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A spiritual fixed point theorem

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Abstract
It is true that there is no life or any existence without mathematical relationships. The entire universe is formed and bound by many simple and complex mathematical relationships. Biological life springs from chemistry, structured by the mathematical relationships of atoms which are due to electrically charged particles. In every phase, there exist some mathematical relationship in creation of matters and biological life of the universe.
On the other hand, a human being passes through different kind of miseries in the way of life and distributes his misery around and makes surroundings miserable. Knowing this reality, Siddhartha Gautam (Lord Buddha) discovered the technique of Vipassana Meditation. To come out from the miseries, meditation had been practiced by many Yogis (Saints) since a long time ago. The modern health science has also accepted that meditation helps to achieve peace and harmony within the mind. Siddhartha Gautam discovered the technique, practiced himself and taught to others to eradicate their miseries. In this paper, we indicate the connection of mathematical relationship with Vipassana Meditation.

Keywords: Fixed point, convergence, meditation, Sheela, Samaadhi, Pragya

Introduction

Mathematics has many branches. Among them, one is functional analysis. Functional Analysis deals with the behavior of functions with their properties and characteristics of the functions. The historical roots of functional analysis lie in the study of spaces of functions and the formulation of properties of transformations of functions, operators between function spaces. Functional analysis also has some branches, one of important branch is the Fixed Point Theory. Fixed points are the roots of the linar and non-linear functions. Hence in in general we can say that fixed point theory is a mathematical theory which gives the solutions of non-linear functions. Fixed point theory is playing a very important role to provide the exact and approximate solutions of various disciplines of pure and applied mathematics. It provides tools for the existence of solutions of the functions.

Some applications of fixed point theory in the fields are:
Decision making, Neural networking, logic programming, Economics, Engineering, Chemical reactions, neutron transport theory, Approximation Problems, Analysis, Image compression, Googles Page rank algorithms, Differential functions etc. and many more.
A simple definition of fixed point for the function is

Definition 1 *Let X be a set. Let T be a function on X i. e.* $T : X \rightarrow X$. *If Tx = x for any x in X, then we say that x is the fixed point of the function T.*

Assume that $x \in X$ is a fixed point of T, then if x_0 is any point in X the sequence $x_{n+1} = T(x_n)$, n = 0, 1, 2,... converges to the unique fixed point.

Example 1 *Let us take* $X = {}^{[1, \ 2]}$ *and* $f(x) = x^3 - x - 1$. *Let us set it as*

$$x = T(x) = (1+x)^{\frac{1}{3}}$$

The sequence $x_{n+1} = \left(1 + x_n\right)^{\frac{1}{3}}$ will converge to a fixed point of X. For this, let us consider a point $x_0 = 1.3 \in (1,2)$ then,

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$$x_0 = 1.3, \quad x_1 = 1.320006122, \quad x_2 = 1.323822354, ..., x_{11} = 1.324717957, \quad x_{12} = 1.324717957, ...$$

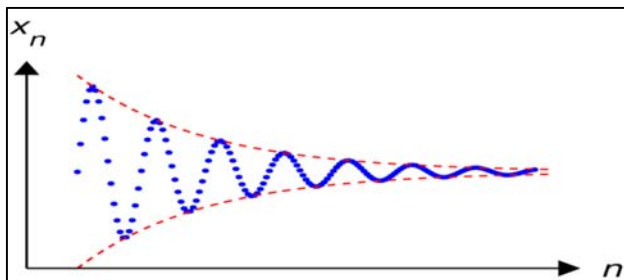We say that the point 1.324717957 is the fixed point of the function T.

If we take any point in the interval (1, 2) and put in the sequence, then after some or more iteration we obtain the fixed point of the function.

The author claims that the same process is applied in our daily life for taking any kind of decision in our mind. Here by decision we mean the solution of the problem.

Let us make this fact clear with this example.

Suppose a person has a difficult situation (problem) and needs the solution of that problem in his / her mind. What he / she does ? he / she starts thinking taking a probable solution in his / her mind. He/ she puts the probable solution in the problem (in mathematics we say function) to get the exact solution. If this is not suitable solution, then he / she takes another point of probable solution which is due to previous solution. He/ she continuous the same process. After some or many iterations, finally he /she come to a final conclusion (solution) of the problem. This final conclusion can be supposed to be the fixed point of the given problem or situation which we say the exact solution of the problem. Knowingly or unknowingly the process of the fixed point iteration is being applied in our daily life for taking a decision for the problem.

The simple and easy way to establish fixed point of the given function is that, one has to show that the contractive condition of the function possesses cauchy sequence criteria and the metric space is supposed to be complete where the cauchy sequence converges. Cauchy sequence is a sequence where the elements become arbitrary close to each other as the sequence progresses. More precisely, given any small positive distance, all but a finite number of elements of the sequence are less than that given distance from each other. With this covering criteria, we become able to establish fixed point of the given function. But in mathematics there are various ways, conditions, properties to establish fixed point results, we do not describe it here which is beyond of our scope.

If we compare with the Cauchy sequence, the same process of sequencing is also applied for taking a decision because our mind is concentrated pointing out to the probable solution and finally to the exact solution. In meditation, the process of concentration of the mind is applied. We narrow our thoughts pointing out to the respiration and fixed point in the body. So, fixed point theory not only gives the solution of outer world, it provides the solution of all problems of the inner world of human kind. It is helpful in decision making of the daily life and in meditation.
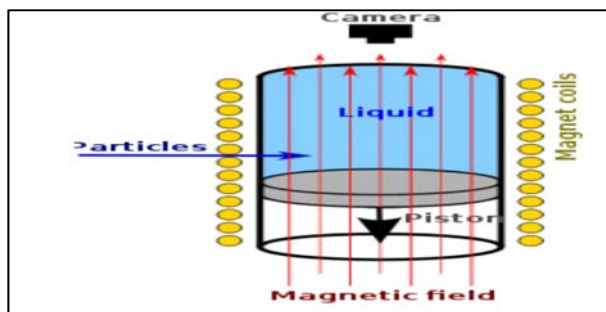
The nature of the meditation is to concentrate the thoughts into a point. The process of concentrating the mind is exactly the same as the Cauchy sequence progresses for the convergence

At first, let us be introduced with the subject Neuroscience. Any or all of the sciences such as neurochemistry and experimental psychology which deal with the structure or the function of the nervous system and brain is neuroscience.

In 1943, Warren Mcculloch and Walter Pitts [6] created a computational model for neural networks based on mathematics and algorithms. An neural network is a computational simulation of a biological neural network. These models represent the real life behavior of neurons and the electrical message they produce between input, processing by the brain and the final output from the brain.

Note that fixed point theory is widely used to get the solution of the functional relations of the neural networks.

In 1952, American physicist and neurobiologist Prof. Donald Arthur Glaser [2] of California University, Berkely investigated that every material (solid, liquid and gas) are composed with eclectically charged particles through bubble chamber used in subatomic particles and was awarded by Nobel Prize in 1960. The bubble chamber enabled him to observe activities and lifetimes of the particles.

But this truth was early known to Hindu Rishi-Munis (saints) as this fact is clear from Vedas. Siddharth Gautam reinvestigated this truth through meditation and expressed that not only our body but every material (solid, liquid or gas) is composed of electrically charged particles. This truth can be self-realized by meditation also.

After re-investigation of the technique of Vipassana Meditation, Siddharth Gautam (Lord Buddha) expressed his feeelings by following verses. These verses make us clear that our body which is made of solid, liquid and gas is the galaxy of electrically charged particles and the nature of these particles is to arise and pass away.

Yo cha vassa sasatam jeeve apssam udayabbayam |
Ekaham jeevitam seyyo passato udayabbayam ||

It is better to survive even for single year watching (realizing), arising and passing of electrically charged particles in the body than to survive one hundred years without watching the phenomenon which happens in the body of a human being.

Anichchavat sankhara uppadavayadhammino |
Upajjhitwa Nirujjhanti tesaam vupa samosukho ||

All the sankharas are impermanent and their nature is to arise and pass away. One arise and pass away but that becomes the cause of rise of another sankhara. Their eradication is sukham (peace and harmony).

Here by eradication we mean, not to react over the sankhara which are due to electrically charged particles.

Meditation

A vipassana meditator knows very well that our body is composed of electrically charged particles i.e. wavelets. These wavelets create shuttle sensation and gross sensation in our body. The shuttle sensation are due to Raaga (Craving) and gross sensations are due to Dwesh (Aversion). Siddhartha Gautam (Lord Buddha) says that, the causes of all kinds of miseries are due to Raaga and Dwesha. Raaga is the strong attachment with desires. Dwesha is the negative aspect of Raaga i.e. strong disattachment with desires. Note that, the desires are not the root cause of miseries, the root cause of misery is the attachment with desires.

Meditation does not solve our desires. It solves miseries caused by raaga and dwesha.

A human being has many more desires in his/her life. He/she wants to fulfil all desires. They think that fulfilment of desires make them happy, satisfied and cheerful. But the situation turns such a way that our all desires are not fulfilled or we can say that most of our desires remain un-fulfilled, only a little are fulfilled. Due to this, we become so miserable. If we observe these facts very minutely, we will be able to know that there are strong attachments with the desires. These strong attachment with the desires are the root cause of our miseries.

Siddhartha Gautam realized the truth that strong attachment with desires cause more misery and less the attachment with the desires cause less the misery for an individual.

So he investigated the technique of vipassana meditation with a long and difficult effort whose purpose was to reduce the raaga and dwesha for the relief of misery of human being. This process helps for the purification of mind. Total purification of mind results to achieve nibbanic peace (enlightment).

For Vipassana Meditation there are three major parts for consideration.

1) **Sheela:** It helps in creating positive aspect of thinking and helps to reduce the rate of wandering of the mind.
2) **Samadhi:** It is the state of intense concentration of mind. In this stage one is able to know the sensation in the body which make realize that whole body is galaxy of electrically charged particles.
3) **Pragya:** It is the state of mind knowing the arising of raaga and dwesha with the help of gross and shuttle sensations in the body.

Sensations are due to electrically charged particles in the body. These particles help to create craving and aversion. The nature of these electrically charged particles are to arise and pass away. These are constantly changing. Knowing this truth one has not to create craving and aversion for such trivial phenomena. Prajna helps to remain in equilibrium. If a human being possesses all these three things, he gets the real solution of his live. he dwells in peace and harmony and makes surroundings peaceful.
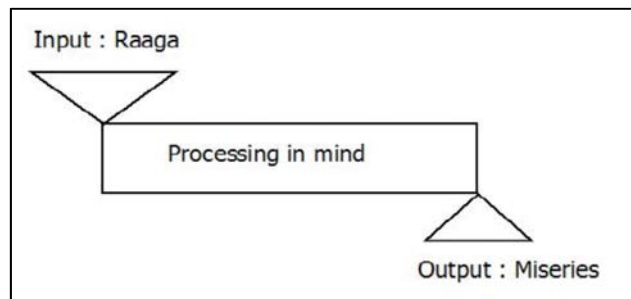
Vishwa Vipassanacharya Shree Satya Narayan Goenka says

Ganga Yamuna Saraswati Sheela Samadhi Gyan |
Teenoka Sangam Hoye Prakate Pad Nirvan ||

The true streams Ganga, Yamuna and Saraswati are Sheela, Samadhi and Prajna. If these streams converge then nibbana (enlightment) menifests.

Mathematical Relationship

The author has a long experience (16 years) of Vipassana Meditation and research interest in fixed point theory. The author always use to feel that there is a relationship between mathematics (fixed point theory) and vipassana meditation as he used to sit in meditation. On the basis of vipassana meditation, Attachment function, mind space, contraction principle (Samadhi) and the role of Pragya are defined. The role of Pragya is to watch the arising and passing away the electrically charged particles in the body. These particles create craving and aversion in the form of shuttle and gross sensations in the body which are the main causes of miseries. So one has to stay or practice to stay in equilibrium and not react over these sensations. The sensations are impermanent in nature and trivial. This truth can be realised in Samadhi (The state of intense concentration of the mind).

The functional relationship of raaga can be expressed by following figure.

Some definitions

Mind: *Mind is a set of thoughts. Thoughts are messages due to electrically charged particles (neurons) and neurons are activated by desires and sense organs.*

Attachment: *Let X be a non-empty set of thoughts. A be function from $X \times X \to [0,1]$ such that for all $x, y, z \in X$ we have the following*

i) $A(x, y) = 0$ implies $x = y$.
ii) $A(x, y) = A(y, x)$
iii) $x = 0 = y$ implies $A(x, y) = 0$

Then A is called the attachment function and the pair (X, A) is called the mind space.

Samadhi: *Let (X, A) be a mind space. A function* $T : X \rightarrow X$ *is said to be Samadhi if*

$$A(Tx, Ty) \leq kA(x, y) \quad for \quad some \quad 0 \leq k < 1$$

for all $x, y \in X$

Vipassana meditating Principle: *Let (X, A) be a mind space with Sheela and* $T : X \rightarrow X$ *satisfies*

$$A(Tx, Ty) \leq kA(x, y) \quad for \quad some \quad 0 \leq k < 1$$

with Pragya for all $x, y \in X$, then T has a unique solution in the mind. More over, for any any $x_0 \in X$ the sequence of iterates converge to the unique solution in the mind.

Note that only Samadhi, which is the state of intense concentration may make us feel peace and harmony. Peace and harmony make us happy and cheerful. But this will not purify our mind. Pragya plays very important role for the purification of mind. Total purification of mind provides us the glimpses of nibbanic peace within us.

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