Design of acceptance sampling plan for life tests based on percentiles using Weibull-Poisson distribution

V Kaviyarasu and P Fawaz

Abstract
In this article a new acceptance sampling procedure is developed for the Weibull Poisson Distribution (WPD). This study deals Weibull-Poisson with three parameter distribution, when the life test is truncated at a specified life time percentile. The sample size, OC values, OC Curves and Producer’s ratios were computed. The plan parameters and their measures are studied in the proposed continuous probability distribution which suited to manufacturing industries towards the selection of samples. Few illustrations were provided for the readymade selection of plan and tables are developed for the reliability sampling plan.

Keywords: Producer’s risk, truncated life test, operating characteristic curve

1. Introduction
In this modern era, quality plays a major role to improve the customer satisfaction and their loyalty towards retaining. A Quality process is the transformation of input into output to maintain or improve the products quality. Acceptance sampling (AS) is one of the major areas of Statistical Quality Control, which is used to sentence about the lot quality. According to AS is a sampling technique from which a decision is made whether to accept or reject a lot based on the sample inspection. It is generally categories as attribute and variable sampling plan. Attribute sampling plan is used when the quality characteristic s involved as attribute in nature whereas, the quality characteristics are measured on a continuous scale then such a sampling plan is called as variable sampling plan.

In certain circumstances, the experiments were carried out to check weather lifetime of a product may follow a life test experiments. A sampling technique used to sentence about the disposition of lots based on the product lifetime such a sampling plan are called as Reliability Sampling Plan. Here a notable characteristic of a plan may follows a lifetime random variable, which can be a continuous in nature such as exponential, weibull, lognormal etc., it can be used to determine the life of the product.


Percentiles gives more information regarding a lifetime distributions than the mean lifetime. When the lifetime distributions is symmetric, the 50th percentile or the median in case of skewed lifetime distributions and more generalized measure in case of symmetrical distributions is equivalent to the mean life. Hence, developing acceptance sampling plans based on percentiles of a life distribution can be treated as a generalization of developing acceptance sampling plans based on the mean life of items.
Balakrishnan et al. (2007) [4] considered the problem of acceptance sampling assuming that the lifetime of a product follows the generalized BS distribution. But Lio et al. (2009) [12] explained the sampling plans based on mean life may not satisfies the consumers’ expectations on quality, since small difference in mean may cause a significant change in variance, but it can take the experiment to a lower percentile than the true one, that is when the mean life is considered only few lots can be accepted with a small change in mean and that does not ensures the consumers’ expectations. Hence the percentiles are highly considerable by the engineers. According to Rao, and Kantam (2010) [14] AS plans for truncated life tests based on the log-logistic distribution for percentiles. Rao et al. (2012) [15] also developed AS plans for percentiles based on the Inverse Rayleigh Distribution. Rao et al. (2014) [13] further proposed AS plans based on the percentiles of Exponentiated half log logistic distribution. Kaviyarasu and Fawaz (2017) [11] developed Certain Studies on Acceptance Sampling Plans for Percentiles Based on the Modified Weibull Distribution.

In this paper a life time of the product is assumed to follow Weibull-Poisson Distribution (WPD) which was introduced by Lu and Shi (2012) [17] and the plan parameters were designed and developed for the attribute acceptance sampling. Suitable tables were obtained and values were developed for the producer’s risk is fixed at certain level and OC curve is drawn. Numerical illustrations are given for the better use of this plan.

2. Weibull-Poisson Distribution

Weibull-Poisson distribution is a compound distribution, it contain three Parameters which is used for life testing as the shape of the failure rate is flexible, it can be decreasing, increasing, upside-down bathtub-shaped or unimodel. According to Lu and Shi (2012) [17] the probability density function of WPD is given by,

\[ f(x; \theta) = \frac{\beta \alpha x^{\alpha-1}}{1-e^{-\lambda}} \exp(-\beta e^{\alpha x})e^{-x}, x > 0 \]  

(1)

Where, \( \theta = (\alpha, \beta, \lambda) \), \( \alpha >0 \) is the shape parameter and \( \beta >0 \) is the scale parameter of Weibull distribution and \( \lambda >0 \) is the Poisson parameter. It is noted that, WPD reduces to two parameter Weibull distribution as \( \lambda \) tends to 0 and the density function of WP distribution is monotone decreasing if \( 0 < \alpha \leq 1 \). The mean and variance of WPD is as follows,

\[ E(X) = (e^\lambda - 1)^{-1} \int_0^\lambda e^y \left[ - \beta^{-1} (\log(y) - \log(\lambda)) \right]^{\frac{1}{\alpha}} dy \]

and

\[ Var(X) = (e^\lambda - 1)^{-1} \int_0^\lambda e^y \left[- \beta^{-1} (\log(y) - \log(\lambda)) \right]^{\frac{2}{\alpha}} dy - [E(X)]^2 \]

Further, they derived the properties of WPD and its cumulative distribution function of WPD is given by,

\[ F(x; \theta) = (e^\lambda e^{-\beta x} - e^\lambda)(1 - e^{-\beta})^{-1} \]

Where \( \theta = (\alpha, \beta, \lambda) \)

Percentile Estimator

The 100th percentile or the \( q \)th quantile of any distribution is given by,

\[ P_q(T \leq t_q) = q \]

\[ \Rightarrow t_q = \left\{ - \beta^{-1} \log(\lambda^{-1} \log q(1-e^{-\lambda}) + e^\lambda) \right\}^{1/\alpha} \]

Assuming when \( \alpha = 1 \). Then \( t_q \) becomes \( t_q = \left\{ - \beta^{-1} \log(\lambda^{-1} \log q(1-e^{-\lambda}) + e^\lambda) \right\} \}

\( t_q \) and \( q \) are directly proportional. Let,

\[ \psi = - \log((\lambda^{-1} \log q(1-e^{-\lambda}) + e^\lambda)] \]

\[ t_q = \frac{1}{\beta} \psi \Rightarrow \beta = t_q / \psi \]

(3)

Replacing the scale parameter \( (\beta) \) by (3), one can get the cumulative distribution function of WPD as,

\[ F(t) = (e^\lambda e^{-t/t_q} - e^\lambda)(1 - e^{-\lambda})^{-1}. \quad t > 0, \theta > 0 \]

Letting

\[ \delta = t/t_q \]

\[ F(t; \delta) = (e^\lambda e^{-t/t_q} - e^\lambda)(1 - e^{-\lambda})^{-1}. \quad t > 0, \theta > 0 \]

3. WPD for life testing using Percentiles

AS plan is an inspection procedure used to determine whether to accept or reject a specified lot. Since the success or failure are experienced in frequent tester in larger sized lots and the parameter follows binomial distribution with parameter \( (n, c, p) \). The following assumptions are considered for the construction of ASP through WPD percentiles,

(1) Let the proposed single sampling plan is said to follow binomial distribution with parameter \( (n, c, p) \).
(2) Let $p$ be the failure for the probability observed during specified time $t$ is obtained through, $p = F(t; \delta_0)$

(3) Let $c$ be the acceptance number and the number of failures is less than $c$ for the specified time $t$ the decision is accept the lot and it have, $F(t; \delta) \leq F(t; \delta_0) \Leftrightarrow t_q \geq t_{q_0}$

### 3.1 Designing of AS plan through WPD percentiles for life testing

Single sampling plan (SSP) is the basic and most widely used acceptance sampling plan. In SSP, samples are selected at random from a lot and inspected; if the number of defective is greater than the prefixed acceptance number ($c$) then reject the lot otherwise accept it. This plan is parameterized by the sample size ($n$) and acceptance number ($c$). Since the output is conforming or non-conforming, the random variable follows binomial distribution B ($n, c, p$). The detailed works on determination these plan parameters are given by Schilling and Neubauer (2009) [16].

This procedure is developed for single sampling plan whose parameter $p$ is assumed to follow WPD with parameter $\delta_0=t/tq_0$

where, $t$ and $tq_0$ are the specified test duration and specified 100$q$th percentile respectively.

According to Cameron (1952) [5], the smallest size $n$ can be obtained by satisfying,

$$ \sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} \leq 1 - p^* $$

Where $p^*$ is the probability of rejecting a bad lot and $(1-p^*)$ is the consumer’s risk. Since $p=F(t, \delta_0)$ depends on $\delta_0$, it is sufficient to specify $\delta_0$.

### 3.2 Operating Characteristic Function

The operating characteristic function of the sampling plan $B(n, c, p)$ gives the probability of accepting the lot $L(p)$ with

$$ L(p) = \sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} $$

The producer’s risk ($\alpha$) is the probability of rejecting a lot when $t_q > t_{q_0}$ and for the given producer’s risk ($\alpha$), $p$ as a function of $d_q$ should be evaluated from the condition by Cameron (1952) [5] as

$$ \sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} \geq 1 - \alpha $$

Where $p=F(t, \delta_0)$ and $F(.)$ can be obtained as a function of $d_q$. For the sampling plan developed, the $d_{0.50}$ values are obtained at the producer’s risk $\alpha=0.05$.

### 3.3 Construction of the Table

Step 1: Find the value of $\psi$ by fixing $\lambda=2$ and $q=0.50$.

Step 2: Set the evaluated $\psi$, $c=0$ and $t/tq_0=0.07, 0.09, 0.11, 0.13, 0.15, 0.17, 0.19, 0.21, 0.23$ and 0.25.

Step 3: Find the smallest value of $n$ satisfying

$$ \sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} \leq 1 - p^* $$

Where $p^*$ is the probability of rejecting the bad lot.

Step 4: For the obtained $n$ value find the ratio $d_{0.50}$ such that

$$ \sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} \geq 1 - \alpha $$

Where, $\alpha=0.05$, $p = F(t, \delta_0) = \frac{1}{d_q}$ and $d_q = t_{q_0}/t_q$.

### 3.4 Example

Suppose an experimenter is interested to use the proposed plan for the inspection of the super energy saver LED bulb product. When a lifetime of this product follows the Weibull-Poisson distribution with $\lambda=2$. Experimenter wants to run the experiment for 4000 hrs. Further, the laboratory has the testers to actual percentile life time $t_{0.50}=2000$ hrs, $c=1$, $\alpha=0.05$, $\beta=0.05$, then, $y=0.506763$s calculated from the equation derived under percentile estimator and the ratio, $t_{0.50}=0.07$ from Table 2 the minimum sample size for the given information is obtained as $n=59$. The respective probability of acceptance values $(n, c, t_{0.50}) = (59, 1, 0.07)$ with $p^* = 0.95$ under WPD are given in the following Table 4.

<table>
<thead>
<tr>
<th>$t_q/tq_0$</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
<th>20</th>
<th>24</th>
<th>28</th>
<th>32</th>
</tr>
</thead>
<tbody>
<tr>
<td>L(p)</td>
<td>0.0501</td>
<td>0.3116</td>
<td>0.664</td>
<td>0.8784</td>
<td>0.9386</td>
<td>0.9631</td>
<td>0.9754</td>
<td>0.9825</td>
<td>0.9869</td>
<td>0.9898</td>
</tr>
</tbody>
</table>
Fig 1: Shows the OC curves for the sampling plan \((n, c=1, t/t_{0.500})\) with \(p^*\) = 0.95 for \(\delta = 1\).

This shows that if the actual 50th percentile is equal to the required 10th percentile \((t_{0.1}/t_{0.1}) = 1.00\) the producer’s risk is approximately 0.9499 (1 - 0.0.501). The producer’s risk is almost equal to 0.05 or less when the true 50th percentile is greater than or equal to 12 times the specified 50th percentile.

From Table 3, one get the values of \(d_{0.50}\) for different choices of \(c\) and \(t/t_{0.50}\) in order to assert that the producer’s risk is less than or equals 0.05. In this example, the value of \(d_{0.50}\) should be 8 for \(c = 1, t/t_{0.50} = 0.07\) and \(p^* = 0.95\). This means the product can have a 50th percentile life of 12 times the required 50th percentile in order to find the plan under the above acceptance sampling plan procedure the product is accepted with probability of at least 0.95.

### Table 1: Minimum sample sizes necessary to assert the 50th percentile to exceed a given values, \(t_{q_{0.50}}\), with probability \(P^*\) and the corresponding acceptance number \(c\), for weibull – Poisson distribution using the Binomial Approximation

<table>
<thead>
<tr>
<th>(p^*)</th>
<th>(c)</th>
<th>(0.07)</th>
<th>(0.09)</th>
<th>(0.11)</th>
<th>(0.13)</th>
<th>(0.15)</th>
<th>(0.17)</th>
<th>(0.19)</th>
<th>(0.21)</th>
<th>(0.23)</th>
<th>(0.25)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>0</td>
<td>29</td>
<td>23</td>
<td>18</td>
<td>15</td>
<td>14</td>
<td>12</td>
<td>11</td>
<td>10</td>
<td>9</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>48</td>
<td>38</td>
<td>31</td>
<td>27</td>
<td>23</td>
<td>20</td>
<td>19</td>
<td>17</td>
<td>16</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>66</td>
<td>53</td>
<td>43</td>
<td>36</td>
<td>32</td>
<td>28</td>
<td>25</td>
<td>24</td>
<td>22</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>84</td>
<td>65</td>
<td>54</td>
<td>46</td>
<td>40</td>
<td>36</td>
<td>33</td>
<td>29</td>
<td>27</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>100</td>
<td>79</td>
<td>66</td>
<td>56</td>
<td>48</td>
<td>44</td>
<td>40</td>
<td>36</td>
<td>33</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>117</td>
<td>92</td>
<td>75</td>
<td>65</td>
<td>57</td>
<td>50</td>
<td>45</td>
<td>42</td>
<td>38</td>
<td>36</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>133</td>
<td>105</td>
<td>87</td>
<td>73</td>
<td>65</td>
<td>58</td>
<td>52</td>
<td>48</td>
<td>44</td>
<td>41</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>149</td>
<td>117</td>
<td>96</td>
<td>82</td>
<td>73</td>
<td>64</td>
<td>59</td>
<td>53</td>
<td>49</td>
<td>46</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>165</td>
<td>129</td>
<td>106</td>
<td>91</td>
<td>80</td>
<td>71</td>
<td>64</td>
<td>58</td>
<td>54</td>
<td>51</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>180</td>
<td>141</td>
<td>117</td>
<td>106</td>
<td>90</td>
<td>78</td>
<td>71</td>
<td>64</td>
<td>60</td>
<td>55</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>195</td>
<td>153</td>
<td>127</td>
<td>109</td>
<td>95</td>
<td>85</td>
<td>77</td>
<td>70</td>
<td>65</td>
<td>60</td>
</tr>
<tr>
<td>0.95</td>
<td>0</td>
<td>37</td>
<td>29</td>
<td>24</td>
<td>21</td>
<td>18</td>
<td>15</td>
<td>14</td>
<td>13</td>
<td>12</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>59</td>
<td>46</td>
<td>38</td>
<td>33</td>
<td>29</td>
<td>25</td>
<td>23</td>
<td>21</td>
<td>19</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>79</td>
<td>61</td>
<td>50</td>
<td>43</td>
<td>38</td>
<td>34</td>
<td>30</td>
<td>28</td>
<td>26</td>
<td>24</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>97</td>
<td>76</td>
<td>63</td>
<td>53</td>
<td>46</td>
<td>42</td>
<td>38</td>
<td>35</td>
<td>31</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>115</td>
<td>90</td>
<td>75</td>
<td>64</td>
<td>55</td>
<td>50</td>
<td>44</td>
<td>41</td>
<td>38</td>
<td>35</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>132</td>
<td>104</td>
<td>86</td>
<td>73</td>
<td>64</td>
<td>57</td>
<td>52</td>
<td>47</td>
<td>43</td>
<td>40</td>
</tr>
</tbody>
</table>

"54"
<table>
<thead>
<tr>
<th>$p^*$</th>
<th>0.99</th>
<th>0.95</th>
<th>0.9</th>
<th>0.07</th>
<th>0.09</th>
<th>0.11</th>
<th>0.13</th>
<th>0.15</th>
<th>0.17</th>
<th>0.19</th>
<th>0.21</th>
<th>0.23</th>
<th>0.25</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>693</td>
<td>692</td>
<td>665</td>
<td>459</td>
<td>423</td>
<td>420</td>
<td>388</td>
<td>303</td>
<td>296</td>
<td>232</td>
<td>232</td>
<td>207</td>
<td>191</td>
</tr>
<tr>
<td></td>
<td>693</td>
<td>665</td>
<td>665</td>
<td>423</td>
<td>401</td>
<td>388</td>
<td>332</td>
<td>257</td>
<td>253</td>
<td>200</td>
<td>200</td>
<td>165</td>
<td>149</td>
</tr>
<tr>
<td></td>
<td>693</td>
<td>665</td>
<td>665</td>
<td>459</td>
<td>401</td>
<td>388</td>
<td>296</td>
<td>200</td>
<td>196</td>
<td>133</td>
<td>133</td>
<td>110</td>
<td>94</td>
</tr>
<tr>
<td></td>
<td>693</td>
<td>665</td>
<td>665</td>
<td>665</td>
<td>423</td>
<td>388</td>
<td>303</td>
<td>200</td>
<td>196</td>
<td>133</td>
<td>133</td>
<td>110</td>
<td>94</td>
</tr>
<tr>
<td></td>
<td>693</td>
<td>665</td>
<td>665</td>
<td>665</td>
<td>665</td>
<td>423</td>
<td>388</td>
<td>303</td>
<td>200</td>
<td>196</td>
<td>133</td>
<td>133</td>
<td>110</td>
</tr>
<tr>
<td></td>
<td>693</td>
<td>665</td>
<td>665</td>
<td>665</td>
<td>665</td>
<td>665</td>
<td>423</td>
<td>388</td>
<td>303</td>
<td>200</td>
<td>196</td>
<td>133</td>
<td>133</td>
</tr>
<tr>
<td></td>
<td>693</td>
<td>665</td>
<td>665</td>
<td>665</td>
<td>665</td>
<td>665</td>
<td>665</td>
<td>423</td>
<td>388</td>
<td>303</td>
<td>200</td>
<td>196</td>
<td>133</td>
</tr>
<tr>
<td></td>
<td>693</td>
<td>665</td>
<td>665</td>
<td>665</td>
<td>665</td>
<td>665</td>
<td>665</td>
<td>665</td>
<td>423</td>
<td>388</td>
<td>303</td>
<td>200</td>
<td>196</td>
</tr>
</tbody>
</table>

Table 2: Minimum ratio of true $d_{0.50}$ for the acceptability of a lot for the Weibull–Poisson Distribution and producer’s risk of 0.05.

Please note that the table contains numerical data and is not meant to be read out in full. It is a representation of the data in a tabular format.
In this paper reliability sampling single sampling plan is proposed based on the Weibull-Poisson Distribution in order to make a decision of the lot. Here, it's assumed that the life time of the product may follow a Weibull-Poisson Distribution which is useful for easy selection of tables and useful in reliability study analysis. The development of the new plan can be extended for other proposed plan can be useful for the industrial shop floor conditions through various quality characteristics which are tailor made to protect both consumers and producers. The plan obtained the minimum sample size and OC values of producer's risk. The sampling plan which may wider open for future research.

5. References


Table 3: Operating Characteristic values of the sampling plan (n, c = 1, t/tq0 0.50) for a given P* under Weibull – Poisson distribution.

4. Conclusion

In this paper reliability sampling single sampling plan is proposed based on the Weibull-Poisson Distribution in order to make a decision of the lot. Here, it’s assumed that the life time of the product may follow a Weibull-Poisson Distribution which is useful for easy selection of tables and useful in reliability study analysis. The development of the new plan can be extended for other sampling plan which may wider open for future research.