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Comparative analysis of different methods for solving fuzzy transportation problem

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Abstract
A variety of methods are proposed for solving fuzzy transportation problems but all the methods in literature, demonstrate the parameters by normal fuzzy numbers. In most of the work it is quite impossible to limit the membership function to the normal form. So in this paper a generalized approach for fuzzy numbers for transportation problem is proposed. In proposed work, a new approach is presented for solving real life fuzzy transportation problems by assuming that a decision maker is uncertain about the accurate values of the demand, availability as well as the transportation cost of the given product. In this paper Vogel's Approximation Method (VAM) is introduced which provide optimum initial solution rather than northwest corner method and intuitive low cost method. In the proposed method demand, availability as well as the transportation cost of the product are represented by generalized trapezoidal fuzzy numbers. The proposed VAM method is very easy to understand and easy to apply on real life fuzzy transportation problems for the decision makers.

Keywords: Ranking function, fuzzy numbers, Vogel's Approximation Method (VAM)

Introduction

In the field of either mathematics or economics, transportation fundamentals is a name assigned to the study of optimal transportation and allocation of resources. Transportation theory is a platform to provide a robust framework to meet the deadlines. The fundamental transportation problem was first developed by Hitchcock [1]. Transportation problem is a special kind of linear programming problem (LPP) in which goods are transported from a set of sources to a set of destinations subject to the supply and demand of the sources and destinations respectively [2]. Such that the total cost of transportation is minimized. There are two types of transportation problem: one is balanced and other one is unbalanced. When demand and supply are equal, problem is balanced otherwise unbalanced.

In general, transportation LPP is simplified with the assumption that the decision parameters such as unit transportation cost, requirement and availability. But in day to day life applications, stock, requirement and unit transportation cost may be uncertain due to various factors [3]. The estimate data may be represented by fuzzy numbers or in terms of crisp values. Fuzzy numbers may be normal, general, triangular and trapezoidal. Ranking method is used to change the fuzzy number into crisp form. In daily life, there are many differing situations due to uncertainty in judgments, lack of evidence etc. Sometimes it is quite impossible to get relevant precise data for the cost parameter [4]. This type of uncertain data is never well represented by random variable selected from a probability distribution. Fuzzy number may represent this data. The objective of fuzzy transportation problem is to find the minimum transportation cost of some commodities through a capacitated network when the stock and requirement of nodes and the capacity and cost of edges are represented as fuzzy numbers [6-8].

An initial basic feasible solution (IBFS) for the transportation problem can be procured by using the north-west corner method (NWCM) and least cost method and Vogel's approximation method. VAM is most useful for determining the optimal solution for the given fuzzy transportation problem. A fuzzy transportation problem is a problem in which the transportation expenditures, demand and supply quantities are belongs to fuzzy set. This paper is organized as follows: In Section 2, generalized trapezoidal fuzzy numbers topology is presented. In Section 3, Different methods to find initial basic feasible solution for transportation problem is presented. Also u-v method is described to find the optimal final solution. The conclusions are discussed in Section 4.

Generalized trapezoidal fuzzy number

A new concept for ranking of generalized trapezoidal fuzzy number is presented using trapezoid as reference point. This Ranking methods map fuzzy number directly in to the real line.

Let A= (p1, q1, r1, s1; ω1) and B= (p2, q2, r2, s2; ω2) be two generalized trapezoidal fuzzy numbers then the following steps to differentiate A and B:

Step 1: Find ω = minimum (ω1, ω2)

Step2: Find R (A) =ω [(p1+q1+r1+s1)/4] and R (B) =ω [(p2+q2+r2+s2)/4].

Case 1: If R (A) > R (B), then A>B i.e. minimum (A, B) =B

Case 2: If R (A) < R (B), then A<B i.e. minimum (A, B) =A

Case 1: If R (A) = R (B), then A=B i.e. minimum (A, B) =A=B

Different method

Any optimize solution for a given transportation problem is solved in two phases:

1. Finding the initial basic feasible solution in first phase.
2. Second phase involves optimization of the initial basic feasible solution (IBFS) obtained in phase 1.

In order to find the initial basic feasible solution we can use North-west corner cell method, least cost cell method and Vogel's approximation method (VAM).In order to optimizing initial basic feasible solution u-v method is used in this paper.

Problem: For the balanced fuzzy transportation problem given below, find the fuzzy quantity of the product transported from each source to various destinations so that the total fuzzy transportation cost is minimum.

		Destination				Supply
		B1	B2	B3	B4	
Source	A1	3	1	7	4	250
	A2	2	6	5	9	350
	A3	8	3	3	2	400
Demand		200	300	350	150	

The above problem can be solved by above mentioned three methods in order to get IBFS.

Method 1: North-west Corner Method

Firstly, it is cleared that the problem should be balanced otherwise it is made balanced problem by adding dummy row as well column. But here no need to add any dummy cell.

200 / 3	50 / 1	7	4	250	0
2	250 / 6	100 / 5	9	350	0
8	3	250 / 3	150 / 2	400	0
200	300	350	150		
0	0	0	0		

Fig 1: Problem solved by north-west corner method

Total cost of transportation is = (200*3) + (50*1) + (250*6) + (100*5) + (250*3) + (150*2) = 3700 RS.

To optimize this initial basic solution u-v method is applied over this problem. First find out the u values (u1, u2, u3) for rows and v values (v1, v2, v3, v4) for columns.

Mathematical formula to find u-v values:

Step 1: $U_i + V_j = C_{ij}$ and m+n-1= No. of allocated cells=>3+4-1=6

Assume u1=0 & Cij values are given in problem (3, 1, 6, 5, 3, and 2). By using the above relations values are u1=0, v1=3, v2=1, u2=5, v3=0, u3=3, v4=-1.

Step 2: Calculate penalties for unallocated cell $P_{ij}=U_i + V_j - C_{ij}$

	v1=3	v2=1	v3=0	v4=-1		
u1=0	200 (-) / 3	50 (+) / 1	7	4	250	0
u2=5	(+) 2	250 (-) / 6	100 / 5	9	350	0
u3=3	8	3	250 / 3	150 / 2	400	0
	200	300	350	150		
	0	0	0	0		

C13=0+0-7=-7, C14=0-1-4=-5, C21=5+3-2=6, C24=5-1-9=-5, C31=3+3-8=-2, C32=3+1-3=1

Step 3: If we get zero or less than zero, optimality is reaching and find out the maximum value among all Cij. So C21 gives the maximum positive value and it is the starting point of basic cell. We draw a closed path from this cell and assign negative & positive values shown in above figure. Now by adding and subtracting the new values matrix is shown below. Also repeat step 1 & 2.

C11=0-3-3=-6, C13=0+0-7=-7, C14=0-1-4=-5, C31=3-3-8=-8, C32=3+1-3=1

	v1=-3	v2=1	v3=0	v4=-1		
u1=0	3	250 / 1	7	4	250	0
u2=5	200 / 2	50 (-) / 6	100 (+) / 5	9	350	0
u3=3	8	(+) 3	250 (-) / 3	150 / 2	400	0
	200	300	350	150		
	0	0	0	0		

Step 4: Again with the same procedure calculated new Cij values are

C11=0-2-3=-5, C13=0+1-7=-6, C14=0+0-4=-4, C22=4+1-6=-1, C24=4+0-9=-5, C31=2-2-8=-8. Now all values are negative optimality is reached. So total cost can be calculated by this new matrix.

	v1=-2	v2=1	v3=1	v4=0
u1=0	3	250 / 1	7	4
u2=4	200 / 2	6	150 / 5	9
u3=2	8	50 / 3	200 / 3	150 / 2

Total transportation cost is= (250*1) + (200*2) + (150*5) + (50*3) + (200*3) + (150*2) = 2450 RS.

Method 2: Least Cost Method

The Least Cost Method (LCM) is second method used to obtain the initial basic feasible solution for the given transportation problem. Here, the allocation begins with the cell which has the minimum cost. The lower cost cells are chosen over the highercost cell with the aim to have the least cost of transportation.

3	250 / 1	7	4	250	0
200 / 2	50 / 6	100 / 5	9	350	0
8	3	250 / 3	150 / 2	400	0
200	300	350	150		
0	0	0	0		

Total transportation cost= (250*1) + (200*2) + (50*6) + (100*5) + (250*3) + (150*2) = 2500 RS.

Method 3: Vogel's Approximation Method:

The difference of minimum cost and next to minimum cost against each row is known as penalty.

Calculate the maximum penalty and find cell with minimum cost to assign maximum possible unit.

	Destination				Supply	Penalty				
Source	3	250 / 1	7	4	250	2	3			
	200 / 2	6	150 / 5	9	350	3	1	1	1	5
	8	50 / 3	200 / 3	150 / 2	400	1	1	1	0	3
Demand	200	300	350	150						
Penalty	1	2	2	2						
		2	2	2						
		3	2	7						
		3	2							
			2							

Total cost is = (250*1) + (200*2) + (150*5) + (50*3) + (200*3) + (150*2) = 2450 RS.

Conclusion

In this proposed work, the transportation costs are taken as generalized fuzzy numbers. Mathematical formulation of fuzzy transportation problem is discussed with suitable numerical example for different basic feasible solution finding methods. Numerical example shows that using this formulation one can have the optimal solution as well as the crisp and fuzzy optimal total cost with a particular method. To illustrate the north-west corner method, least cost method and VAM method a numerical example is solved and transportation cost is calculated. All three methods give total transportation cost are 3700, 2500 and 2450 respectively. In all above three methods which provides the initial basic feasible solution, VAM is quite better to calculate total cost. But to calculate the optimize transportation cost, here a u-v method is introduced. By applying u-v method over north-west corner method we reduce the cost from 3700 Rs to 2450 Rs. It is concluded that if u-v method is applied over VAM method, we can further reduce its transportation cost from 2450 Rs. However, believing that the results obtained in this paper gives us the optimum cost for the fuzzy transportation problems.

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