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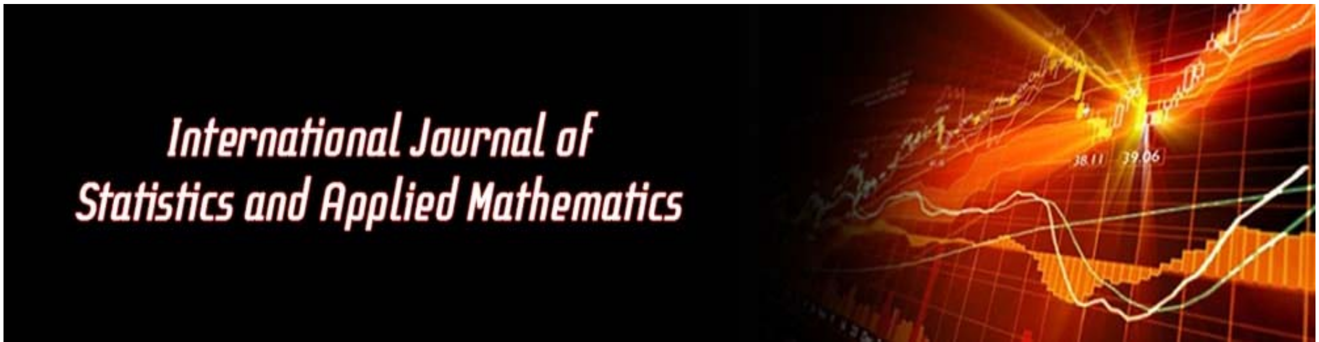

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Determination of temperature or heat distribution (Potential Function) over multiply connected infinite medium

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Abstract
The evaluation of heat conduction is determined through integral equation method. It is found that the flux, derivative of potential function, having Cauchy type singularity at crack tips. There are closed form expressions for temperature distribution.

Keywords: Fourier transform, integral equations, flux intensity factor, potential theory

1. Introduction

The "Potential Theory" arose from the fact that the fundamental forces of nature were believed to be derived from potentials which satisfied Laplace equation. It is interesting to investigate that: How the various potential function behave on approaching and crossing the smooth or fractured boundary? In the present research endeavour we try to find out the behavior of flux and temperature in steady state conditions for multiply connected body.

We know that the heat flow follows the following empirical laws: (a) Heat flows from higher to lower temperature. (b) The amount of heat required to produce a given temperature in a body is proportional to the mass of the body and temperature change. (c) The rate at which heat flows through an area is proportional to the area and to the temperature gradient normal to area. The constants of proportionality in (b) and (c) are called specific heat c and thermal conductivity k of the material, respectively. Thus the equation, which Temperature function $T(x,y)$ satisfies, comes out to be (for steady state)

$$\frac{k}{\rho c}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)T(x,y)=0 \tag{1.1}$$

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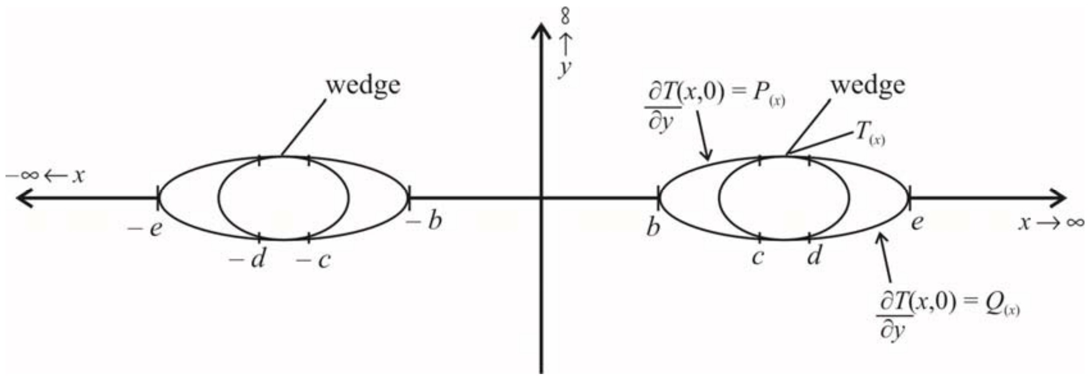
Above is Laplace equation. Hence we say that temperature behaves as potential function. P is mass density of the medium. Four Griffith cracks are opened by two equal and uniform wedge placed symmetrically with respect to *y*-axis. Wedges are heated at temperature $T(x,0)$. Then we are to find $T(x,y)$ and its derivative i.e. flux across *x*-axis or *y*-axis. The function satisfies the conditions given below.

The distribution of heat is such that it is not function of *z* which gives plane-strain condition. It is also assumed that the elastic and thermal properties of medium does not change with heat. It is also assumed that temperature (heat) T and flux $\frac{\partial T}{\partial x}, \frac{\partial T}{\partial y}$ vanish as $\sqrt{x^2+y^2}\to\infty$. There is no heat source/sink in the medium.

Heat transfer is due to conduction. There is no convection and radiation. The cracks occupy the region $y = 0$, $b < |x| < c,\ d < |x| < e.$ Over the wedge heat is prescribed.



The temperature distribution is through conduction only.

$$T(x,0) = \begin{cases} 0, 0 \le |x| \le b \\ T(x), c \le |x| \le d \\ 0,\ e \le |x| < \infty \end{cases} \quad (1.2)$$

and

$$\frac{\partial T(x,0)}{\partial y} = \begin{cases} P(x), b < |x| < c \\ Q(x), d < |x| < e \end{cases} \quad (1.3)$$

The temperature (heat conduction) satisfy the Laplace's equation given in (1.1). Thus we see that $T(x,y)$ is a potential function. The solution space is $(-\infty,\infty) \cup (-\infty,\infty)$. The cracks are also symmetrically opened with respect to *x*-axis. Making use of symmetries (geometrical and thermal properties) the solution space is reduced to $[0,\infty) \cup [0,\infty)$ i.e., first quadrant only.

There are very few problems over heat distribution. But there is good amount of work done over Thermal-stress. Oleziak [1] had solved for thermal stress caused by penny-shaped crack. Florence and Goddier [2] too, solved for penny shaped crack with linear thermo-elasticity. Shail [3] had solved for thermo-elastic problem in steady state in infinite isotropic solid. Kassir and Sih [4] solved for thermal stress for external circular crack.

There are few more thesis submitted over thermo-elasticity Chandra [5], Sorout [6], Singh [7], Singh [8]. Singh [9] solved for heat conduction in simply connected domains.

The plan of the paper is as follows: Section-1 contains the definition and history of potential function along with Temperature as potential functions. The section-2 formulates, reduces and solves the integral equation. Section-3 evaluate the potential function and its derivative in the form of temperature and flux. Section 4 considers a special case of Temperature and Flux. Section-5 concludes as discussion and conclusion. The references are in the last.

2. Formulation reduction and solution of integral equation

The solution of Laplace equation (1.1) is obtained by using Fourier cosine transform with respect to '*x*' as

$$R_c(\xi) = \int_0^\infty R(x)\cos(\xi x)dx$$

with usual inversion. We assume the solution as

$$T(x,y) = \int_0^\infty A(\xi)e^{-\xi y}\cos(\xi x)d\xi \quad (2.1)$$

Then,

$$\frac{\partial T(x,y)}{\partial y} = -\int_0^\infty \xi A(\xi)e^{-\xi y}\cos(\xi x)d\xi \quad (2.2)$$

Thus the boundary conditions (1.2) and (1.3) reduce to

$$\int_0^\infty A(\xi)\cos(\xi x)d\xi = \begin{cases} 0, x \in I_1 \\ T(x) \in I_3 \\ 0, x \in I_5 \end{cases} \quad (2.3)$$

And

$$\int_0^\infty \xi A(\xi)\cos(\xi x)d\xi = -\begin{cases} P(x), \ x\in I_2 \\ Q(x), \ x \in I_4 \end{cases}$$

$$I_1 = [0,b], I_3 = [c,d], I_5 = [e,\infty); \ \ I_2 = (b,c), I_4 = (d,e) \tag{2.4}$$

The equations (2.3) – (2.4) are quintuple-integral equation. The unknown $A(\xi)$ will be determined by solving the above mixed-boundary value problem.

Solution

We assume the solution, see Kushwaha [10]

$$A(\xi) = \frac{2}{\pi\xi}\left[\left\langle \int_b^c g_1(t) + \int_d^e g_2(t) - \int_0^b T'(t)\right\rangle \sin(\xi t)dt\right] \tag{2.5}$$

When (2.5) is substituted in (2.3) and using the integral

$$\int_0^\infty \frac{\sin(\xi t)\cos(\xi t)}{\xi}d\xi = \begin{cases} \pi/2, t > x \\ \pi/4, t = x \\ 0, t < x \end{cases}$$

Then it satisfies it if

$$\int_b^c g_1(t)dt = -T(c) \tag{2.6}$$

$$\int_d^e g_2(t)dt = -T(d) \tag{2.7}$$

Substituting (2.5) in (2.4) and then using the method of Kushwaha [10] or alternately we can use Kushwaha and Awasthi [11]

$$g_1(t) = \frac{2}{\pi^2}\frac{\Delta(t)}{\theta(t)}, t \in I_2 \tag{2.8}$$

$$g_2(t) = -\frac{2}{\pi^2}\frac{\Delta(t)}{\theta(t)}, t \in I_4 \tag{2.9}$$

$$\left.\begin{aligned} &\theta(t) = \left\{\left|t^2 - b^2\right|\left|c^2 - t^2\right|\left|d^2 - t^2\right|\left|e^2 - t^2\right|\right\}^{1/2} \\ &\Delta(t) = \Delta_0(t) + tR + M \\ &\Delta_0(t) = \left\langle \int_b^c P(y) - \int_d^e Q(y)\right\rangle \frac{y\theta(y)}{y^2 - t^2}dy + \int_c^d T'(\alpha)\frac{\alpha d\alpha}{\alpha^2 - t^2} \end{aligned}\right\} \tag{2.10}$$

where R and M are two arbitrary constants to be determined through (2.6) – (2.7).

3. Physical Quantities

Temperature (Potential)

The temperature $T(x,0)$ is obtained through the value of integral in left hand side of (2.3) and is given as

$$T(x,0) = \begin{cases} \int_x^c g_1(t)dt - T(c) \\ \int_x^e g_2(t)dt \end{cases} \tag{3.1}$$

FLUX (Derivative of Potential)

The flux $\frac{\partial T(x,0)}{\partial y}$ is obtained through the value of integral in left hand side of (2.4) and using (2.5) there and then evaluating the integrals, we get

$$\frac{\partial T(x,0)}{\partial y} = \begin{cases} \dfrac{\Delta(x)}{\pi\theta(x)}, x \in I_1 \\ -\dfrac{\Delta(x)}{\pi\theta(x)}, x \in I_3 \\ \dfrac{\Delta(x)}{\pi\theta(x)}, x \in I_5 \end{cases} \tag{3.2}$$

Where $\theta(x)$ and $\Delta(x)$ are defined in (2.10). We see that in (3.2) flux has Cauchy type or square root singularity at crack tips. We define flux intensity factor as these are defined for stresses as stress-intensity factor of crack tips.

$$T_b = \lim_{x \to b^-} \sqrt{b-x}\,\frac{\partial T(x,0)}{\partial y} \tag{3.3}$$

$$T_c = \lim_{x \to c^+} \sqrt{x-c}\,\frac{\partial T(x,0)}{\partial y} \tag{3.4}$$

$$T_d = \lim_{x \to d^-} \sqrt{d-x}\,\frac{\partial T(x,0)}{\partial y} \tag{3.5}$$

$$T_e = \lim_{x \to e^+} \sqrt{x-e}\,\frac{\partial T(x,0)}{\partial y} \tag{3.6}$$

Thus using (3.2) in (3.3) – (3.6) we get,

$$T_b = \frac{\Delta(b)}{\pi\ n_1(b)} \tag{3.7}$$

$$T_c = -\frac{\Delta(c)}{\pi\ n_1(c)} \tag{3.8}$$

$$T_d = -\frac{\Delta(d)}{\pi\ n_2(d)} \tag{3.9}$$

$$T_e = \frac{\Delta(e)}{\pi\ n_2(e)} \tag{3.10}$$

Where

$$n_1(x) = \sqrt{2x(c^2-b^2)(d^2-x^2)(e^2-x^2)} \tag{3.11}$$

$$n_2(x) = \sqrt{2x(x^2-c^2)(x^2-b^2)(e^2-d^2)} \tag{3.12}$$

4. Special Case

We take one special case. Let

$$T(x) = T_0 = \text{constant temperature} \tag{4.1}$$

$$P(x) = Q(x) = p_0 = \text{constant flux,} \tag{4.2}$$

(4.1) gives that, $\dfrac{dT(x)}{dx} = 0$ (4.3)

Using (4.1) - (4.3) into (2.10) wet get

$$\Delta_0(t) = p_0\left[s_1 + s_2\left|t^2-b^2\right| + s_3\left|t^2-b^2\right|\left|t^2-c^2\right| + s_4\left|t^2-b^2\right|\left|t^2-c^2\right|\left|t^2-d^2\right| + \frac{\pi}{2}\theta^2(t)\right] \tag{4.4}$$

$$\left.\begin{aligned} s_1 &= \left(\int_b^c - \int_d^e\right) y\theta_3(y)dy, \qquad & s_2 &= \left(\int_b^c - \int_d^e\right) y\theta_2(y)dy \\ s_3 &= \left(\int_b^c - \int_d^e\right) y\theta_1(y)dy, \qquad & s_4 &= \left(\int_b^c - \int_d^e\right) \frac{y}{\theta(y)}dy \end{aligned}\right\} \tag{4.5}$$

$$\left.\begin{aligned}&\theta_1(y)=(e^2-y^2)^{1/2},\theta_2(y)=\left|d^2-y^2\right|^{1/2}\theta_1(y)\\&\theta_3(y)=\left|c^2-y^2\right|^{1/2}\theta_2(y)\end{aligned}\right\} \tag{4.6}$$

The constants R and M in second of (2.10) are given as, after using (2.6) – (2.7),

$$R=\frac{s_{12}}{s_{13}},M=-\frac{s_{14}}{s_{13}},\ s_{14}=s_{11}s_9-\frac{p_0}{\pi}s_8s_6,\ s_{13}=s_6s_{10}-s_9s_7$$

$$s_{12}=s_{11}s_{10}-\left(\frac{p_0}{\pi}s_8+t_0\right)s_7,\ s_{11}=T_0-\frac{p_0}{\pi}s_5,\ s_{10}=\int_d^e\frac{dt}{a+1}$$

$$s_9=\int_d^e\frac{t^2dt}{\theta(t)},\ s_8=s_1s_{10}+s_2\int_d^e\frac{(t^2-b^2)dt}{\theta(t)}+s_3\int_d^e\frac{(t^2-b^2)(t^2-c^2)}{(t^2-d^2)(e^2-t^2)}dt+s_4\int_d^e\theta(t)dt$$

$$s_7=\int_b^c\frac{dt}{\theta(t)},s_6=\int_b^c\frac{t^2dt}{\theta(t)}$$

$$s_5=s_1s_7+s_2s_6+s_3\int_b^c\left(\frac{(t^2-b^2)(c^2-t^2)}{(d^2-t^2)(e^2-t^2)}\right)^{1/2}dt+s_4\int_b^c\theta(t)dt$$

Thus the closed form expressions for temperature distribution and flux too, at $y = 0$, are evaluated.

5. Conclusion and Discussion

Thus we determined the potential function (Temperature or heat) over multiply connected isotropic body by using the integral equation method. The method used for heat distribution can be extended to the analysis for crack opening due to heat. The distribution of flux across *x*-axis for region $x \in I_1 \cup I_3 \cup I_5$ is determined and it is found that flux has square root singularity at crack tips. The singularity at crack tips, it seems, may generate plastic region around crack tips. This type of problems will be discussed in future. The temperature distribution along *x*-axis for $x \in I_2 \cup I_4$ is smooth, i.e.; there is no singularity anywhere in the region. This method can be extended to orthotropic medium, too.

6. Reference

1. Oleziak Z, Sneddon IN. The distribution thermal stress in an infinite elastic solid containing a penny-shaped crack, Archive for Rational Mechanics and Analysis. 1960; 4(1):238-254.
2. Florence AL, Goddier GN. The linear thermo elastic problem of uniform heat flow disturbed by a penny-shaped crack, International Journal of Engineering Science. 1963; 1(4):533-540.
3. Shail R. Some steady state thermo-elastic stress distributions in the viscinity of an external crack in an infinite solid, International Journal of Engineering Science. 1968; 6(12):685-694.
4. Kassir MK, Sih GC. Thermal stresses in solid weakened by an external circular crack, International Journal of Solids and structures. 1969; 5(4):351-367.
5. Chandra U. Two dimensional mixed boundary value problems in a strip, A Ph.D. Thesis, Agra University, Agra, 1987.
6. Sorout PS. Mixed boundary value problems in orthotropic medium due to heated wedge, A Ph.D. Thesis, Agra University, Agra, 1993.
7. Singh H. Mixed Boundary Value Problems in Orthotropic Medium due to Thermal Stress, A Ph.D. Thesis, Dr. B.R.A. University, Agra, 1999.
8. Singh A. The stress-intensity factors due to n-Griffith cracks opened by thermal stress in Orthotropic Medium, A Ph.D. Thesis, Dr. B.R.A. University, Agra, 2001.
9. Singh R. The disturbance caused in heat flow due to the presence of Griffith-cracks in an isotropic rectangle, A Ph.D. Thesis, Dr. B.R.A. University, Agra, 2015.
10. Kushwaha PS. Some Two Dimensional Crack Problems in the Mathematical Theory of Elasticity, A Ph.D. Thesis, I.I.T., Bombay, 1975.
11. Kushwaha PS, Awasthi SN. n-Griffith Cracks in an elastic strip, Acta Ciencia Indica. 1979; 5(3):136-140.