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Subhash Kumar Sharma
Assistant Professor, Department of Electronics, MGPG College, Gorakhpur, Uttar Pradesh, India

Umesh Kumar Gupta
Associate Professor, Department Of Mathematics, MGPG College, Gorakhpur, Uttar Pradesh, India

Correspondence
Subhash Kumar Sharma
Assistant Professor, Department of Electronics, MGPG College, Gorakhpur, Uttar Pradesh, India

An approach to mathematical modelling of mechanical and electrical systems and engineering

Subhash Kumar Sharma and Umesh Kumar Gupta

Abstract
This paper explains electrical, mechanical and electromechanical systems. In case of system Mathematical model plays an important role to give response. In Accordance of it examples of Mechanical, Electrical systems are represented by mathematical model; In different types of Mathematical model i.e. Mechanical System by Differential Equation Model, Electrical system by State-Space Model and other System by Transfer Function Model.

Keywords: mathematical modelling, electrical, mechanical and electromechanical systems and their behaviour

Introduction

What is mathematical modelling? Models describe our beliefs about how the world functions. In mathematical modelling, we translate those beliefs into the language of mathematics. This has many advantages

1. Mathematics is a very precise language. This helps us to formulate ideas and identify underlying assumptions.
2. Mathematics is a concise language, with well-defined rules for manipulations.
3. All the results that mathematicians have proved over hundreds of years are at our disposal.
4. Computers can be used to perform numerical calculations.

There is a large element of compromise in mathematical modelling. The majority of interacting systems in the real world are far too complicated to model in their entirety. Hence the first level of compromise is to identify the most important parts of the system. These will be included in the model, the rest will be excluded. The second level of compromise concerns the amount of mathematical manipulation which is worthwhile. Although mathematics has the potential to prove general results, these results depend critically on the form of equations used. Small changes in the structure of equations may require enormous changes in the mathematical methods. Using computers to handle the model equations may never lead to elegant results, but it is much more robust against alterations.

What objectives can modelling achieve?

Mathematical modelling can be used for a number of different reasons. How well any particular objective is achieved depends on both the state of knowledge about a system and how well the modelling is done. Examples of the range of objectives are:

1. Developing scientific understanding - through quantitative expression of current knowledge of a system (as well as displaying what we know, this may also show up what we do not know);
2. test the effect of changes in a system;
3. aid decision making, including

(i) tactical decisions by managers;
(ii) Strategic decisions by planners.

Classifications of models: When studying models, it is helpful to identify broad categories of models. Classification of individual models into these categories tells us immediately some of the essentials of their structure. One division between models is based on the type of outcome they predict. Deterministic models ignore random variation, and so always predict the same outcome from a given starting point. On the other hand, the model may be more statistical in nature and so may predict the distribution of possible outcomes. Such models are said to be stochastic. 1 A second method of distinguishing between types of models is to consider the level of understanding on which the model is based. The simplest explanation is to consider the hierarchy of organisational structures within the system being modelled. For animals, one such hierarchy is:

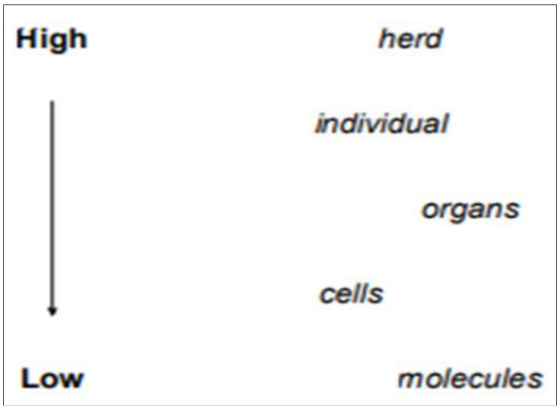


Fig 1: Hierarchy for animals

A model which uses a large amount of theoretical information generally describes what happens at one level in the hierarchy by considering processes at lower levels these are called mechanistic models, because they take account of the mechanisms through which changes occur. In empirical models, no account is taken of the mechanism by which changes to the system occur. Instead, it is merely noted that they do occur, and the model trys to account quantitatively for changes associated with different conditions. The two divisions above, namely deterministic/stochastic and mechanistic/empirical, represent extremes of a range of model types. In between lie a whole spectrum of model types. Also, the two methods of classification are complementary. For example, a deterministic model may be either mechanistic or empirical (but not stochastic). Examples of the four broad categories of models implied by the above method of classification are:

Table 1: A deterministic model may be either mechanistic or empirical (but not stochastic). Examples of the four broad categories of models implied by the above method of classification are:

	Empirical	Mechanistic
Deterministic	Predicting cattle growth from a **regression relationship** with feed intake	Planetary motion, based on Newtonian mechanics (**differential equations**)
Stochastic	Analysis of **variance** of variety yields over sites and years	Genetics of small populations based on Mendelian inheritance (**probabalistic equations**)

One further type of model, the system model, is worthy of mention. This is built from a series of sub-models, each of which describes the essence of some interacting components. The above method of classification then refers more properly to the sub-models: different types of sub-models may be used in any one system model. Much of the modelling literature refers to ’simulation models’. Why they are not included in the classification? The reason for this apparent omission is that ’simulation’ refers to the way the model calculations are done - i.e. by computer simulation. The actual model of the system is not changed by the way in which the necessary mathematics is performed, although our interpretation of the model may depend on the numerical accuracy of any approximations.

Stages of modelling

It is helpful to divide up the process of modelling into four broad categories of activity, namely building, studying, testing and use. Although it might be nice to think that modelling projects progress smoothly 2 from building through to use, this is hardly ever the case. In general, defects found at the studying and testing stages are corrected by returning to the building stage. Note that if any changes are made to the model, then the studying and testing stages must be repeated. A pictorial representation of potential routes through the stages of modelling is:

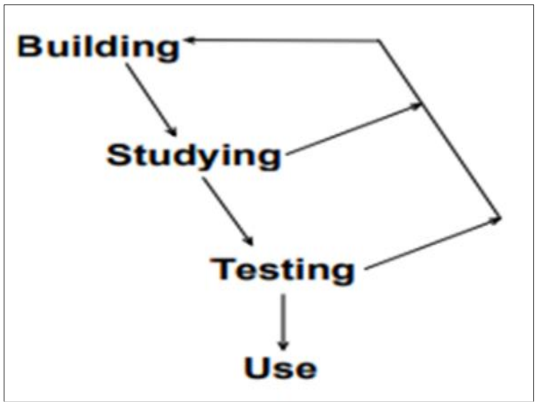


Fig 2: A pictorial representation of potential routes through the stages

System is used to describe a combination of component which may be physical or may not. Mathematical model describes the system in terms of mathematical concept. The process of developing mathematical Model is known as Mathematical Modelling. Modelling is the process of writing a differential equation to describe a physical situation. The basis for mathematical model is provided by the fundamental physical laws that govern the behaviour of system. It uses laws like Kirchhoff’s law for electrical system, Newton’s law for mechanical system. Modelling of any system can help us to study effect of different of component and to make Prediction about Behaviour. Modelling can be divided into two parts i.e. First Principle Model and empirical model given in figure below.

- First principle model that seeks to calculate a physical quantity starting directly from established laws of physics without making any assumptions. Example-Electronic structure of atoms
- An Empirical modelling refers to any kind of modelling based on empirical observations rather than mathematically describable relationships of the system modelled

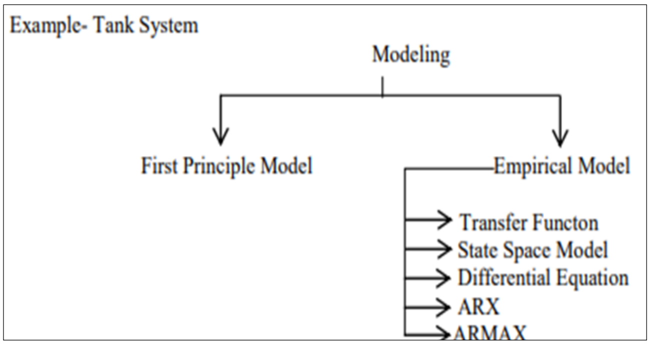


Fig 3: Gives types and subtypes of modelling

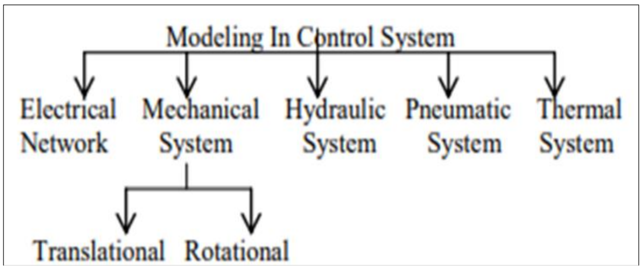


Fig 4: Types of control system are given in

Electrical System

The Electrical System is made with element like Resistor, Capacitor and Inductor we can use mathematical modelling to check the behavior of any electrical system. 1. Resistor: Resistor is a passive two-terminal electrical component that implements electrical resistance as a circuit element. The current through a resistor is in direct proportion to the voltage across the resistor's terminals. This relationship is represented by Ohm's law:

$$I = \frac{V}{R}$$

Where I is the current through the conductor in units of amperes, V is the potential difference measured across the conductor in units of volts, and R is the resistance of the conductor in units of ohms. The ratio of the voltage applied across a resistor's terminals to the intensity of current in the circuit is called its resistance, and this can be assumed to be a constant (independent of the voltage) for ordinary resistors working within their ratings.

Fig 5: Resistor

Capacitor: Capacitor is passive electrical component used to store energy electrostatically in an electric field. The forms of practical capacitors vary widely, but all contain at least two electrical conductors separated by a dielectric; for example, one common construction consists of metal foils separated by a thin layer of insulating film. When there is a potential difference across the conductors, an electric field develops across the dielectric, causing positive charge to collect on one plate and negative charge on the other plate. Energy is stored in the electrostatic field.

Fig 6: Capacitor

Voltage across Resistor given by

$$\frac{1}{C}\int I_C\,dt$$

Inductor: An inductor is characterized by its inductance, the ratio of the voltage to the rate of change of current, which has units of Henry (H). Many inductors have a magnetic core made of iron or ferrite inside the coil, which serves to increase the magnetic field and thus the inductance.

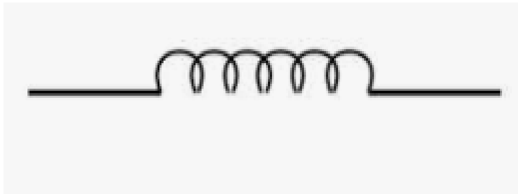


Fig 7: Inductor

Current I_L through inductor is given by

$$\frac{1}{L}\int E_L\,dt$$

Example of Electrical System

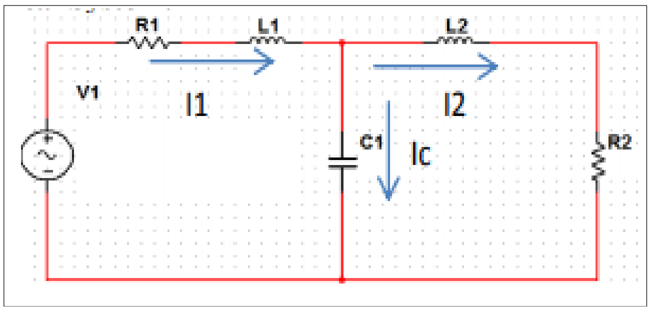


Fig 8: Electrical System

By Applying KCL $I_C = I_1 - I_2$

$$C\frac{dV}{dt} = I_1 - I_2$$

By Applying KVL at inductor 1

$$V_i = R_1 I_1 + L_1\frac{dI_1}{dt} + V_c$$

$$\frac{dI_1}{dt} = \frac{1}{L_1}(V - R_1 I_1 - V_c)$$

By Applying KVL At Inductor 2

$$V_c = L_2 \frac{dI_2}{dt} + R_2 I_2$$

$$\frac{dI_2}{dt} = \frac{1}{L_2}(V_c - R_2 I_2)$$

State Space Representation is given by

$$\begin{bmatrix} \dot{V_c} \\ \dot{I_1} \\ \dot{I_2} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{C1} & \frac{1}{C1} \\ -\frac{1}{L1} & -\frac{R1}{C1} & 0 \\ \frac{1}{L2} & 0 & \frac{R2}{L2} \end{bmatrix} \begin{bmatrix} V_c \\ I_1 \\ I_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L1} \\ 0 \end{bmatrix} [V_i]$$

$$V_o = [0\ 0\ R2] \begin{bmatrix} V_c \\ I_1 \\ I_2 \end{bmatrix}$$

Mechanical System

Elements of Mechanical System

1. Mass

- A Force applied to the mass produces an acceleration of the mass.
- The reaction force fm is equal to the product of mass and acceleration and is opposite in direction to the applied force in term of displacement y, a velocity v, and acceleration a, the force equation is

$$F = Ma = M\dot{v} = M\ddot{x}$$

2. Spring

- The reaction force F on each end of the spring is the same and is equal to the product of stiffness k and the amount of deformation of the spring.
- End C has a position Yc and end D has a position Yd measured from the respective equilibrium positions. The force equation, in accordance with the Hok's law is F = k (Yc -Yd)
- If the end D is stationary, then Yd = 0 and the above equation reduces to F = kYc

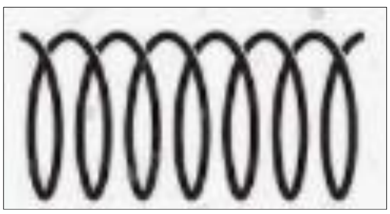


Fig 9: Spring

3. Damper

- The reaction damping force FB is approximated by the product of damping B and the relative velocity of the two ends of the dashpot.
- The direction of this force depend on the relative magnitude and direction of the velocity Dye and Dyf.

$$F_b = B(V_e - V_f) = B(\dot{D}_{y_e} - \dot{D}_{y_f})$$

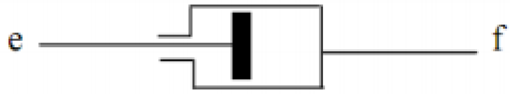
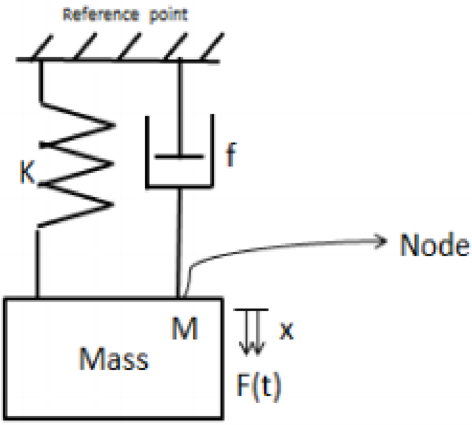


Fig 9: Damper

Mechanical System Example

It is required to set up equation for system on application of force F(t) to mass m. The resulting displacement of mass being x

$$\text{Inirtial force} = M\frac{d^2x}{dt^2}$$
$$\text{Viscous Force} = f\frac{dx}{dt}$$
$$\text{Spring Restoring Force} = Kx$$



By Applying Nodal Analysis We Get

$$M\frac{d^2x}{dt^2} + f\frac{dx}{dt} + kx = f(t)$$

By Assuming initial conditions are zero and taking Laplace Transforms We have

$$Ms^2X(s) + fsX(s) + kX(s) = F(s)$$

Transform Equation can be given as

$$G(s) = \frac{1}{Ms^2 + fs + K}$$

Transfer function

It has been shown already that the input and output of a linear system in general, is related by a linear or a set of linear differential equations. Such relationships are capable of completely describing the system behaviour in the presence of a particular input excitation and known initial conditions.

Differential equation of Eq. (A) is seldom used in its original form for the analysis and design of control systems. To obtain the transfer function of the linear system that is represented by Eq.(A), we simply take the Laplace transform on both sides of the equation, and assume zero initial conditions. The result is

The transfer function between r(t) and c(t) is given by

$$(s^n + a_n s^{n-1} + \ldots + a_2 s + a_1)C(s) = (b_{m+1}s^m + b_m s^{m-1} + \ldots + b_2 s + b_1)R(s) \quad \textbf{(A)}$$

$$G(s) = \frac{C(s)}{R(s)} = \frac{b_{m+1}s^m + b_m s^{m-1} + \ldots + b_2 s + b_1}{s^n + a_n s^{n-1} + \ldots + a_2 s + a_1} \quad \textbf{(B)}$$

We can summarize the properties of the transfer function as follows:

1. Transfer function is defined only for a linear time-invariant system. It is meaningless for nonlinear systems.
2. The transfer function between an input variable and an output variable of a system is defined as the Laplace transform of the impulse response. Alternately, the transfer function between a pair of input and output variables is the ratio of the Laplace transform of the output to the Laplace transform of the input.
3. When defining the transfer function, all initial conditions of the system are set to zero.
4. The transfer function is independent of the input of the system.
5. Transfer function is expressed only as a function of the complex variable s. It is not a function of the real variable, time, or any other variable that is used as the independent variable.

Transfer Function (Multivariable Systems)

The definition of transfer function is easily extended to a system with a multiple number of inputs and outputs. A system of this type is often referred to as the multivariable system. In a multivariable system, a differential equation of the form of Eq. (A) may be used to describe the relationship between a pair of input and output variables. When dealing with the relationship between one input and one output, it is assumed that all other inputs are set to zero. Since the principle of superposition is valid for linear systems, the total effect on any output variable due to all the inputs acting simultaneously is obtained by adding up the outputs due to each input acting alone. A number of examples is appropriate to illustrate the concept of transfer function.

Conclusion

In Order to understand the behaviour of systems, Mathematical Models are needed. These are simplified representations of certain aspects of real system. Such a model is created using equations to describe the relationship between input and output of system and can then be used to enable prediction to be made of the behaviour of a system under specific condition.

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References

1. Zibo Z, Naghdy F. Application of Genetic Algorithms to System Identification, IEEE Int. Conf. On Evolutionary Computation 1995;2:777-787.
2. Vos P. Assessments of applied mathematics and modelling: using a laboratory like environment. In W. Blum, P. L. Galbraith, H. Henn, & M. Niss (Eds.), Modelling and applications in mathematics education. The 14th ICMI study. New York: Springer 2007, 441-448.
3. Vos P. What is ‘authentic’ in the teaching and learning of mathematical modelling? In G. Kaiser, W. Blum, R. Borromeo Ferri, & G. Stillman (Eds.), Trends in Teaching and Learning of Mathematical Modelling. New York: Springer 2011, 713-722.
4. Vos P. Assessment of modelling in mathematics examination papers: ready-made models and reproductive mathematising, In G. Stillman, G. Kaiser, W. Blum, & J. Brown (Eds.), teaching mathematical modelling: Connecting to research and practice (pp. 479-488). Springer: New York 2013.
5. Wagner J. The unavoidable intervention of educational research: A framework for reconsidering researcher-practitioner cooperation. Educational Researcher 1997;26(7):13-22.
6. Wake G. Considering workplace activity from a mathematical modelling perspective. In W. Blum, P. L. Galbraith, H.-W. Henn, & M. Niss (Eds.), Modelling and applications in mathematics education. New York, NY: Springer 2007, 395-402.
7. Blomhøj M. Mathematical modeling: A theory for practice. In B. Clarke *et al*. (Eds) International perspectives on learning and teaching mathematics. Sweden: National Center for Mathematics Education 2004, 145-159.
8. Blomhøj M, Højgaard Jensen T. Developing mathematical modeling competence: Conceptual clarification and educational planning. Teaching Mathematics and its Applications 2003;22(3):123-138.
9. Blum W, Galbraith P, Henn H, Niss M (Ed.). Modelling and applications in mathematics education—The 14th ICMI Study. New York: Springer 2007.
10. Blum W *et al*. ICMI Study 14: Applications and modelling in mathematics education—Discussion Document. Educational Studies in Mathematics 2003;51:149-171.
11. Borba M, Bovo A. Modelagem em sala de aula de matema´tica: interdisciplinaridade e pesquisa em biologia. Revista de 2002;8(6-7):27-33.
12. Borba M, Villarreal M. Humans-with-media and the reorganization of mathematical thinking: Information and communication technologies, modelling, experimentation and visualization. New York: Springer Science? Business Media (Mathematics Education Library) 2005.
13. Nagrath and Gopal, Control System Engineering.
14. W. Bolton, Mechatronics, Pearson Publication.
15. R.K. Bansal, Matlab and its application in engineering, Pearson Publication.
16. Dr. Imtiaz Hussain, Mathematical Modelling of Liquid Level Systems.