20] if $f^{-1}(V)$ is rgwα-closed in X, for every closed set V in Y.

Definitions 2.4: A map f :(X, τ) → (Y, σ) is said to be
i. irresolute [21] if $f^{-1}(V)$ is semi-open in X, for every semi-open set V in Y.
ii. ω-irresolute [13] if $f^{-1}(V)$ is ω-closed in X, for every ω-closed set V in Y.
iii. rgα-irresolute [5] if $f^{-1}(V)$ is rgα-closed in X, for every rgα-closed set V in Y.
iv. rgwα-irresolute [20] if $f^{-1}(V)$ is rgwα-closed in X, for every rgwα-closed set V in Y.

Definition 2.5: A bijective function f: (X, τ) → (Y, σ) is called
(i) generalized homeomorphism (g-homeomorphism) [9] if both f and $f^{-1}$are g-continuous,
(ii) w-homeomorphism [13] if both f and $f^{-1}$are w-continuous.
(iii) w*-homeomorphism [13] if both f and f^{-1} are w- irresolute,
(iv) gc-homeomorphism [9] if both f and $f^{-1}$are gc-irresolute,
(v) rwg-homeomorphism [16] if both f and $f^{-1}$are rwg-continuous,
(vi) rgα-homeomorphism [16] if both f and $f^{-1}$are rgα – continuous.
(vii) wgrα-homeomorphism [23] if both f and $f^{-1}$are wgrα – continuous

3. rgwα-homeomorphisms in Topological Spaces

We introduce the following definition

Definition 3.1: A bijection f: (X, τ) → (Y, σ) is called regular generalized weakly α-homeomorphism (briefly, rgwα-homeomorphism) if f and f^{-1} are rgwα-continuous.
The family of all homeomorphism and rgwα-homeomorphism of a topological space
(X, τ_1) onto itself is denoted by h(X, τ) and rgwα-h(X, τ) respectively.

Example 3.2: Let X=Y={a, b, c, d, e}, τ ={X, ϕ, {a}, {d}, {e}, {a, d}, {a, e}, {d, e}, {a, d, e} and σ ={Y, ϕ, {a},{d},{e}, {a, d}, {a, e},{d, e},{a, d, e }. Let map f: X→Y defined by f(a)=c, f(b)= a, f(c)=b, f(d)=d, f(e)=e, then f bijection, rgwα-continuous and f^{-1} is rgwα- continuous. Therefore f is rgwα-homeomorphism.

Theorem 3.3: Every homeomorphism is a rgwα-homeomorphism, but not conversely.
Proof: Let f: (X, τ) → (Y, σ) be a homeomorphism. Then f and f^{-1} are continuous and f is bijection. As every continuous function is rgwα-continuous, we have f and f^{-1} are rgwα-continuous. Therefore f is rgwα-homeomorphism.
The converse of the above Theorem need not be true, as seen from the following example.

Example 3.4: Let X=Y={a, b, c, d, e}, τ ={X, ϕ, {a}, {d}, {e}, {a, d},{a,e},{d,e},{a,d,e}} and σ ={Y, ϕ, {a},{d},{e}, {a,d}, {a,e},{d,e},{a,d,e }. Let map f: X→Y defined by f(a)=c, f(b)= a, f(c)=b, f(d)=d, f(e)=e, then f is bijection, rgwα-continuous and f^{-1} is rgwα- continuous. Therefore f is rgwα-homeomorphism but it is not homeomorphism as closed set F ={a,c,e} in Y, then f^{-1} (F)={a, b, e} in X which is not closed set in X.

Theorem 3.5: For any topological space (X, τ), h(X, τ) ⊆ rgwα-h(X, τ).
Proof: Let us consider f ∈ h(X, τ), then by the definition of homeomorphism f and f^{-1} are continuous. Since every continuous function is rgwα-continuous so f and f^{-1} are rgwα-continuous map. Now by the definition of rgwα-homeomorphism we can say that f ∈ rgwα-h(X, τ).

Theorem 3.6: A bijection f: (X, τ) → (Y, σ) be rgwα-homeomorphism and (X, τ) and (Y, σ) are $\tau_{rgw\alpha}$- space, then f is homeomorphism.
Proof: It follows from fact that in $\tau_{rgw\alpha}$- space, every rgwα-closed set is closed.

Theorem 3.7: Every α-homeomorphism is a rgwα-homeomorphism but not conversely.
Proof: Let f: (X, τ) → (Y, σ) be a α-homeomorphism. Then f and f^{-1} are α-continuous and f is bijection. As every α-continuous function is rgwα-continuous, we have f and f^{-1} is rgwα-continuous. Therefore f is rgwα-homeomorphism.
The converse of the above Theorem is not true in general as seen from the following example.

Example 3.8: Consider X=Y={a,b,c,d,e}, τ ={X,ϕ,{a},{d},{e},{a,d},{a,e},{d,e},{a,d,e}} and σ ={Y, ϕ, {a},{d},{e}, {a,d}, {a,e},{d,e},{a,d,e }. Let map f: X→Y defined by f(a)=c, f(b)= a, f(c)=b, f(d)=d, f(e)=e, then f is rgwα- homeomorphism but not α-homeomorphism, as closed set F= {a,c,e} in Y, then $f^{-1}(F)$={a,b,e} in X which is not α-closed set in X.

Theorem 3.9: Every w- homeomorphism (resp. r-homeomorphism, β- homeomorphism, rw- homeomorphism, rs- homeomorphism, rα- homeomorphism, wα-homeomorphism, gα- homeomorphism, rgα-homeomorphism) is a rgwα-homomorphism but not conversely.

Proof: Proof similar to the theorem 3.7.

Example 3.10: Let X=Y={a,b,c,d,e}, τ ={X, ϕ,{a},{d},{e},{a,d},{a,e},{d,e},{a,d,e}} and σ ={Y, ϕ, {a},{d},{e}, {a,d}, {a,e},{d,e},{a,d,e }. Let map f: X→Y defined by f(a)=c, f(b)= a, f(c)=b, f(d)=d, f(e)=e, then f is rgwα- homeomorphism but not w- homeomorphism r-homeomorphism, β- homeomorphism, rw- homeomorphism, rs- homeomorphism, rα- homeomorphism, wα-homeomorphism, gα- homeomorphism, rgα- homeomorphism as closed set F= {b,c,d,e} in Y, then $f^{-1}(F)$={a,c,d.e} in X which is rgwα-closed but not r-closed, w-closed, β-closed, rs-closed and rα-closed set in X. and closed set F={b,c,d} in Y $f^{-1}(F)$ ={a,b,d} which is not rw-closed, wα-closed, gα- closed, rgα- closed set in X.

Theorem 3.11: Every rgwα-homeomorphism is a gβ-homeomorphism but not conversely.

Proof: Let f: (X, τ) → (Y, σ) be rgwα-homeomorphism. Then f and f^{-1} are rgwα-continuous and f is bijection. Since every rgwα-continuous function is gβ-continuous, we have f and f^{-1} are gβ-continuous. Therefore f is gβ-homeomorphism.

The converse of the above Theorem is not true in general as seen from the following example.

Example 3.12: Consider X=Y= {a,b,c}, τ={X, ϕ,{a},{b,c}} and σ ={Y, ϕ, {a}}. Let map f: X→Y defined by f(a)=b, f(b)= a, f(c)=c, then f is gβ- homeomorphism but not rgwα-homeomorphism as a closed set F={b,c} in Y, $f^{-1}(F)$= f^{-1}{b,c}= {a,c} which is not rgwα-closed set.

Remark 3.13: rgwα-homeomorphism and g-homeomorphism are independent as seen from the following example.

Example 3.14: Let X=Y= {a,b,c}, τ={X, ϕ,{a},{b,c}} and σ ={Y, ϕ, {a}}. Let map f: X→Y defined by f(a)=b, f(b)= a, f(c)=c, then f is g- homeomorphism but not rgwα-homeomorphism, as a closed set F={b,c} in Y, $f^{-1}(F)$= f^{-1}{b,c} ={a,c} is not rgwα-closed set.

Example 3.15: Let X= {a,b,c,d}and Y={a,b,c}, τ={X, ϕ,{a},{b},{a,b},{a,b,c}} and σ ={Y, ϕ, {a},{b,c}}. Let map f: X→Y defined by f(a)=b, f(b)= a, f(c)=a, f(d)=c then f is rgwα- homeomorphism not g-continuous, as a closed set F={a} closed set in Y $f^{-1}(F)$={b} which is not g-closed set in X.

Remark 3.16: rgwα-homeomorphism and αg-homeomorphism (resp. wg- homeomorphism, rg-homeomorphism, gr- homeomorphism, gpr- homeomorphism rwg- homeomorphism and rgw- homeomorphism) are independent as seen from the following example.

Example 3.17: Let X=Y= {a,b,c}, τ={X, ϕ,{a},{b,c}} and σ ={Y, ϕ, {a}}. Let map f: X→Y defined by f(a)=b, f(b)= a, f(c)=c, then f is αg-homeomorphism, wg- homeomorphism, rg- homeomorphism, gr- homeomorphism, gpr-homeomorphism rwg- homeomorphism and rgw-homeomorphism but not rgwα-homeomorphism as a closed set F={b,c} in Y $f^{-1}(F)$= f^{-1}{b,c} ={a,c} is not rgwα-closed set.

Example 3.18: Let X= {a,b,c,d} and Y= {a,b,c}, τ={X, ϕ,{a},{b},{a,b},{a,b,c}} and σ ={Y, ϕ, {a},{b,c}}. Let map f: X→Y defined by f(a)=b, f(b)= a, f(c)=a, f(d)=c then f is rgwα- homeomorphism but not αg-homeomorphism, wg-homeomorphism, rg- homeomorphism, gr- homeomorphism, gpr- homeomorphism rwg- homeomorphism and rgw-homeomorphism as a closed set F={a} closed set in Y $f^{-1}(F)$={b} which is not αg-closed, wg-closed, rg-closed, gr-closd, gpr-closed, rgw-closed, rwg-closed set.

Theorem 3.19: Let the function f: (X, τ) → (Y, σ) be a one-one, onto, rgwα-continuous map. Then the following statements are equivalent:

(1) The function f is a rgwα-open map.
(2) The function f is a rgwα-homeomorphism.
(3) The function f is a rgwα-closed map.

Proof: it follows directly from definitions.

Theorem 3.20: Let the function f: (X, τ) → (Y, σ) and g : (Y, σ) → (Z, φ) are rgwα-homeomorphism, then composition of these two function as gof : (X, τ) → (Z, φ) is also rgwα-homeomorphism.

Proof: Let G be rgwα-open in Z. Now $(gof)^{-1}(G)$ = $f^{-1}(g^{-1}(G))$ = $f^{-1}(G)$, where V = $g^{-1}(G)$. By given, if G us rgw-open in Z and g is rgw-homeomorphism, then $g^{-1}(G)$ = H is rgwα-open in Y. Again, as given g−1(G) is open in Y and f is rgwα-homeomorphism, then $f^{-1}(g^{-1}(G))$ is rgwα-open in X. Therefore, gof is continuous. Similarly we can show that $(gof)^{-1}$ is also continuous. Hence, by the definition gof is rgwα-homeomorphism.

By the above result we can proof that

Theorem 3.21: The rgwα-homeomorphism is an equivalence relation in the family of all topological spaces.

Proof: It follows from above theorem.

Theorem 3.22: Let (X, τ) be a topological space, then the collection rgwα-h(X, τ) forms a group under the composition of functions.

Proof: Let a binary operation o': rgwα-h(X, τ) × rgwα-h (X, τ) → rgwα-h(X, τ) defined by f o' g = gof for all f, g ∈ rgwα-h(X, τ), where gof : X → X is composite maps of f and g such that (gof)(x) = g(f (x)) for all x in X. By the theorem 3.8, gof ∈ rgwα-h(X, τ). The following properties hold by the collection rgwα-h(X, τ).

(1) Associativity: Since the composition of two rgwα-homeomorhism is again rgwα-homeomorhism, with this easily we can prove it is associative.

(f o' g)o' h = f o'(g o' h) for all f, g, h ∈ rgwα-h(X, τ)

(2) Existence of identity: Since the identity map i_X: X → X is also rgwα-homeomorphism, then easily we can say that for all element f ∈ rgwα-h(X, τ), there exists an element i_X such that

f o' i_X = i_X o' f = i_X

(3) Existence of inverse: We know that the composition of maps is associative and the identity map i_X : (X, τ) → (X, τ) belonging to rgwα-h(X, τ) servers as the identity element. If f ∈ rgwα-h(X, τ), then f^{-1} ∈ rgwα-h(X, τ) such that f of^{-1} =

$f^{-1}of = i$ and so inverse exists for each element of rgwα-h(X, τ).
Therefore, (rgwα-h(X, τ), o) is a group under the operation of composition of maps.

Theorem 3.23: If f: (X, τ) → (Y, σ) is rgwα-homeomorphism, then f induces an isomorphism between rgwα- h(X, τ) and rgwα-h(Y, σ) i.e. rgwα-h(X, τ) ≅ rgwα-h(Y, σ).
Proof: Using the map f, we define a map ψ_f : rgwα- h(X, τ) → rgwα- h(Y, σ) by $\psi_f(h) = f ohof^1$ for every h ∈ rgwα− h(X, τ_1). Then ψ_f is bijection. Further, for all h_1, h_2 ∈ ψ_f, $\psi_f(h_1oh_2) = f\ o(h_1\ oh_2)of^{-1} = (f\ oh_1of^{-1})o(f\ oh_2\ of^{-1}) = \psi_f(h_1)o\psi_f(h_2)$. Therefore ψ_f is a homeomorphism and so it is an isomorphism induced by f.

Definition 3.24: A bijection f: (X, τ) → (Y, σ) is said to be rgwα*-homeomorphism if both f and f^{-1} are rgwα-irresolute. We say that spaces (X, τ) and (Y, σ) are rgwα*-homeomorphic if there exists a rgwα*-homeomorphism from (X, τ) onto(Y, σ).
We denote the family of all rgwα*-homeomorphism of a topological space (X, τ) onto itself by rgwα*-h(X, τ).

Theorem 3.25: Every rgwα*-homeomorphism is a rgwα-homeomorphism but not conversely.
Proof: Let f: (X, τ) → (Y, σ) be an rgwα*-homeomorphism. Then f and f^{-1} are rgwα-irresolute and f is bijection. By Theorem 5.3 in [20], f and f^{-1} are rgwα-continuous. Therefore f is rgwα-homeomorphism.
The converse of the above Theorem is not true in general as seen from the following example.

Example 3.26: Consider X={a,b,c,d,e}, Y={a,b,c,d} τ = {X, ϕ, {a}, {d}, {e}, {a,d}, {a,e}, {d,e}, {a,d,e}} σ ={Y, ϕ,{a},{b}, {a,b},{a,b,c}}, Let map f: X→Y defined by, f(a)=b, f(b)=c, f(c)=d, f(d)=a, f(e)=d then f is rgwα-homeomorphism but f is not rgwα*- homeomorphism, as rgwα-closed set F= {a, b} in Y, then $f^{-1}(F)$={a,d} in X, which is not rgwα-closed set in X.

Theorem 3.27: Every rgwα*-homeomorphism is gβ-homeomorphism but not conversely.
Proof: Proof follows from Theorems 3.24 and 3.11.

Example 3.28: Consider X=Y= {a,b,c}, τ={X, ϕ,{a},{b,c}} and σ ={Y, ϕ, {a}}. Let map f: X→Y defined by f(a)=b, f(b)= a, f(c)=c, then f is gβ- homeomorphism but not rgwα*-homeomorphism as a rgwα-closed set F={b,c} in Y, $f^{-1}(F)= f^{-1}${b,c}= {a,c} which is not rgwα-closed set.

Remark 3.29: rgwα*-homeomorphism and w*-homeomorphism are independent as seen from the following example.
Example 3.30: Let X=Y={a,b,c,d,e}, τ ={X, ϕ,{a},{d},{e},{a,d},{a,e},{d,e},{a,d,e}} and σ ={Y, ϕ, {a},{d},{e}, {a,d}, {a,e},{d,e},{a,d,e }}. Let map f: X→Y defined by f(a)=f(b)=b, f(c)=c, f(d)=d, f(e)=e, then f is rgwα*-homeomorphism but not w*- homeomorphism as w-closed set F= {b,c,d,e} in Y, then $f^{-1}(F)$={a,c,d.e} in X which is rgwα-closed but not w-closed closed set in X.

Example 3.31: Let X=Y= {a,b,c}, τ={X, ϕ,{a},{b,c}} and σ ={Y, ϕ, {a}}. Let map f: X→Y defined by f(a)=b, f(b)= a, f(c)=c, then f is w*-homeomorphism but not rgwα-homeomorphism, as a closed set F={b,c} in Y, $f^{-1}(F)= f^{-1}${b,c} ={a,c} is not rgwα-closed set.

Theorem 3.32: Let f : (X, τ) → (Y, σ) and g : (Y, σ) → (Z, η) are rgwα*- homeomorphisms, then their composition g ∘ f : (X, τ) → (Z, η) is also rgwα*- homeomorphism.
Proof: Let U be a rgwα-open set in (Z, η). Since g is rgwα-irresolute, $g^{-1}(U)$ is rgwα-open in (Y, σ). Since f is rgwα-irresolute, $f^{-1}(g^{-1}(U)) = (g \circ f)^{-1}(U)$ is rgwα-open set in (X, τ). Therefore g ∘ f is rgwα-irresolute. Also for a rgwα-open set G in (X, τ), we have (g ∘ f) (G) = g(f(G)) = g(W), where W = f (G). By hypothesis, f(G) is rgwα-open in (Y, σ) and so again by hypothesis, g(f (G)) is a rgwα-open set in (Z, η). That is (g ∘ f) (G) is a rgwα-open set in (Z, η) and therefore $(g \circ f)^{-1}$ is rgwα-irresolute. Also g ∘f is a bijection. Hence g ∘ f is rgwα*-homeomorphism.

Theorem 3.33: The set rgwα*-h(X, τ) is a group under the composition of maps.
Proof: Define a binary operation ψ: rgwα*-h(X, τ) × rgwα*-h(X, τ) → rgwα*- h(X, τ) by f ∗ g = g ∘ f for all f, g ∈ rgwα*h(X, τ) and ∘ is the usual operation of composition of maps. Then by Theorem 3.7, g ∘ f ∈ rgwα*-h(X, τ). We know that the composition of maps is associative and the identity map I: (X, τ) → (X, τ) belonging to rgwα*-h(X, τ) serves as the identity element.
If f ∈ rgwα*-h(X, τ), then f^{-1} ∈ rgwα*-h(X, τ) such that $f \circ f^{-1} = f^{-1} \circ f = I$ and so inverse exists for each element of rgwα*-h(X, τ). Therefore (rgwα*- h(X, τ), ∘) is a group under the operation of composition of maps.

Theorem 3.34: Let f: (X, τ) → (Y, σ) be a rgwα*-homeomorphism. Then f induces an isomorphism from the group rgwα*-h(X, τ) onto the group rgwα*-h(Y, σ).
Proof: Using the map f, we define a map Ψ_f : rgwα*-h(X, τ) → rgwα*-h(Y, σ) by $\Psi_f(h) = f \circ h \circ f^{-1}$ for every h ∈ rgwα*-h(X, τ). Then Ψ_f is a bijection. Further, for all h_1, h_2 ∈ rgwα*-h(X,τ), $\Psi_f(h_1 \circ h_2) = f \circ (h_1 \circ h_2) \circ f^{-1} = (f \circ h_1 \circ f^{-1}) \circ (f \circ h_2 \circ f^{-1}) = \Psi f(h_1) \circ \Psi_f(h_2)$. Therefore Ψ_f is a homeomorphism and so it is an isomorphism induced by f.

Theorem 3.35: If f : (X, τ) → (Y, σ) is homeomorphism, then there exists isomorphism between rgwα− h(X, τ) and rgw−h(Y, σ) i.e. rgwα−h(X, τ) ≅ rgwα − h(Y, σ).
Proof: Using the map f, we define a map ψ_f : rgwα− h(X, τ) → rgwα− h(Y, σ) by $\psi_f(h) = f\ ohof^1$ for every h ∈ rgwα− h(X, τ). Then ψ_f is bijection. Further, for all h_1, h_2 ∈ ψ_f, $\psi_f(h_1oh_2) = f\ o(h_1\ oh_2)of^{-1} = (f\ oh_1of^{-1})o(f\ oh_2\ of^{-1}) = \psi_f(h_1)o\psi_f(h_2)$. Therefore ψ_f is a homeomorphism and so it is an isomorphism induced by f.

Definition 3.36: A function f: (X, τ) → (Y, σ) is said to **contra rgwα-irresolute** if f^{-1} (V) is rgwα-closed in (X, τ) for every rgwα-open set V of (Y, σ).

Lemma 3.37: Let two function f : (X, τ) → (Y, σ) and g : (Y, σ) → (Z, φ) defined on topological spaces (X, τ) and (Y, σ) respectively, then
(1) If functions f and g are contra rgwα-irresolute, then the composition gof is also rgwα-irresolute.
(2) If function f is rgwα-irresolute (resp. contra rgwα-irresolute) and g are contra rgwα-irresolute (resp. rgwα-irresolute), then the composition function gof is contra rgwα-irresolute.

Definition 3.38: A bijection f: (X, τ) → (Y, σ) is called contra regular generalized weakly α-homeomorphism (briefly, conra rgwα-homeomorphism) if f is contra rgwα-irresolute and f^{-1} are rgwα-irresolute.

The family of all contra rgwα-homeomorphism of a topological space (X, τ) onto itself is denoted by con- rgwα-h(X, τ). i. e.

Con-rgwα-h(X, τ) = {f: f (X, τ) → (X, τ) is a contra rgwα-irresolute bijection and f^{-1} is rgwα-irresolute}

For a topological space (X, τ), we construct alternative groups, say rgwα-h(X, τ) ∪ con-rgwα-h(X, τ).

Theorem 3.39: If (X, τ) be a topological space, then union of two collections, rgwα-h(X, τ) ∪ con− rgwα-h(X, τ), forms a group under the composition of functions.

Proof: Let us B_X = rgwα-h(X, τ) ∪ con− rgwα-h(X, τ). A binary operation $w_X : B_X \times B_X \to B_X$ is well defined by w_X (a, b) = boa, where boa : X → X is the composite function of the functions a and b. Indeed, let (a, b) ∈ B_X ; if a ∈ rgwα-h(X, τ) and b∈ con−rgwα-h(X, τ), then boa : (X, τ)→ (X, τ) a contra rgwα-irresolute bijection and $(boa)^{-1}$ is also contra rgwα-irresolute and so w_X (a, b) = boa ∈ rgwα-h(X, τ) ⊂ B_X, if a ∈ rgwα-h(X, τ) and b ∈ rgwα-h(X, τ) then boa : (X, τ) → (X, τ) is a rgwα-irresolute bijection and so w_X (a, b) = boa ∈ rgwα-h(X, τ) ⊆ B_X, if a ∈ con − rgwα-h(X, τ) and b ∈ con − rgwα-h(X, τ), then boa: (X, τ) → (X, τ) is a rgwα-irresolute bijection and $(boa)^{-1}$ is also rgwα-irresolute and so w_X (a, b) = boa ∈rgwα-h(X, τ) ⊂ B_X is a ∈ con−rgwα-h(X, τ) and b ∈ rgwα-h(X, τ) then boa: (X, τ) → (X, τ) is a contra rgwα-irresolute bijection and $(boa)^{-1}$ is also rgwα-irresolute and and so w_X (a, b) = boa ∈ con − rgwα-h(X, τ) ⊆ B_X. By the similar arguments, it is claimed that the binary operation w_X: $B_X \times B_X \to B_X$ satisfies the axiom of group; for the identity element e of B_X, e = 1_X : (X, τ) → (X, τ). Thus the pair (B_X, w_X) forms a group under the composition of functions, i.e., rgwα-h(X, τ) ∪ con − rgwα-h(X, τ) is a group.

Theorem 3.40: The homeomorphism group h(X, τ) is a subgroup of rgwα-h(X, τ) ∪ con −rgwα-h(X, τ).

Proof: By Theorem 3.11, it can be show that h(X, τ) is a subgroup of rgw-h(X, τ) ∪ con −rgw-h(X, τ).

Theorem 3.41: Let f : (X, τ) → (Y, σ) is rgwα-continuous and collection of functions f is defined as G_g (f) = {(x, y) ∈ X × Y : y = f (x)}, where X × Y is product topological space and G_g (f) is called rgwα-graph f. Then the following properties are satisfied:

(1) G_g (f), as a subspace of X × Y, rgwα-homeomorphism to X.

(2) If Y is rgwα-Housdrofff space, then G_g (f) is rgwα-closed in X × Y.

Proof:

(1) Consider the function g: X → G_g (f) is defined by g(x) = (x, f (x)) for each x ∈ X is rgwα-continuous and g^1 is also rgwα-continuous. It is obvious that g is an injective function. Let P and Q are an arbitrary neighbourhood x ∈ X and (x, f (x)) in G_g(f) respectively. So, there exists two rgwα-open sets U and V in X and Y respectively containing x and f (x) for which (U × V) ∩ G_g (f) ⊂ E and U ⊂ P and f (U) ⊂ V. Let N = (U × V) ∩ G_g (f), then (x, f (x)) ∈ N and x ∈ g^{-1} (N) ⊂ U ⊂ P. This shows that g^{-1} is rgwα-continuous. Therefore, g(U) ⊂ (U × V) ∩ G_g (f) ⊂ Q. Hence g is rgwα-continuous which means that g is a rgwα-homeomorphism.

(2) Let (x, y) ∉ Gg (f). Then y_1 = f (x) = y. By hypothesis, there exist disjoint rgwα-open sets V_1 and V in Y such that y_1 ∈V_1, y ∈ V. Since f is rgwα-continuous, there exists an open set U in X containing x such that f (U) ⊂ V_1. Then g(U) ⊂ U × V_1. It follows from this and the fact that V_1 ∩ V = φ that (U × V) ∩ G_g (f) = φ.

4. Conclusion

In this paper we have introduced and studied the properties of rgwα-Homeomorphism and rgwα*-Homeomorphisms. Our future extension is to study Locally rgwα-Homeomorphism and Locally rgwα*- Homeomorphism in Topological Spaces.

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